Abstract
This paper proposes an alternative to standard cardinal tournaments. The analysis contrasts "hybrid" cardinal tournaments to standard cardinal tournaments and piece rates. It shows that providing for partial insurance against common uncertainty via a hybrid tournament (in which the weights on absolute and group average performances are not equal) is always better for the principal than providing for full insurance against common uncertainty via a standard tournament (with equal weights), or than providing for no insurance at all via piece rates. Hybrid tournaments increase the principal's profit because the agents exert more effort in equilibrium.

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1. Introduction

Following the seminal work of Lazear and Rosen (1981), Holmström (1982), Green and Stokey (1983), Nalebuff and Stiglitz (1983) and Malcomson (1984), the current literature on relative performance evaluation has two strains. One strain has focused on rank-order, ordinal tournaments (i.e., tournaments based on rank with fixed prizes), and the other one on cardinal tournaments. The rationale behind both strains is that relative performance evaluation is useful only when agent activities are subject to common shocks, in which case individual performance is not a \textit{sufficient statistic} for individual effort. The focus of this paper is on the second strain of literature. In particular, Knoeber (1989), Knoeber and Thurman (1994 and 1995), Tsoulouhas (1999), Tsoulouhas and Vukina (1999 and 2001), Wu and Roe (2005, 2006), Tsoulouhas and Marinakis (2007), Vandegrift, Yavas and Brown (2007) and Marinakis and Tsoulouhas (2009ab) have focused on cardinal tournaments with payments taking the form \( b + \beta (x_i - \bar{x}) \), where \( x_i \) is agent output and \( \bar{x} \) is average output, and contrasted these schemes with standard linear piece rate schemes of the form \( b + \beta x_i \).

The intuition is that by the \textit{strong law of large numbers}, \( \bar{x} \) provides an informative signal about the value of common shocks. Note that Lazear and Rosen focused on rank-order tournaments, however, such tournaments are informationally wasteful when data on the agents’ cardinal performance are available (Holmström (1982)). Cardinal tournaments are popular in several occupations or industries where cardinal performance data are available (e.g., salesmen contracts, physician contracts with HMOs, agricultural contracts, promotion tournaments and annual salary raises for faculty), partly because they are simple to design and easy to implement and enforce, even though linear schemes are only proxies of optimal non-linear schemes.

Relative performance evaluation via tournaments constitutes a Pareto move because the principal uses the available information more efficiently. By removing common uncertainty from the responsibility of agents, and by charging a premium for this insurance, the principal increases his profit without hurting the agents. Moreover, by providing this type of insurance, tournaments enable the principal to implement higher power incentives than under piece rates. However, it is not a priori clear whether the principal should provide full or partial

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2 To some extend, the non-linearity of the theoretically optimal contract is due to the fact that contracts accomodate all possible events. Holmström and Milgrom (1987), however, have argued that schemes that adjust compensation to account for rare events may not provide correct incentives in ordinary high probability circumstances. Nevertheless, the optimal contract can be linear under specific assumptions (see Holmström 1979).
insurance against common shocks. Insulating agents fully may lead them to exert less effort than if the principal only provides partial insurance. There is a tradeoff; the principal can increase his profit by charging for the insurance against common shocks, but too much insurance can be detrimental to efforts.

Unlike the current literature on cardinal tournaments, the analysis in this paper considers tournaments of the form \( b + \beta x_i - \gamma \bar{x} \), which have been largely overlooked. In this paper we call these compensation mechanisms "hybrid" tournaments, that is, tournaments in which the weights on absolute performance and group average performance are not equal, entailing a partial filtering of common uncertainty from the responsibility of agents. The analysis shows that providing for partial insurance against common uncertainty via a hybrid tournament is always better for the principal than providing for full insurance against common uncertainty via a standard tournament or than providing for no insurance at all against common uncertainty via piece rates. The principal can induce the agents to exert more effort by subjecting them to some common uncertainty. This is because the agents work harder in order to insure themselves against bad realizations of the common shocks.

2. Model
There is one principal and a finite number of homogeneous agents \( n \). Each agent \( i \) produces output according to the production function \( x_i = a + e_i + \eta + \varepsilon_i \), where \( a \) is the agent’s known ability, \( e_i \) is his effort, \( \eta \) is a common shock inflicted on all agents and \( \varepsilon_i \) is an idiosyncratic shock. Both shocks follow independent normal distributions with zero means and finite variances \( \text{var}(\eta) = \sigma_\eta^2 \) and \( \text{var}(\varepsilon_i) = \sigma_{\varepsilon_i}^2 \), \( \forall i \). Each agent’s effort and the subsequent realizations of production shocks are private information to him, but output obtained is publicly observed. The price of output is normalized to 1 so that output produced by the agents is revenue to the principal. Inline with Lazear and Rosen (1981), agent preferences are represented by a CARA utility function \( u(w_i, e_i) = -\exp \left( -r w_i + \frac{1}{2} a e_i^2 \right) \), where \( r \) is the agent’s coefficient of absolute risk aversion and \( w_i \) is his compensation. Note that the cost of effort decreases with agent ability and is measured in monetary units. This utility function is widely used in the literature.

The principal compensates the agents for effort based on their outputs by using a cardinal tournament or a piece rate scheme. The general form of such a scheme is \( w_i = b + \beta x_i - \gamma \bar{x} \), where \( b \) is called the base payment. In this form, the scheme will be called a hybrid cardinal tournament (H). By setting \( \gamma = \beta \) we obtain the standard cardinal tournament (T)

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\[ w_i = b + \beta(x_i - \bar{x}), \]
where \( \beta \) is called the bonus factor and \( \beta(x_i - \bar{x}) \) is the bonus (penalty) for above (below) average performance. Thus, under hybrid tournaments the weights on absolute and average group performance are unequal, whereas under standard tournaments they are equal. By setting \( \gamma = 0 \) we obtain the piece rate scheme (R) \( w_i = b + \beta x_i \), where \( \beta \) is called the piece rate. Both types of tournaments evoke relative performance evaluation, however, piece rates amount to absolute performance evaluation. Hybrid and standard tournaments we also call two-part piece rate tournaments.

3. Payment Scheme

The principal can optimally determine parameters \((b, \beta, \gamma)\) by backward induction. The hybrid tournament can be expressed as

\[
\begin{align*}
    w_i^H & = b + \beta x_i - \gamma \bar{x} = b + \left( \beta - \frac{\gamma}{n} \right) x_i - \frac{\gamma}{n} \sum_{j \neq i} x_j = \\
    & = b + (\beta - \gamma) \alpha + (\beta - \gamma) \eta + \left( \beta - \frac{\gamma}{n} \right) (e_i + \varepsilon_i) - \frac{\gamma}{n} \sum_{j \neq i} (e_j + \varepsilon_j).
\end{align*}
\]

Thus, if \( \gamma = \beta \), the standard tournament satisfies

\[
\begin{align*}
    w_i^T & = b + \beta(x_i - \bar{x}) = b + \beta \left( \frac{n-1}{n} x_i - \frac{1}{n} \sum_{j \neq i} x_j \right) = \\
    & = b + \beta \left[ \frac{n-1}{n} (e_i + \varepsilon_i) - \frac{1}{n} \sum_{j \neq i} (e_j + \varepsilon_j) \right],
\end{align*}
\]

and if \( \gamma = 0 \), the piece rate scheme satisfies

\[
    w_i^R = b + \beta x_i = b + \beta (a + e_i + \eta + \varepsilon_i).
\]

Equation (1) indicates that each agent’s wage depends on agent efforts, as well as on the realizations of common and idiosyncratic uncertainties. By contrast, (2) shows that standard tournaments fully insulate agents from common shocks. In addition, the total wage bill, \( \Sigma w_i \), under standard tournaments is fixed and equal to \( nb + \beta (\Sigma x_i - n \bar{x}) = nb \). Thus, standard tournaments allow the principal to fix his total wage costs; relative performance evaluation will then determine each agent’s share of the fixed pie. Further, under piece rates, each agent is again exposed to common shocks (see (3)). The reason why we call the first type of tournaments "hybrid" is therefore obvious, in that agents bear part of the common shocks, whereas standard tournaments filter them away.
The agent’s expected utility is

\[ EU = -\exp \left\{ -r \left[ \frac{[b + (\beta - \gamma)a] + (\beta - \frac{\gamma}{n}) e_i - \frac{r}{n} \sum_{j \neq i} e_j - \frac{e_i^2}{2n} - \left( \beta - \frac{\gamma}{n} \right)^2}{\frac{r}{2} \left[ \left( \beta - \frac{\gamma}{n} \right)^2 + (n - 1) \left( \frac{\gamma}{n} \right)^2 \right] \sigma_e^2 + \frac{r}{2} (\beta - \gamma)^2 \sigma_n^2} \right] \right\}, \]  

(4)

where the expression in square brackets is the certainty equivalent compensation of the agent.\(^4\) Observe that \( EU \) rises with increases in expected payments and reductions in implemented effort and the variance of payments. To ensure the compatibility of the scheme with agent incentives to perform, the principal calculates the effort that maximizes (4).\(^5\) Hence,

\[ e_i^H = a \left( \beta - \frac{\gamma}{n} \right). \]  

(5)

Effort under standard tournaments or piece rates is obtained by setting \( \gamma = \beta \) or \( \gamma = 0 \), respectively.

The principal is endowed with the bargaining power. To ensure the compatibility of the scheme with participation incentives, the principal selects the value of the base payment, \( b \), that satisfies the agent’s individual rationality constraint with equality so that he receives no rents. For ease of exposition we normalize the agent’s reservation utility to \(-1\),\(^6\) hence, his individual rationality constraint \( EU = -1 \) implies that the base payment as a function of \( \beta \) and \( \gamma \) satisfies

\[ b^H = \left[ \frac{n - 1}{n} \left( \beta - \frac{\gamma}{n} \right) \gamma - \frac{1}{2} \left( \beta - \frac{\gamma}{n} \right)^2 - (\beta - \gamma) \right] a + \]  

\[ + \frac{r}{2} \left[ \left( \beta - \frac{\gamma}{n} \right)^2 + (n - 1) \left( \frac{\gamma}{n} \right)^2 \right] \sigma_e^2 + \frac{r}{2} (\beta - \gamma)^2 \sigma_n^2. \]  

(6)

Then, given condition (5), the principal maximizes expected total profit

\[ ET \Pi = n (Ex_i - nEw_i) = (1 - \beta + \gamma) \left( 1 + \beta - \frac{\gamma}{n} \right) na - nb. \]  

(7)

Substituting the individual rationality constraint (6) into (7), maximizing with respect to \( \beta \) and \( \gamma \), and substituting back into (6) yields:

\[ \beta^H = \frac{na\sigma_n^2 + a\sigma_e^2}{(n - 1)a\sigma_e^2 + \left[ a + r\sigma_e^2 + nr\sigma_n^2 \right] \sigma_n^2}, \]  

(8)

\(^4\) The equation follows from \( E[\exp(-rw_i + \frac{1}{2} r e_i^2)] = \exp[\mu + \frac{\sigma^2}{2}] \), when \(-rw_i + \frac{1}{2} r e_i^2 \sim N(\mu, \sigma^2)\).

\(^5\) Note that the objective function is concave, therefore, first-order conditions are necessary and sufficient.

\(^6\) Note that the analysis is directly applicable to any (negative) normalization other than \(-1\).
\[ H = \frac{n \alpha \sigma^2}{(n-1) \alpha \sigma^2 + [a + r \sigma^2 + n \rho \sigma^2]^2} \]  

(9)

\[ b^H = -\frac{1}{2} a^2 - (1 - n)^2 a \sigma^4 + \frac{(n-1)2a - (n-1)n \rho \sigma^2 + r \sigma^2}{[(n-1) \alpha \sigma^2 + n \rho \sigma^2 \sigma^2 + r \sigma^2 + a \sigma^2]^2} \]  

(10)

Thus, the principal’s expected profit under a hybrid tournament is

\[ ET\Pi^H = \frac{1}{2} n a \frac{(n-1)3a \sigma^2 + 2n \rho \sigma^2 \sigma^2 + 3a \sigma^2 + 2r \sigma^4}{(n-1) \alpha \sigma^2 + n \rho \sigma^2 \sigma^2 + r \sigma^2 + a \sigma^2}. \]  

(11)

If \( \gamma = \beta \), standard tournaments satisfy

\[ \beta^T = \frac{na}{(n-1) \alpha + n \rho \sigma^2}, \]  

(12)

\[ b^T = \frac{1}{2} \frac{(n-1) \alpha^2}{(n-1) \alpha + n \rho \sigma^2}, \]  

(13)

\[ ET\Pi^T = na + \frac{a^2}{2} \frac{n(n-1)}{(n-1) \alpha + n \rho \sigma^2}. \]  

(14)

If \( \gamma = 0 \), piece rates satisfy

\[ \beta^R = \frac{a}{a + r(\sigma^2 + \sigma^2)^2}, \]  

(15)

\[ b^R = \frac{a^2}{2} \frac{r(\sigma^2 + \sigma^2) + 3a}{[r(\sigma^2 + \sigma^2) + a]^2}, \]  

(16)

\[ ET\Pi^R = na + \frac{a^2}{2} \frac{n}{a + r(\sigma^2 + \sigma^2)}. \]  

(17)

Conditions (11), (14) and (17) indicate that whereas expected profit under standard tournaments is independent of \( \sigma^2 \eta \) (because standard tournaments insulate agents from common shocks), expected profit under hybrid tournaments or under piece rates is negatively dependent on \( \sigma^2 \eta \). This is because effort \( e_i \) is negatively dependent on \( \sigma^2 \eta \). However, both \( \beta^H \) and \( \gamma^H \) are positively dependent on \( \sigma^2 \eta \). It can be shown that \( \beta^T > \beta^H > \beta^R \). Also note that \( b^T > b^R \) because expected bonus under standard tournaments is zero, therefore agents are expected to be compensated for effort via the base payment. Lastly, the relationship of \( b^H \) to \( b^T \) depends on the parameters.

\footnote{As shown below, the removal of part or all of common uncertainty under tournaments enables the principal to implement higher-power incentives.}
4. Dominant Scheme
The principal’s decision about which scheme to offer depends on expected profits. As shown below, first, hybrid tournaments always dominate both standard tournaments and piece rates provided that common uncertainty is not zero, which is the main finding of the paper. Second, consistent with the Lazear and Rosen (1981) finding, standard tournaments dominate piece rates provided that common uncertainty is sufficient to warrant insurance provision against common shocks. Interestingly, the magnitude of common uncertainty required is only a fraction of idiosyncratic uncertainty. By contrast, piece rates are dominant over simple tournaments when common uncertainty is significantly small relative to idiosyncratic, and they are dominant over all forms of tournaments when common uncertainty is zero. The implication of this finding is that in practice tournaments, especially of the hybrid type, should normally be favored over piece rates. This is because empirical research (Knoeber and Thurman, (1995)) provides some evidence that the magnitude of common uncertainty is approximately equal to that of idiosyncratic.

Proposition 1 If $\sigma^2_\eta > 0$, then: (i) the hybrid tournament $w^H_i = b^H + \beta^H x_i - \gamma^H \pi$ always Pareto dominates the standard tournament $w^T_i = b^T + \beta^T (x_i - \pi)$ and the piece rate scheme $w^R_i = b^R + \beta^R x_i$; (ii) the standard tournament $w^T_i = b^T + \beta^T (x_i - \pi)$ Pareto dominates the piece rate scheme $w^R_i = b^R + \beta^R x_i$ iff

$$\sigma^2_\eta > \frac{1}{n-1} \sigma^2_\varepsilon. \quad (18)$$

If $\sigma^2_\eta = 0$, then the piece rate scheme $w^R_i = b^R + \beta^R x_i$ Pareto dominates both the hybrid and the standard tournaments.

Proof. Statement (i): If $\sigma^2_\eta > 0$, in solving for the hybrid tournament, conditions (8) and (9) imply $\beta^H \neq \gamma^H \neq 0$ (to be precise, $\beta^H > \gamma^H > 0$), given that the number of agents is finite. Therefore, constraints $\gamma = \beta$ or $\gamma = 0$ would be binding if the principal solved for either the standard tournament or the piece rate scheme. Also note that the individual rationality constraints are always binding. Thus, hybrid tournaments, being subject to one less binding constraint, are always expected to be more profitable for the principal.

Statement (ii): The proof is straightforward by using equations (14) and (17).

If $\sigma^2_\eta = 0$, then the optimal $\gamma^H = 0$, thus, piece rates are dominant. ■

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8This is in accord with Marinakis and Tsoulouhas (2009a).
9If the number of agents converged to infinity, $\lim \beta^H = \lim \gamma^H$.
10Note that if the number of agents converged to infinity, the standard tournament would always be better than the piece rate, assuming $\sigma^2_\eta > 0$ and given a finite variance $\sigma^2_\varepsilon$. 
Note that, given $\beta^H > \gamma^H$, the expected bonus under a hybrid tournament is positive (whereas the expected bonus under a standard tournament is zero). Also note that while tournaments filter away common shocks from the responsibility of agents, they expose them to the idiosyncratic shocks of other agents (in contrast to piece rates which do the opposite). However, with a large number of agents, idiosyncratic shocks cancel out and $\bar{x}$ provides an informative signal about the value of common shocks. Condition (18) indicates that a large number of agents strengthens the dominance of tournaments from the principal’s perspective.

**Corollary 2** If $\sigma^2_\eta > 0$, then $e_i^H > e_i^T$ and, if condition (18) holds, $e_i^T > e_i^R$.

**Proof.** The proof of $e_i^H > e_i^T$ is straightforward. The proof of $e_i^T > e_i^R$ is also straightforward under condition (18).\(^{11}\)

Thus, the removal of part or all of common uncertainty under tournaments enables the principal to implement higher-power incentives. Specifically, agents exert more effort under hybrid than under standard tournaments, and more effort under standard tournaments than under piece rates.

5. Conclusions

Tournament theory has long argued that tournaments dominate piece rate schemes in the presence of relatively large common shocks that affect agent performance, when agents are risk averse. The analysis focuses on cardinal tournaments but, unlike the current literature, it emphasizes what we in this paper call "hybrid" tournaments, entailing a partial filtering of common uncertainty from the responsibility of agents. The analysis shows that providing for partial insurance against common uncertainty via a hybrid tournament is always better for the principal than providing for full insurance via a standard tournament or than providing for no insurance at all via piece rates. Agents are shown to exert more effort under hybrid tournaments. However, if transaction costs (of determining the optimal parameters) force the principal to use a simpler scheme, then standard tournaments are dominant provided that the variance of the common shock is larger than a fraction of the variance of the idiosyncratic shock. By contrast, if there is no common uncertainty, consistent with the Lazear and Rosen (1981) finding, the dominant scheme is the piece rate. The policy implication of our analysis is that principals who often use standard cardinal tournaments (for instance, processing companies in the agricultural sector, HMOs and physicians, car manufacturers and salesmen etc.) should consider switching to hybrid tournaments.

\(^{11}\)Thus, condition (18) ensures that both the standard tournament dominates the piece rate and $e_i^T > e_i^R$. 
References


