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A Flexible Non Linear Model to Test the Expectation Hypothesis of Interest Rates

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Abstract

Conventional approaches to examining the expectation hypothesis of interest rates assume a parametric linear specification among variables. In contrast, this paper tests the hypothesis using a flexible nonlinear inference approach proposed by Hamilton (2001). We examine the impact of the nonlinearity of interest rates to explain the variability of risk premia on market rates. It is assumed that the term structure of interest rates can be identified by two factors, the risk-free rate and its volatility. The results of the linearity test against nonlinear alternatives suggest that there is clear evidence of nonlinearity. Our empirical study shows that correctly accounting for the nonlinearity of the term structure of interest rates may explain the variability of risk premia and the specific characteristics of interest rate dynamics on the U.S. market.

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1. Introduction

The expectations hypothesis (EH) of the term structure of interest rates implies that yields spread between the long rate and short rate are an optimal predictor of future changes in short rates over the life of the "long bond." This hypothesis has been central to empirical and theoretical work on fixed incomes. Indeed, over the last twenty years, very short-term interest rates have served as instruments for central banks as new inflation-targeting monetary policies have been adopted. Today, it is clear that short-term interest rates are largely influenced by monetary policy. Changes in these policies impact on risk premium values and thus affect the expectations of financial market investors.

In this context, the lack of empirical verification of the hypothesis of rational expectations is problematic. It is generally attributed to high variability in the term premia or the non-rationality of investors' expectations. Most empirical tests [Shiller (1979), Fama (1984), Campbell and Shiller (1987), Gerlach (1996)...] have rejected the different methods of explaining this theory. However, all these studies have been based on the linear regression of excess return on the yield spread. Several reasons could rationalize the hypothesis of nonlinearity in the equilibrium adjustment process between long-term and short-term interest rates. Anderson (1997) suggests that transaction costs may differ according to the maturity of the bond in question and may well change over time. Fama (1990) confirms the idea of strong correlation between long-term and short-term interest rate movements. He suggests that the risk premia may also be time-varying and exhibit nonlinear behaviour. Bekeart, Hodrick and Marshall (1997) used a more restrictive regime switching framework to argue that the failure of the expectations hypothesis is due to fewer high interest rate regimes than expected. Regime-switching that affects the interest rate spread is an important factor that may cause severe deformity to bond yield curves. They conclude that the adjustment from these regime shifts can also be nonlinear. Two other considerations have often been put forward to reject the likelihood of a linear adjustment of the spread between long and short interest rates. The first is related to the phenomenon of smooth interest rates applied by central banks (Mankiw and Miron 1986, Mc Callum 1994, Woodford 1996). Mankiw and Miron (1986) argue that the results of expectation theory tests are sensitive to the mode of intervention on the money market by the authorities. They remind us that the U.S. Federal Reserve relinquished the interest rate smoothing objective between October 1979 and October 1982, adopting a policy of partial smoothing at the end of this period. Mankiw and Miron argue that this change in operating procedure will render interest rate behaviour more favourable to the expectations theory after October 1979. They conclude that these structural changes to monetary policies cannot be taken into account by the linear models of rational expectations. The second consideration is related to the difference between the volatility of short-term and long-term interest rates. Tsavalis and Wickens (1998) argue that taking the risk premium variable into account makes the data consistent with the hypothesis of rational expectations in the United States. Indeed, the failure to incorporate this effect into the model leads to the nonlinearity of movements in short-term interest rates. Ahn, Dittmar and Gallant (2002) suggest that ignoring nonlinearity is a theoretical drawback that limits the empirical performance of term structure models. They propose a quadratic term structure model in which the bond yield is a quadratic function of underlying state variables. Their approach is relatively flexible and more powerful than the affine models in explaining bond price movements in the U.S. market.

This paper contributes to the literature by studying the expectation hypothesis of interest rate markets using a flexible, nonlinear model developed by Hamilton (2001). Following Longstaff and Schwartz (1992), and Kim (2003) we develop a flexible model in which the yield is an unrestricted function of two state variables: the risk-free rate and its volatility. The flexible approach provides a valid test of the null hypothesis of linearity against a broad

range of alternative nonlinear models, consistently estimating what the nonlinear function looks like, and making a formal comparison of alternative nonlinear models.
The rest of the paper is organized as follows. Section 2 develops a nonlinear test of rational expectations based on the flexible nonlinear model proposed by Hamilton (2001). Section 3 describes the data used in this study and presents our results. The last section concludes.

2. Flexible Nonlinear Inference Approach, Hamilton (2001)

The methodology used by this paper is based on the parametric nonlinear inference approach developed by Hamilton (2001). This approach further develops a new test of the null hypothesis of linearity based on the Lagrange multiplier principle and small-sample confidence intervals based on numerical Bayesian methods. It is considered as flexible in the sense that no assumption is imposed initially on the functional form of the term structure. The basic idea underlying the flexible regression model approach suggested by Hamilton (2001) is not only to view the endogenous variable as a realization of a stochastic process but also to consider the functional form of the conditional mean function itself as the outcome of a random process.

2.1. The expectations hypothesis of interest rate term structure

The expectations hypothesis of the term structure of interest rates states that the yield on a long bond is equal to the average expectation of the short yield over the life of the long bond, plus a constant risk premium. Assuming that short-term rates will remain constant in the future, the long rate equals the short rate (plus a constant risk premium). However, while short-term rates are expected to rise, the long rate will exceed the current short rate plus the risk premium constant, so as to provide the same predictable performance. Therefore, the shape of the yield curve reflects market expectations for short-term interest rates in the future. The reason for the variation in interest rates on bonds with different maturities is that since long term securities are perfect substitutes for short-term securities, a change in a long-term security yield means that short-term interest rates will be expected to have different values in the future. Thus, the rational anticipation states that:

$$\boldsymbol{y}_{k,t} = \frac{1}{\boldsymbol{k}}\left[\sum_{j=1}^{k} \boldsymbol{E}_t \boldsymbol{y}_{1,t+j-1}\right] + \theta_{k,t} \tag{1}$$

Where $\boldsymbol{y}_{k,t}$ is the yield to maturity of an $\boldsymbol{k}$-period zero coupon at time $\boldsymbol{t}$. $\theta_{k,t}$ is a term premium, and $\boldsymbol{E}_t$ is the expectation conditional on the information at date $\boldsymbol{t}$. The nonlinearity of the term premium may be a cause for the nonlinearity of the relationship between interest rate movements and the ***spread*** between long and short rates. The risk premium is often ignored because it is unobservable and is supposed to be empirically insignificant in this equation. Equation (1) implies that investors have no particular preference between investing in long-term government securities and rolling over an investment at short-term rates. Empirical studies by Fama (1984), Fama and Bliss (1987) and Campbell and Shiller (1991) strongly argue that the classical theory of rational expectations is no longer adequate to explain interest rate dynamics since it assumes a constant risk premium. Indeed, this theory affirms that the yield curve is fully derived from current market expectations and any investor exposed to the risk gets a constant premium.

2.2. Theoretical methodology framework

The flexible approach to inference developed by James Hamilton (2001) offers a new possibility to explain the anomaly of the risk premium on interest rate markets. He proposes a model in the form:

$$y_t = \mu(x_t) + \varepsilon_t \quad (2)$$

Where ε_t is an iid error term ~ ***N (0, σ2)*** and $\mu(x_t)$ is a function[1] vector of dimension $k \times 1$. The functional form $\mu(x_t)$ is unknown and is generated from a Gaussian moving average process. This approach provides a new test for nonlinearity based on the principle of Lagrange multipliers. The proposed statistic value depends on a number of parameters that are defined under specific assumptions. The calculation of the Lagrange multiplier requires priors on the magnitude of the unidentified parameters.

The aim is to estimate the conditional expectation of y_t given the vector of observable variables (X_t). This corresponds to:

$$E(y/x)=\mu(X_t) \quad (3)$$

The term $\mu(Xt)$ is the conditional expectation function.

Hamilton (2001) considers that $\mu(.)$ is governed by a random process. At each $x_t = \tau$, the function $\mu(x_t)$ is considered as a Gaussian random variable of mean: $\alpha_0 + \alpha'\tau$ and of variance: λ^2. Moreover, α_0, α and λ are also parameters to be estimated. When $\mu(x_t)$ is evaluated at any point, it becomes an unobservable random variable. Hamilton (2001) considers that the process $\mu_{fnt}(x_t)$ follows: $\mu_{fnt}(x_t) = x'_t\beta + \lambda m(g \otimes x_t)$, where g and λ are scalars.

To obtain the form of $\mu(X_t)$, we must first characterise the random process $m(Xt)$. In the following, we show how Hamilton (2001) described the random process. Consider the case of a single explanatory variable. Let ***[a, b]*** be a closed interval in IR. The distribution of the interval $[a-\omega, b+\omega]$ describes the distribution of the explanatory variable X: ($x_1, ..., x_N$) with the extremities: $x_1 = a - \omega$ and $x_N = b + \omega$ and where ω is a parameter of distance to normalize. We assume that $x_i = x_{i-1}\Delta N$ for $i = 2, ..., N$. Suppose that each observation x_i (observable variable) is generated by a variable $e(x_i)$ having a normal distribution. $e(x_i)$ is independent of $e(x_j)$ for each $i \neq j$. Once the interval $a \leq xi \leq b$ is defined, then we have all the elements needed to construct a random variable $m_N(x_i)$. To ensure that $m_N(x_i) \rightarrow N(0,1)$, Hamilton introduced the concept of constant proportionality as follows:

$$m_N(x_i) = (1 + 2\omega/\Delta_N)^{-1/2} \sum_{j=\omega/\Delta_N}^{\omega/\Delta_N} e(x_{i+j}) \quad (4)$$

Where $m_N(x_i)$ is correlated with $m_N(x_j)$ for each $|x_i - x_j| \leq 2\omega$. If these two terms are not correlated, then the distance between two observations of a given variable is sufficiently distant. We discuss below the implications of the extent of correlation on the identification of nonlinearity.

Given $x \in [a,b]^2$, the function of the random variable can be constructed as follows:

$$m(x) \in (2\omega)^{-1/2}[W(x+\omega) - W(x-\omega)] \quad (5)$$

Where W (.) is a Wiener process. Any realization of the function $m(.)$ is continuous and not differentiable.

We can now introduce the expression of $m(x)$ in the conditional expectation function $\mu(x)$. We assume that $\mu(x)$ is governed by other parameters, including scalars: g and λ multiplied by the value of x and the function $m(x)$. We thus deduce that:

$$\mu_{fnt}(x_t) = x'_t\beta + \lambda m(g \otimes x_t) \quad (6)$$

The value of x_t can be generated from $\mu(x)$. In this study, we consider that interest rates are generated by two factors, the risk-free rate and the volatility of risk-free rate. Thus, our model can be expressed by:

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[1] This function may include lagged dependent variables.

[2] With $m(x) \in R$.

$$\boldsymbol{y_{nt}} = \alpha_0 + \alpha_1 \boldsymbol{y_{1t}} + \alpha_2 \widehat{\vartheta}^2_{t+1/t} + \sigma[\varsigma\, \boldsymbol{m}(\boldsymbol{g_1 y_{1t}}, \boldsymbol{g_2}\widehat{\vartheta}_{t+1/t})] + \upsilon_t \quad (7)$$

Where $\widehat{\vartheta}^2_{t+1/t}$ is the variance of $\boldsymbol{y_{1,t+1}}$, conditional on information at date $\boldsymbol{t}$. The error term is expressed in terms of σ time υ, and the parameter λ is expressed in terms of σ time ζ.

If the parameter ζ is zero, the function $\boldsymbol{\mu(x)}$ becomes linear and equation (6) is reduced to a linear function. The parameter $\boldsymbol{g_i}$ ***(i = 1,2)*** measures the distance between two points and determines the covariance between random variables. If the parameter g ~ ∞, it is impossible to distinguish the nonlinear ***m (g (X))*** compound of the dependent variable $\boldsymbol{y_t}$ from the error term since ***mg(X)*** and the error term are correlated. For ***g~0***, the contribution becomes impossible to distinguish from α_0. If the value of the $\boldsymbol{i^{th}}$ element of ***g*** is equal to ***0***, the conditional expectation function is linear in $\boldsymbol{x_i}$ since the variables are perfectly correlated. In the following, we explain how Hamilton (2001) adopted a measure of distance between two points to determine the correlation between random variables. Consider a function of nonlinear conditional expectation ***μ (X)*** relative to two variables. We assume that:

$$\boldsymbol{E}[\mu(\boldsymbol{x_t}) - \alpha_0 - \alpha_1 \boldsymbol{x}_1][\mu(\boldsymbol{x_s}) - \alpha_0 - \alpha_1 \boldsymbol{x_s}] = 0 \quad (8)$$

$$\textbf{\textit{For}} \quad \boldsymbol{H}_{12} = \left[\sum_{\boldsymbol{i}=1}^{\boldsymbol{k}} \boldsymbol{g}_{\boldsymbol{i}}^2 (\boldsymbol{x_{it} x_{is}})\right]^{1/2} \geq 1$$

Where $\boldsymbol{g_1}, \boldsymbol{g_2}, \ldots \boldsymbol{g_k}$[3] represent the weighting parameters for each explanatory variable. $\boldsymbol{H_{12}}$ are scalar that measure the distance between two observations $\boldsymbol{x_{is}}$ and $\boldsymbol{x_{it}}$. This measure represents the sum of the differences between the observations of two variables weighted by the parameters $\boldsymbol{g_i}$ ***(i=1,…,K)***. The correlation increases as $\boldsymbol{H_{12}}$ increases. The correlation is perfect when the distance between two points is zero (i.e. $\boldsymbol{g_i} = \boldsymbol{0}$). In this particular case, the relationship between independent and dependent variables is perfectly linear. So, the correlation characterizes the degree of linearity between two variables. The correlation of the process ***μ (X)*** for ***k = 2*** is given by:

$$\textbf{\textit{Corr}}(\mu(\boldsymbol{x_s}), \mu(\boldsymbol{x_t})) = \boldsymbol{H}_2(\boldsymbol{h_{st}}) \ \text{ if } 0 \leq \boldsymbol{h_{st}} \leq 1 \quad (9)$$

$$\textbf{\textit{Where}} \ \boldsymbol{H}_2(\boldsymbol{h_{st}}) = 1 - (2/\pi)[\boldsymbol{h_{st}}(1 - \boldsymbol{h}_{\boldsymbol{st}}^2)^{1/2} + \sin^{-1}(\boldsymbol{h_{st}})]$$

3. Tests and results

3.1 Database and test of the expectation hypothesis

Empirical studies interested in modelling the term structure of interest rates have largely been conducted on the U.S. market [Shiller (1979), Fama (1984), Campbell and Shiller (1987), Gerlach (1996), etc.]. To test the contribution of nonlinear flexible models in explaining the expectations hypothesis in the U.S. market for our study, we collected daily data over the period from 03/08/2001 to 2/02/2007. This was a period of economic expansion in the United States, beginning in 2001 and ending in 2007. The data consisted of short-term interest rates of 1, 3 and 6-month maturities and long-term government bond yields of 3, 5 and 10-year maturities. We assume that the one-month interest rate is the risk-free rate. In table I, we provide the descriptive characteristics of interest rates. The average interest rate returns increase with maturity while the standard deviations exhibit a concave shape.

The expectations hypothesis establishes that an upward sloping yield curve implies that investors expect a rise in interest rates. Such an assumption has been widely tested in the literature by the following linear regression:

[3] $\boldsymbol{g_i}$ ***(i=1,…,K)*** are parameters to be estimated.

$$y(t+1,T) - y(t,T) = \alpha_{\kappa} + \beta_{k}\frac{y(t,T) - y(t,1)}{\kappa - 1} + \varepsilon_{t} \tag{10}$$

The normalization of interest rate ***spread*** by (***1/κ-1***) in equation (10) implies that under the expectations hypothesis the slope coefficient should be equal to unity for any maturity (***n***). In addition, the constant of the model must be zero in the presence of any risk premium. This paper thus sets out to test the hypothesis of rational expectations theory in a nonlinear flexible framework. To our knowledge, the flexible inference approach has been used by Kim (2003) to model the interest rate term structure, but has not been used to test the rational expectations hypothesis.

Table II presents the results of rational expectations expressed in equation (10). The estimated slope coefficients express negative and insignificant values for most maturities, with the exception of the 1-month and 10-year maturity interest rates where the slope coefficient is positive but appears insignificant and lower than the unity. However, estimates of the constant coefficients α, are significantly positive for all maturities. The coefficients of determination R^2 appear very low for all maturities. We deduce that only a small proportion of excess bond returns is explained by the interest rate spread.

From these results we reject the expectation hypothesis over the period of the study. In the next section we test the contribution of nonlinear flexible inference models to explain the risk premium anomaly in the U.S. interest rate market.

3.2 Risk premium anomaly test with flexible nonlinear inference model

We assume that the 1-month interest rate is the risk-free rate, as is usual in the literature, and consider its volatility as the conditional variance of the spot rate. To describe time variation in the volatility of interest rates, we use the generalized autoregressive conditional heteroskedasticity (GARCH) framework. In line with Brenner, Harjes, and Kroner (1996) and Hamilton and Kim (2002), we model the conditional variance of the risk-free rate as a function of both the interest rate level and lagged squared interest rate innovations:

$$\begin{aligned} y_{1t} &= c + \phi y_{1,t-1} + \varepsilon_t, \\ \varepsilon_t \left| \Omega_{t-1} \right. &\rightarrow N(0, \vartheta^2_{t/t-1}), \\ \vartheta^2_{t+1/t} &= \omega_0 + \omega_1 \varepsilon^2_{t-1} + \omega_2 \vartheta^2_{t-1/t-2} \end{aligned} \tag{11}$$

Maximum likelihood estimates of these equations are as follows, with conventional standard errors in parentheses:

$$\begin{aligned} y_{1t} &= \underset{(0.0058)}{0.0123} + \underset{(0.0022)}{0.9897}\, y_{1,t-1} + \varepsilon_t, \\ \vartheta^2_{t+1/t} &= \underset{(9.38\times10^{-5})}{0.0003} + \underset{(0.0351)}{0.5814}\, \vartheta^2_{t-1/t-2} + \underset{(0.0446)}{0.4185}\, \varepsilon^2_{t-1} \end{aligned} \tag{12}$$

Below, we use the fitted values of the conditional variance of the risk-free rate $\widehat{\vartheta}^2_{t+1/t}$ to test the expectation hypothesis.

Before applying the flexible model developed by Hamilton (2001), we need to test the linearity of the relationship between interest rates and the two state variables, namely the risk-free rate and its volatility. The results of linearity against the alternative hypothesis of nonlinearity are reported in Table III. Large values of the statistic ν^2 support the rejection of the null hypothesis that the relation between bond yields and the two factors, the risk-free rate and its volatility, is linear.

Estimates of the parameters of the flexible nonlinear equation (6) by the maximum likelihood and their standard deviations are reported in Table VI. The risk-free rate has a significant positive effect on all the rates considered. However, the volatility of risk-free rate has a negative effect on short-term rates and a positive effect on long-term rates.

The coefficient ζ is significantly different from zero, which confirms the results of the LM test and implies that the nonlinear component makes a significant contribution to all bond yields.

In line with Kim (2003), to examine the shape of the nonlinear function ***μ (.)***, we fix the value of $\hat{\vartheta}^2_{t+1/t}$ equal to its sample mean $\overline{\hat{\vartheta}^2_{t+1/t}}$, and we evaluate the expectation for various values of y_{1t}. Figures 1a-1f plot ***μ (.)*** as a function of y_{1t} along with 95% bound probability for 3-month, 6-month, 1-year, 3-year, 5-year and 10-year rates. While the figures 1a, 1b, 1c indicate that the relationship between bond yields in the short term (3M, 6M, 1Y) and the risk-free rate is approximately linear, the relation between long rates (5 years, 10 years) and the risk-free rate seems to be nonlinear. These findings confirm those of Ahn and Gao (1999) which indicate that nonlinearity increases with time to maturity.

Figures 2a-2f answer the analogous question, setting y_{1t} equal to its simple mean $\bar{y}_{1t}$, and varying the value $\hat{\vartheta}^2_{t+1/t}$ for the 3-month, 6-month, 1-year, 5-year and 10-year bond yields respectively. All the figures indicate that there is a threshold effect in volatility on interest rates but the threshold level differs depending on interest rates. In particular, the threshold effect is significant on the 3-month and 6-month rate and is respectively about 1.0 and 1.2 of the conditional variance. These results imply that the effect of volatility on interest rates is small for low volatilities while relatively higher volatility has a significant impact on interest rates.

We noted three nonlinearity characteristics: a threshold volatility effect on bond yields, interaction between the risk-free rate and its volatility, and convexity. The threshold effect has an impact on the formation of yield curves and may affect the expectations of market investors, which is likely to impact on the validation of expectation hypothesis.

To test the risk premium anomaly by flexible nonlinear inference models, we generated the interest rate from the flexible model and displayed an estimation of the equation (7). In order to examine the flexible model's interest rate simulation performance, we evaluated the correlation between observed and fitted interest rate series. We also regressed observed interest rates on fitted interest rates following this regression $Y_t = \alpha + \beta_{fitted} Y_t + \varepsilon_t$ (Y_t is the observed interest rate).

The results are reported in table V and show a strong correlation between fitted and observed rates. However, the performance of this model deteriorates with interest rate maturity, since we observe relatively lower correlations for 5-year and 10-year interest rates. The coefficient β appears significant and the coefficient of determination exceeds 98% for all the maturities, implying good performance of the model.

In order to test the contribution of the flexible approach to the explanation of the risk premium anomaly, we estimated the equation (7) from fitted interest rates for different maturities. The slope coefficients obtained are depicted in Figure 3. To provide a comparison, this figure also includes the slopes estimated from market data. And to better examine the importance of taking the linearity in the risk premium anomaly test into account, we plotted a third curve that represents slope coefficients of equation (7), estimated from a purely linear model where interest rates depend on two factors: the risk-free interest rate and its volatility. In other words, we assume that the parameter λ is zero in equation (6). Interest rates are generated from the following model: $y_{nt} = \alpha_0 + \alpha_1 y_{1t} + \alpha_2 \hat{\vartheta}^2_{t+1/tt}$.

The results of this regression are also presented in table V. The results shows that for short maturities, the slope coefficients generated from the flexible model converge to the market ones. The most important spread between the estimated coefficients and the market data slopes are those maturing at 3 years. For all the maturities considered, the slopes of the linear model are lower than those estimated from the flexible model. We also note that the flexible model slopes exceed those of the market data except in the case of 1-year rates where the market slope appears to be slightly higher than that generated by the flexible model.

The figure 3 highlights that a combined affine and flexible term structure provides a better explanation of the rational expectations hypothesis. Taking into account the nonlinearity between interest rates and the risk-free rate and its volatility creates slopes that converge towards the market slopes especially for the short rate. The flexible model expresses negative slopes for the 6-month and 1-year rates. The indeterminate form of the function estimated provided a flexible model that outperforms the pure linear one. The slope coefficients of the nonlinear model are very similar to those of the market data, which implies that taking the effect of threshold into account gives a better representation of market characteristics. We suggest that this nonlinear model is a potentially powerful new instrument for identifying nonlinear components in interest rate time series. Thus, the flexible nonlinear model is a promising representation of nonlinearities and a better candidate for hedging or pricing contingent interest rate claims than the affine models.

4. Conclusion

A possible test of the expectations theory is to examine whether the forward rate is an unbiased estimator of future short rates. Fama (1984), Fama and Bliss (1987), and Mishkin (1988) regress spot rate changes on the long-term and short-term interest rate spread. Their results reject the expectation hypothesis. They attribute this theory’s rejection to the risk premium variability and the uncertainty about anticipations. These studies were based on linear regression models linking interest rate evolutions in the spread between long rates and short rates.

Our study uses a flexible nonlinear inference approach proposed by Hamilton (2001) to re-examine the anomaly of the risk premium on the U.S. market. In contrast to conventional parametric methods, the flexible approach allows the data to express whether or not the relation is nonlinear, what the nonlinearity looks like, and whether the relation is adequately described by a specific parametric model. We consider that interest rates depend on two state variables: the risk-free interest rate and its volatility. Our results show a nonlinear relationship between interest rate changes and the long term-short term interest rates spread. We suggest that allowing for nonlinearities in the pricing of bond yields is relatively important for describing the term structure. Thus, taking nonlinearity into account in term structure modelling allows for better categorization of risk premia and captures the fundamental characteristics of interest rate markets.

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Table I: Descriptive interest rate statistics

	Mean	Max.	Min.	S.Dev	Skew.	Kurt.
1M	2.391	5.276	0.013	1.283	0.476	-1.066
3 M	2.472	5.193	0.010	1.559	0.455	-1.150
6 M	2.632	5.338	0.153	1.684	0.433	-1.235
1 Year	2.762	5.865	0.342	1.506	0.357	-1.229
3 Years	3.255	5.264	0.882	1.248	-0.051	-1.106
5 Years	3.696	5.239	1.267	1.113	-0.428	-0.450
10 Years	4.292	5.449	2.084	1.016	-0.903	1.378

Table II: Results of rational expectations test

			Interest rate maturity			
	3 Month	**6 Month**	**1 Year**	**3 Year**	**5 Year**	**10 Year**
β_i	0.075	-0.339	-0.116	-0.175	-0.049	0.082
	(-0.956)	(-0.703)	(-0.308)	(0.151)	(0.690)	(1.130)
α_i	0.005	0.005	0.005	0.001	-0.0003	-0.001
	(14.711)*	(12.645)*	(1.901)*	(4.372)*	(3.112)*	(-7.408)*
IR^2	0.008	0.003	0.002	1.1824E-05	0.002	0.0004

* Significant at 5% level.

Table III: Testing the null hypothesis for linearity

Maturity	LM Statistic	P-Value
3 M	12.7912	0.0000
6 M	9.9531	0.0000
1 year	11.0313	2.083×10^{-9}
3 years	20.8651	1.690×10^{-5}
5 years	14.4790	1.517×10^{-6}
10 years	8.0096	8.580×10^{-6}

Table VI: Estimated by the MLE model of nonlinear flexible.

$$\boldsymbol{y}_{\boldsymbol{nt}} = \alpha_0 + \alpha_1 \boldsymbol{y}_{1\boldsymbol{t}} + \alpha_2 \widehat{\vartheta}^2_{\boldsymbol{t}+1/\boldsymbol{t}} + \sigma[\varsigma\ \boldsymbol{m}(\boldsymbol{g}_1 \boldsymbol{y}_{1\boldsymbol{t}}, \boldsymbol{g}_2 \widehat{\vartheta}^2_{\boldsymbol{t}+1/\boldsymbol{t}}) + \upsilon_{\boldsymbol{t}}]$$

Maturity	$\widehat{\alpha}_0$	$\widehat{\alpha}_1$	$\widehat{\alpha}_2$	$\widehat{\sigma}$	$\widehat{\varsigma}$	$\widehat{\boldsymbol{g}}_1$	$\widehat{\boldsymbol{g}}_2$	Ln
3 M	0.218 (0.023)	0.911 (0.035)	-1.255 (0.581)	0.144 (0.0125)	2.844 (0.084)	2.428 (0.954)	8.494 (0.399)	42.347
6 M	0.927 (0.116)	1.267 (0.034)	-0.836 (0.363)	0.255 (0.0019)	0.901 (0.710)	7.397 (0.299)	2.502 (1.933)	32.452
1 year	1.342 (0.362)	0.326 (0.732)	0.256 (0.255)	0.199 (0.095)	1.698 (0.671)	1.296 (0.692)	6.994 (0.826)	63.026
3 years	1.201 (0.350)	0.823 (0.622)	0.962 (0.105)	0.023 (0.0266)	-0.677 (0.236)	7.489 (2.259)	5.125 (4.054)	95.377
5 years	3.383 (0.249)	0.851 (0.115)	0.162 (0.834)	0.255 (0.2646)	-2.896 (0.267)	1.650 (0.097)	6.499 (0.399)	72.183
10 years	1.933 (0.299)	0.867 (0.152)	0.216 (0.460)	0.964 (0.0068)	-2.952 (0.007)	15.220 (0.821)	17.872 (1.358)	106.024

Table V: Performance of the flexible model

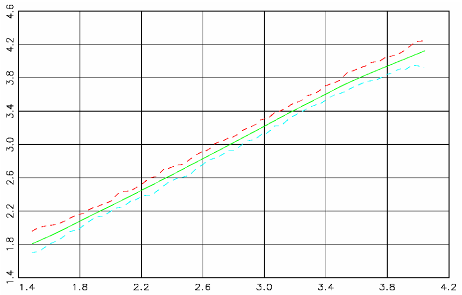
	Correlation between observed and fitted interest rates	Regression of observed on fitted interest rates		
		α	**β**	**R^2**
3 M	0,999	0,637	0,996**	0,999
6 M	0,998	0,278	0,991**	0,994
1 year	0,999	0,462	0,997**	0,989
3 years	0,999	0,973	0,995**	0,987
5 years	0,985	0,399	0,996**	0,989
10 years	0,975	0,379	0,992**	0,983

** Significant at 0.01 level.

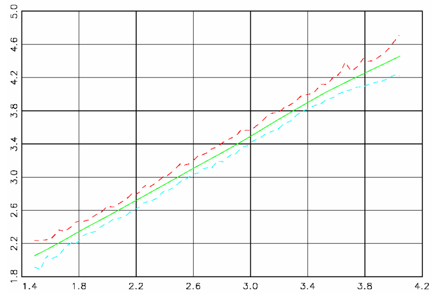
Figure 1a-1f: Effect of risk-free rate on interest rates

The solid line plots the posterior expectation of the function $\alpha_0 + \alpha_1^{'}\boldsymbol{x_t} + \lambda \boldsymbol{m}(\boldsymbol{x_t})$ evaluated at $\boldsymbol{x_t} = (\boldsymbol{y}_1, \overline{\hat{\vartheta}^2_{t+1/t}})'$ as a function of $\boldsymbol{x_1}$ where $\overline{\hat{\vartheta}^2_{t+1/t}} = T^{-1}\sum_{t=1}^{T}\hat{\vartheta}^2_{t+1/t}$. Dashed lines represent 95% probability region.

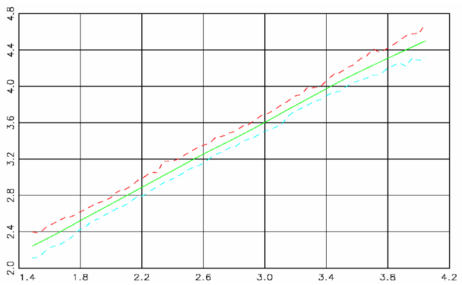
3-month interest rate



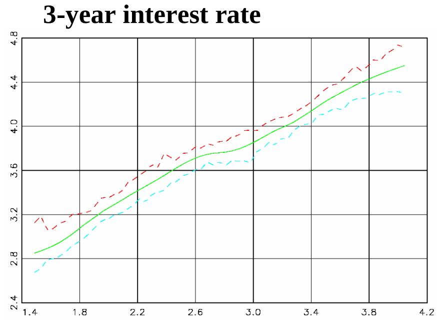
6-month interest rate



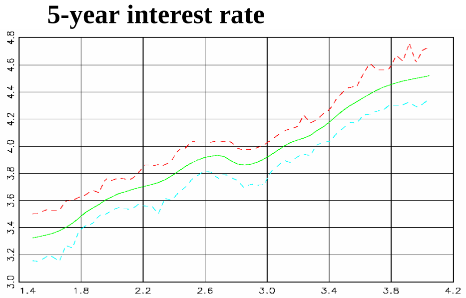
1-year interest rate



3-year interest rate



5-year interest rate



10-year interest rate

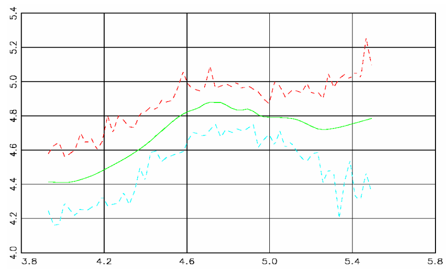


Figure 2a-2f: Volatility effect on interest rates

The solid line plots expectation of the function $\alpha_0 + \alpha_1^{'} \boldsymbol{x_t} + \lambda \boldsymbol{m}(\boldsymbol{x_t})$, evaluated at $\boldsymbol{x_t} = (\overline{\boldsymbol{y}_1}, \widehat{\vartheta}^2_{\boldsymbol{t}+1/\boldsymbol{t}})'$ as function of $\boldsymbol{x_2}$. Dashed lines give 95% probability region.

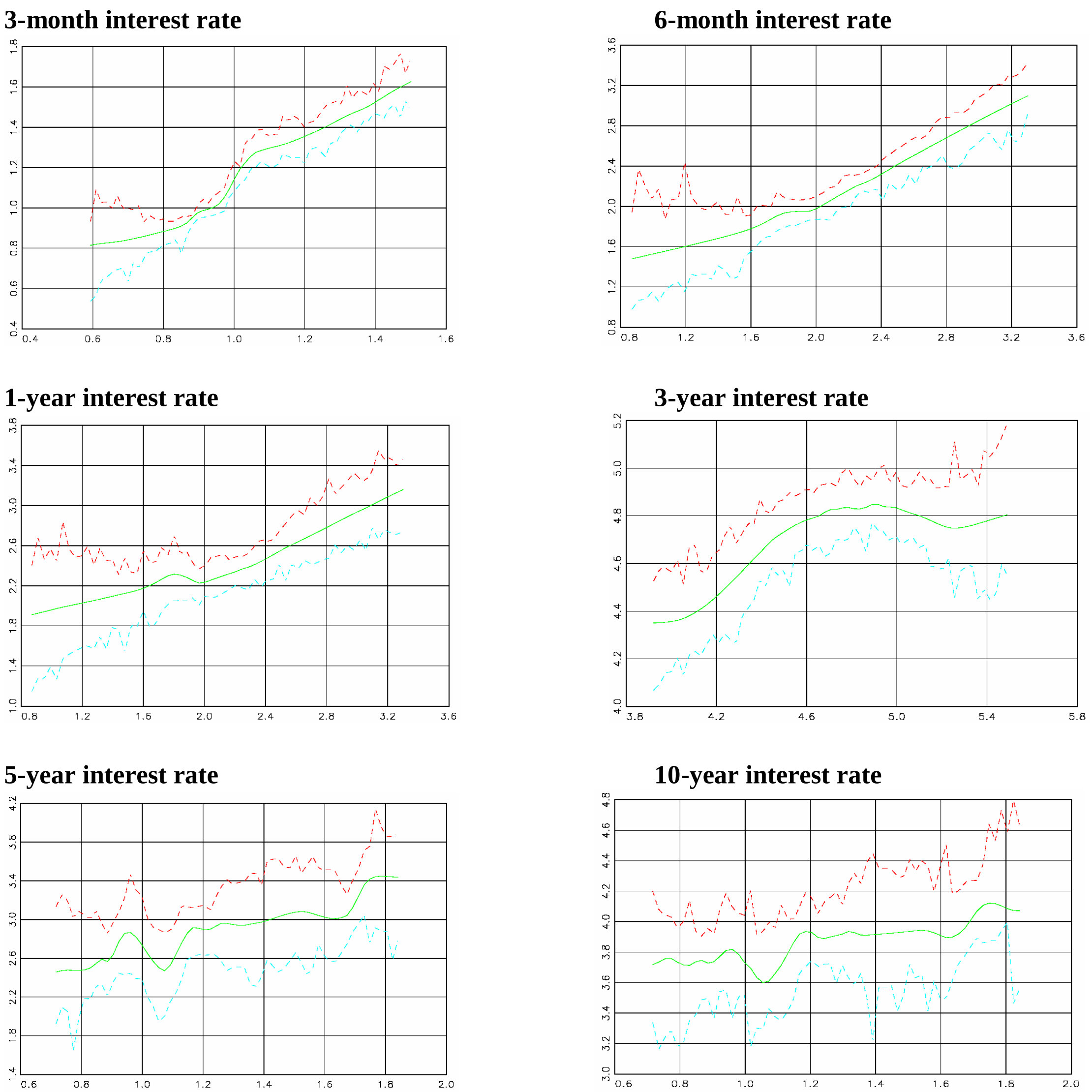


Figure 3: Expectation hypothesis slope coefficients on the U.S. market