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A note on optimal commodity taxation with moral hazard and separable preferences

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Abstract

In this paper we show that differential commodity taxation is superfluous in an economy with moral hazard and separable preferences.

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1. Introduction

Is differential commodity taxation desirable in an economy where moral hazard is present? Arnott and Stiglitz (1986) showed that in general the answer to this question is affirmative. The rationale behind this result is simple: typically an insured individual exerts less effort, to avoid a bad outcome, than what is socially desirable. Therefore, to provide the individual with the incentives to take the appropriate level of care, it is optimal to subsidize those goods which are complementary with the effort level and tax those goods which are substitutes for it. Since taxes and subsidies introduce a wedge between prices and marginal costs, an optimal policy minimizes the welfare costs stemming from the insufficient level of care and the distortions induced by taxes and subsidies.

The reasoning just outlined is based on the presumption that the benefit from altering the level of effort, by changing consumers' prices through commodity taxes, outweighs the cost of tax distortions. Nevertheless, if preferences over consumption goods and the level of effort are separable, and if the probability of random outcomes depends only on the latter, one would expect that any level of effort induced by an optimal policy with taxes could be implemented by lump-sum transfers which do not interfere with incentive compatibility. Hence differential commodity taxation would *not* be required.

In this paper we show that the above intuition is correct. More precisely, we follow Arnott and Stiglitz (1986) and consider a government which seeks to insure individuals against idiosyncratic shocks using income transfers. In addition, the government has the option to levy commodity taxes. Under the assumption that commodities are separable from the level of effort in the utility function, we prove that the solution set of the government's optimal taxation problem does include a policy of taxes and transfers that involves no commodity taxation at all (see Proposition 1 below). Also, we show that if taxes on consumption goods are to be imposed, they do not alter relative prices (see Proposition 2 below). To the best of our knowledge this result has not been derived before in the simple manner we put forward here. Arnott and Stiglitz (1983, p.28) obtained an analogous result, but under more restrictive assumptions. Indeed, they studied the optimal taxation problem by analyzing quite cumbersome first order conditions. In order to render them more manageable, they assumed two consumption goods and additively-separable preferences over these goods. In contrast, our proof dispenses with any characterization of the optimal taxation problem through first order conditions, and we do not assume that preferences over commodities are additively-separable, nor do we restrict the number of consumption goods. Moreover, our assumptions on the probability of random shocks and the cost-of-exerting-effort function are very general.

Note that our Propositions 1 and 2 below may be viewed as a formalization of what Mirrlees refers to as *the no-tax principle*, that is the idea that "When preferences within a group of commodities are independent of the private variables, relative prices for that group should not be distorted by taxation"(Mirrlees, 1995, p.389).

Our result is somewhat related to the literature on competitive equilibria in economies with moral hazard. Indeed, Lisboa (2001) has shown that competitive equilibria are constrained efficient if insurance firms can strategically offer exclusive contracts, and if preferences over consumption and effort are *separable*. In this case taxation of consumption goods would not be required, a finding which is consistent with the result we will prove below[1].

However, under more general assumptions, competitive equilibria are not constrained efficient and differential commodity taxation is thus welfare-improving. Indeed, this result encompasses other type of informational asymmetries, as argued by Greenwald and Stiglitz (1986)[2].

Regarding the approach to proving our results, we should mention that it is close in spirit to Laroque's (2005), even thought the type of informational asymmetries therein contemplated are different from the moral hazard problem we study here[3]. Moreover, in the proof of Propositions 1 and 2 we draw on Lisboa's (2001, p.566) idea of exploiting the duality between utility maximization and expenditure minimization.

The paper is organized as follows: in the second section we describe the economic environment and we formalize our assumptions. In the third section we examine individuals' optimal choices of consumption bundles and effort level, and we set-up the government optimal taxation problem. The fourth section contains our main results on the structure of optimal taxation.

2. The economy

There is a finite number of consumption goods, indexed by $l = 1, ..., L$, and a single non-reproducible good, indexed by $l = 0$. There is a $[0, 1]$-continuum of ex-ante identical individuals. Each individual is endowed with the non-reproducible good which is supplied as a unique input to the production sector. The latter is characterized by a constant returns to scale technology with fixed coefficients and no joint production. Individuals are subject to an idiosyncratic random shock to their endowment of the non-reproducible good. For simplicity, we assume that there are only two realizations of the shock, indexed by $s = 1, 2$. Individual trades in commodities are not observable, whereas the realization of the shocks are observable. Thus the government insures individuals against uncertainty using lump-sum transfers contingent upon idiosyncratic states. In addition, the government can impose linear taxes or subsidies on consumption goods.

The generic consumer has preferences over all the existing goods represented by a continuous utility function $u : \mathbb{R}_+^{L+1} \to \mathbb{R}$ which is assumed to be strictly monotonic and strictly quasi-concave. Moreover, it is assumed that indifference surfaces passing through strictly positive bundles do not intersect the boundary of the consumption set. By virtue of this assumption, it is further assumed that preferences satisfy the following smoothness property[4]

Assumption 1. *$h(p, \bar{u}) = h(p', \bar{u})$ implies $p = \lambda p'$ for $\lambda > 0$*

where $h(p, \bar{u})$ is the compensated demand at prices p and utility level $\bar{u}$.

[1]See, however, Panaccione (2007) for an analysis of the relevance of separability of preferences for the constrained efficiency of equilibria.

[2]We know from Prescott and Townsend (1984) that, when it is possible to restrict consumption patterns, constrained efficient allocations with moral hazard can be decentralized as competitive equilibria. There is no inconsistency with the result just mentioned, since that restriction is not assumed therein; see, for instance, Arnott, Greenwald and Stiglitz (1994) and Stiglitz (1994, p.29).

[3]See also Kaplov (2006), Hellwig (2008) and Gauthier and Laroque (2009).

[4]Assumption 1 is akin to the *directional density* employed by Rader (1976).

Agents optimally choose consumption bundles, supply of the production input, and a level of effort $a \in \mathcal{A}$, where $\mathcal{A}$ is a compact subset of $\mathbb{R}$. By choosing a, the individual incurs a cost (disutility) $c(a)$, where $c : \mathcal{A} \to \mathbb{R}_+$ is assumed to be a continuous function. As usual, the effort is not observable and affects the probability of the realizations of the idiosyncratic shock. Therefore, in what follows $\pi_s(a)$ denotes the probability of state s if effort level a is chosen, where $\pi_s : \mathcal{A} \to (0, 1)$ is assumed to be a continuous function. Preferences over commodities and effort are separable. Thus, the utility of a generic bundle $(x, a) \in \mathbb{R}^{L+1}_+ \times \mathcal{A}$ is given by $u(x) - c(a)$.

Normalized producer prices are $(q, 1) \in \mathbb{R}^{L+1}_{++}$, where q is the price vector of consumption goods and the second component is the normalized price of the non-reproducible input. Note that $q \in \mathbb{R}^L_{++}$ is fixed by virtue of the assumptions on technology[5]. Consumers are subject to linear taxation levied on consumption goods. Hence consumer prices are equal to $q + \tau$, for some $\tau \in \mathbb{R}^L$. In addition to commodity taxation, the government sets state-contingent lump-sum transfers $T_s \in \mathbb{R}$. In what follows, we will refer to the tuple $(\tau, T) = (\tau, T_1, T_2) \in \mathbb{R}^{L+2}$ as a policy. The objective of the government is to choose a feasible policy so as to maximize individuals' ex-ante utility[6]. The government takes into account agents' optimal choices of commodities and effort. In other words, the government maximizes the ex-ante indirect utility of the representative agent.

3. Optimal choices

For any $s = 1, 2$, let us denote by $x_s = (x_{ls}, x_{0s}) \in \mathbb{R}^{L+1}_+$ the generic commodity bundle. The generic agent chooses, ex-ante and simultaneously, the level of effort and the commodity bundles contingent on the realization of idiosyncratic shocks so as to maximize expected utility. When the idiosyncratic state unfolds, the generic individual implements planned purchases of commodities on spot markets. Because preferences are separable, it is easy to see that the optimal choice problem can be decomposed as follows: given a policy (τ, T), each individual chooses a commodity bundle

$$x_s(q, \tau, T_s) = \arg\max \{u(x_s) : (q + \tau) \cdot x_{ls} + x_{0s} \leq \bar{x}_{0s} - T_s\}, \tag{1}$$

where $\bar{x}_{0s}$ is the endowment of the non-reproducible good in state s. Since the budget constraint correspondence is continuous and compact-valued, $x_s : \mathbb{R}^L_{++} \times \mathbb{R}^{L+1} \to \mathbb{R}^{L+1}_{++}$ is a continuous function. Given the optimal commodity bundle $x_s = x_s(q, \tau, T_s)$, each individual chooses the effort $a \in \alpha(q, \tau, T)$, where

$$\alpha(q, \tau, T) = \arg\max \left\{ \sum_s \pi_s(a) u(x_s) - c(a) : a \in \mathcal{A} \right\}. \tag{2}$$

Clearly, by the Maximum Theorem $\alpha : \mathbb{R}^L_{++} \times \mathbb{R}^{L+2} \twoheadrightarrow \mathcal{A}$ is an upper hemi-continuous and compact-valued correspondence. The individuals' ex-ante indirect utility is

$$v(q, \tau, T) = \sum_s \pi_s(a) u(x_s) - c(a) \quad \text{with} \quad x_s = x_s(q, \tau, T_s) \text{ and } a \in \alpha(q, \tau, T), \tag{3}$$

[5]See, e.g., Gale (1955) and Arrow and Hahn (1971, section 2.11).

[6]A policy is said to feasible if it does not result in a budget deficit.

which is a continuous function. The government anticipates individuals' optimal choices, given a policy (τ, T), and thus solves the following problem:

$$\max_{(\tau,T)\in\mathbb{R}^{L+2}} v(q,\tau,T) \text{ s.t. } \tau\cdot\left(\sum_s \pi_s(a)x_{ls}(q,\tau,T_s)\right)+\sum_s \pi_s(a)T_s \geq 0 \text{ for any } a\in\alpha(q,\tau,T)\,.$$

Note that the inequality constraint in the above problem asserts that the government must not run a budget deficit.

4. Optimal taxation

Let $\Phi(q)$ denote the set of optimal policies and assume that $\Phi(q) \neq \emptyset$. The first result we prove is that consumption good taxation is superfluous.

Proposition 1. *There exists $(\hat{\tau}, \hat{T}) \in \Phi(q)$ such that $\hat{\tau} = 0$.*

Proof. The proof is by construction. Pick any $(\tau^*, T^*) \in \Phi(q)$. Construct the policy $(\hat{\tau}, \hat{T})$ as follows:

$$\hat{\tau}=0 \quad \text{and} \quad \hat{T}_s=\bar{x}_{0s}-e(q,u_s^*)\,, \tag{4}$$

where $u_s^* \equiv u(x_s(q,\tau^*,T_s^*))$ and $e(q,u_s^*)$ is the expenditure function at prices $(q,1)$ and utility level u_s^*. From the identity $u(x(p,e(p,u))) \equiv u$ it is easy to see that

$$u(x_s(q,\hat{\tau},\hat{T}_s))=u_s^*\,,$$

and therefore, by (2), we get

$$\alpha(q,\hat{\tau},\hat{T})=\alpha(q,\tau^*,T^*)\,.$$

By definition, $e(q,u_s^*) \leq q\cdot x_{ls}(q,\tau^*,T_s^*)+x_{0s}(q,\tau^*,T_s^*)$. Using the budget constraint in (1), we obtain

$$\bar{x}_{0s}-q\cdot x_{ls}(q,\tau^*,T_s^*)-x_{0s}(q,\tau^*,T_s^*)\geq \tau^*\cdot x_{ls}(q,\tau^*,T_s^*)+T_s^*\,,$$

hence

$$\hat{T}_s=\bar{x}_{0s}-e(q,u_s^*)\geq \bar{x}_{0s}-q\cdot x_{ls}(q,\tau^*,T_s^*)-x_{0s}(q,\tau^*,T_s^*)\geq \tau^*\cdot x_{ls}(q,\tau^*,T_s^*)+T_s^*\,. \tag{5}$$

Therefore, by (5) and by feasibility of (τ^*, T^*), for any $a \in \alpha(q,\hat{\tau},\hat{T}) = \alpha(q,\tau^*,T^*)$ we have that

$$\sum_s \pi_s(a)\hat{T}_s \geq \tau^*\cdot\left(\sum_s \pi_s(a)x_{ls}(q,\tau^*,T_s^*)\right)+\sum_s \pi_s(a)T_s^*\geq 0\,. \tag{6}$$

By (3), $v(q,\hat{\tau},\hat{T}) = v(q,\tau^*,T^*)$, and therefore it follows immediately that $(\hat{\tau},\hat{T}) \in \Phi(q)$. ∎

Proposition 1 above is a general result which does not require assumption 1. We now add assumption 1 and we argue that even if an optimal policy involving taxes and/or subsidies exists, then (i) subsidies are bounded from below; (ii) relative prices of consumption goods are not distorted. That is, there is no differential commodity taxation. Thus, Proposition 2 below comforms to the *no-tax principle* mentioned in the introduction.

Proposition 2. *Under assumption 1, if* $(\tau^*, T^*) \in \Phi(q)$ *then there exists a* $\gamma > -1$ *such that* $\tau^* = \gamma q$.

Proof. Suppose, by way of contradiction, that for any $\gamma > -1$, $\tau^* \neq \gamma q$. That is, $q + \tau^* \neq (1+\gamma)\, q$. By assumption 1,

$$h_s(q, u_s^*) \neq h_s(q + \tau^*, u_s^*).$$

Recall that $u_s^* \equiv u(x_s(q, \tau^*, T_s^*))$. Hence, by strict quasi-concavity of u we must have

$$e(q, u_s^*) < (q, 1) \cdot h_s(q + \tau^*, u_s^*) = q \cdot x_{ls}(q, \tau^*, T_s^*) + x_{0s}(q, \tau^*, T_s^*).$$

As in the proof of Proposition 1, it follows from (4), (5) and (6) that

$$\sum_s \pi_s(a) \hat{T}_s > 0 \tag{7}$$

for any $a \in \alpha(q, \hat{\tau}, \hat{T})$. Now define the correspondence $\sum_s (\pi_s \circ \alpha)\, T_s : \mathbb{R}^{L+2} \twoheadrightarrow \mathbb{R}$ by

$$(\tau, T) \mapsto \left\{ \sum_s \pi_s(a) T_s : a \in \alpha(q, \tau, T) \right\}.$$

Because $\pi_s : \mathcal{A} \to (0,1)$ is continuous and $\alpha : \mathbb{R}^L_{++} \times \mathbb{R}^{L+2} \twoheadrightarrow \mathcal{A}$ is an upper hemi-continuous and compact-valued correspondence, $\sum_s (\pi_s \circ \alpha)\, T_s$ is upper hemi-continuous[7]. Also, note that (7) above implies

$$\sum_s \pi_s(\alpha(q, \hat{\tau}, \hat{T})) \hat{T}_s \subseteq (0, +\infty)\,.$$

Therefore, by upper hemi-continuity one can find a positive ε and construct the policy $\left(\hat{\tau}, \hat{T}^\varepsilon\right) \equiv \left(0, \hat{T}_1 - \varepsilon, \hat{T}_2 - \varepsilon\right)$ so that $\sum_s \pi_s(\alpha(q, \hat{\tau}, \hat{T}^\varepsilon)) \hat{T}_s^\varepsilon \subseteq (0, +\infty)$, that is

$$\sum_s \pi_s(a) \hat{T}_s^\varepsilon > 0$$

for all $a \in \alpha(q, \hat{\tau}, \hat{T}^\varepsilon)$. In other words, the policy $\left(0, \hat{T}_1 - \varepsilon, \hat{T}_2 - \varepsilon\right)$ is feasible. Clearly, (1) implies that $u(x_s(q, \hat{\tau}, \hat{T}_s^\varepsilon)) > u(x_s(q, \hat{\tau}, \hat{T}_s))$. Thus, for all $a \in \alpha(q, \hat{\tau}, \hat{T})$, we have that

$$v(q, \hat{\tau}, \hat{T}^\varepsilon) \geq \sum_s \pi_s(a) u(x_s(q, \hat{\tau}, \hat{T}_s^\varepsilon)) - c(a) > \sum_s \pi_s(a) u(x_s(q, \hat{\tau}, \hat{T}_s)) - c(a) = v(q, \hat{\tau}, \hat{T}).$$

[7] See Aliprantis and Border (2006, Theorem 17.32).

Since $v(q, \hat{\tau}, \hat{T}) = v(q, \tau^*, T^*)$, we get

$$v(q, \hat{\tau}, \hat{T}^{\varepsilon}) > v(q, \tau^*, T^*),$$

which contradicts the hypothesis that $(\tau^*, T^*) \in \Phi(q)$. ■

References

Aliprantis, C.D. and K.C. Border (2006) *Infinite Dimensional Analysis. A Hitchhiker's guide*, 3rd edition, Springer: Berlin.

Arnott, R.J., B. Greenwald and J.E. Stiglitz (1994) "Information and economic efficiency" *Information Economics and Policy* **6**, 77-88.

Arnott, R.J. and J.E. Stiglitz (1986) "Moral hazard and optimal commodity taxation" *Journal of Public Economics* **29**, 434-462.

Arnott, R.J. and J.E. Stiglitz (1983) "Moral hazard and optimal commodity taxation" NBER working paper number 1154.

Arrow, K.J. and F. Hahn (1971) *General Competitive Analysis*, North-Holland: Amsterdam.

Atkinson, A. and J.E. Stiglitz (1980) *Lectures on Public Economics*, McGraw-Hill: New York.

Atkinson, A. and J.E. Stiglitz (1976) "The design of tax structure: direct versus indirect taxation" *Journal of Public Economics* **6**, 55-75.

Gale, D. (1955) "The law of supply and demand " *Mathematica Scandinavica* **3**, 155-169.

Gauthier, S. and G. Laroque (2009) "Separability and public finance" *Journal of Public Economics* **93**, 1168-1174.

Greenwald, B. and J.E. Stiglitz (1986) "Externalities in economies with imperfect information and incomplete markets" *Quarterly Journal of Economics* **101**, 229-264.

Hellwig, M. (2008) "A generalization of the Atkinson and Stiglitz (1976) theorem on the undesirability of nonuniform excise taxation", Technical Report, Max Planck Institute for Research on Collective Goods.

Kaplow, L. (2006) "On the undesirability of commodity taxation even when income taxation is not optimal" *Journal of Public Economics* **90**, 1235-1250.

Laroque, G. (2005) "Indirect taxation is superfluous under separability and taste homogeneity: a simple proof" *Economics Letters* **87**, 141-144.

Lisboa, M.B. (2001) "Moral hazard and general equilibrium in large economies" *Economic Theory* **18**, 555-575.

Mirrlees, J.A. (1995) “Private risk and public action: the economics of the welfare state” *European Economic Review* **39**, 383-397.

Panaccione, L. (2007) “Pareto optima and competitive equilibria with moral hazard and financial markets” *The B.E. Journal of Theoretical Economics* **7** (Topics), Article 37.

Prescott, E. and R. Townsend (1984) “Pareto optima and competitive equilibria with adverse selection and moral hazard” *Econometrica* **66**, 880-890.

Rader, T. (1976) “The welfare loss from price distortions” *Econometrica* **44**, 1253-1257.

Stiglitz, J.E. (1994) *Whither Socialism*, 4th edition, MIT Press: Cambridge, MA.