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Shock persistence in output and the role of stochastic population growth

Tapas Mishra
Department of Economics, Swansea University, Swansea, UK

Alexia Prskawetz
Vienna University of Technology & Vienna Institute of Demography, Austria

Mamata Parhi
Department of Economics, Swansea University, Swansea, UK

Claude Diebolt
BETA, University of Strasbourg, Strasbourg, France

Abstract

This paper illustrates both analytically and empirically that stochastic long-memory in economic growth arises due to the presence of a long-memory in population growth. Specifically, we show that the long-run conditional mean and variances of economic growth are functions of stochastic long-memory in demographic system. This is well-supported by an empirical example.

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1 Introduction

This paper seeks to address one of the important puzzles in economic growth: whether (and how) stochasticity in economic growth is explained by possible stochasticity in the demographic system? Extant literature in demography and economic growth for the past three centuries since Malthus (1798) have impelled us to believe that despite having distinct evolutionary characters, perturbations in demographic system is very likely to induce instability in economic growth in the long run. However, the mechanism involved to explain this phenomenon banked upon stationary system and Markov process without non-linearity assumption (Mishra, Diebolt and Parhi, forthcoming). An alternative route would be to exploit the temporal evolutionary properties of both systems to explain reasons of stochasticity in economic growth. This paper adopts this route and exploits time dynamics of aggregate population to explicate the nature of persistence in economic growth.

Till date the conventional practice in (empirical) economic growth models has been to treat population growth as stationary implying that stochastic shocks to the population series would completely disappear in the long-run and thus would exert no measurable impact on its long-run mean and variance. Statistically, a stationary series may still accommodate long-memory features, however the shock convergence patterns of short-memory and long-memory stationary processes are vastly different (e.g., Bailey, 1986). From economic theoretic and policy perspectives such differences are interesting as they determine economies' speed and pace of growth over time. This is so because, the longer the demographic shocks take time to taper-off, the longer it takes for the economy to stabilize in the long-run. Recent research (e.g., Boucekkine et al., 2002; Azomahou et al., 2009) have rendered similar observations using dynamic overlapping generations and spatial vector autoregressive models but have remained silent on the plausibility of stationary population growth assumption. This paper aims to examine this convention.

Questions may arise then, what contributes to the stochasticity (and non-stationarity) in population and consequently in economic growth? Can population growth be characterized by a long-memory process? Is the observed long-memory in economies' growth in recent studies (e.g., Michelacci and Zaffaroni, 2000) attributable to a possible long-memory in population growth? To answer, we build an analytical framework and show that long-memory in economic growth arises due to the presence of long-memory in population growth. Evidence of long-memory is then provided by empirical examination for a set of countries over the period 1950-2004.

Indeed, population growth is not a purely demographic phenomenon as its evolutionary mechanism is contingent upon the behavioral changes occurring within both the economy and the environment (see e.g., Birdsall et al., 2001). Due to the remarkable adaptive capability and response to subtle environmental and economic variations, human demographic system displays far more complex dynamics than any other natural demographic systems. Unique to its very nature, a concurrent existence of both independent and interactive mechanisms contribute to the persistence of shocks in the demographic system and ultimately lead to stochastic memory features in its evolutionary process.[1] Prskawetz and Feichtinger (1995) for example, showed that the underlying mechanism describing the demographic system is exceedingly complex, characteristically non-linear and may result in a pattern which exhibits chaotical growth dynamics.

Similarly, long-memory in economic growth has been investigated by Michelacci and Zaffaroni (2000) where fractional convergence of output has been possible by stochastic technological shocks and induced by cross-sectional aggregation of growths. Although admirable set of research have stemmed following this tradition, a clear demographic dimension to the explanation of long-memory in economic growth seems to be missing. Although some exceptional recent research (e.g. Gil-Alana, 2003; Mishra, 2008) independently investigated the fractal structure of population, they did not lend further economic insights of the role of fractional population growth in the persistence of economic growth. This paper attempts to explain long-memory in economic growth by possible stochasticity in population growth. Very often though the long memory effects are confused with hysteresis effect, it is important to bear in mind the objective of the paper lies in the stochastic long-memory aspect of demographic-economic growth system. Because the hysteretic effect is a persistence in the series like the long memory effect, nevertheless, the long term behavior of the hysteretic series is very different from the long term behavior of the long memory series. Very importantly, the hysteretic series are not mean reverting whereas the long memory series are (if correctly differenced). A mean reverting long-memory demographic system is the subject of interest in this paper.

Thus, novelty of the paper thrives on the modeling idea of demographic dynamics in a stochastic setting and in our case with a long-memory system. Gil-Alana (2003) only tested if population growth in OECD countries is fractionally integrated without providing economic theoretic reasons: why would this series follow such a pattern. Similarly, Mishra (2008) underlined the relevance of long-memory in demographic system and did some exercise

[1] Shaffer (1987) argues that demographic stochasticity is caused by (i) chance realizations of individual probabilities of death and reproduction in a population and (ii) by environmental stochasticity from a nearly continuous series of small or moderate perturbations.

with an extended Solow-Swan model. Once again, the argument lacked a clear economic and econometric theoretic mechanism which would generate long-memory in demographic system. Instead, our contribution provides an economic demographic link and shows that how long-memory in economic growth is a consequence of stochastic long-memory in population growth. Our contribution is a significant improvement over Gil-Alana (2003) and Mishra (2008) in that we calculate the long-run mean and variance of output as a function of long-memory in population growth. This is important in light of the predictive power of the economic system and adoption of strategies in countering stochastic shocks in the longer run. In the next section (Section 2), we characterize long-memory in demographic and economic growth system and provide analytical framework to investigate the long memory in demography-economic growth system. Section 3 provides econometric methodology and empirical illustration and finally section 4 summarizes the results with policy implications.

2 Characterizing long-memory in population and economic growth

2.1 Long-memory in population growth

To demonstrate that a long-memory in population growth gives rise to a long memory in economic growth, it is necessary to characterize population growth with fractional dynamics. For the purpose, define population growth (n_t) as the difference in fertility (f_t) and mortality (d_t) rates while accounting for net migration rate (m_t) in the economy. This is written as

$$n_t = (f_t - d_t) + m_t \tag{1}$$

Although n_t is normally assumed to be stationary in most empirical growth literature, it is still unknown whether the demographic system (being in continuous interaction with the economy) would tend to converge to a stable long-run equilibrium level. Indeed, while stability (or stationary) assumption is an apparent possibility, it may not be the only possibility. In fact, characterization of n_t with a fractal structure (as in definition 1) allows persistence of shocks with varying convergence patterns of which stationarity could be a limiting case.

Definition 1 *Denote d as the integration parameter lying on the real line, k as the lag length. Now, suppose that n_t is a process with autocovariance* *function* $\gamma(k) \sim C(k)k^{2d-1}$ *as* $k \to \infty$, $C(k) \neq 0$, *where* k *defines the lag between current and distant observations. Then* n_t *is a long-memory process if the autocovariance function decays slowly over time.*

Persistence in population growth as reflected by the slow decaying autocorrelation function in definition (1) may result from a combination of sources. For instance, if the transition of the demographic state is non-stationary, then the presence of a stochastic shock in one state would move over time to succeeding states. The interacting system - in our case - the economy will be arguably perturbed due to transition of stochastic shocks from the past. Other mechanism to generate long memory in population growth could be the non-linear dynamics with short-memory. For instance, the autocorrelation function of a non-linear Markov process can also exhibit long-memory persistence (with convergence).[2]

2.2 Long-memory in output growth

Definition 1 is utilized below to demonstrate how stochastic demographic system may induce volatility in economic growth. Specifically, we show that the conditional mean and variance of $k-$period (or long-run) aggregate output is a function of stochastic memory in population growth. Assume the following economic-demography growth mechanism (EDM):

$$y_t = \gamma n_{t-1} + \eta_t \qquad (2)$$

where $\eta_t \sim iid(0, \sigma_\eta^2)$. Relation (2) implies that past population growth affects current output growth. Lagged - not instantaneous effect - occurs in this relation due to the inevitability that the economy takes time to respond to a shock in n_t necessitating thus the EDM to thrive on the natural feedback effects. The motivation behind the EDM representation in (2) follows from two notable research, viz., Easterlin (1966) and Dasgupta (1995). Easterlin (1966) provided the cornerstone of the widely discussed economic-demographic interactions with feedback effects. He argued that the co-evolutionary pattern of the economic-demographic system determines long-swings in economic growth. Dasgupta (1995) formally presented the feedback mechanism in the form of relation (2) assuming, like most of the conventional literature, the stationarity of population growth.[3] Thus, a stochastic shock in this system will inevitably

[2] Thanks to the anonymous referee who pointed out this aspect.

[3] One may well present the converse case, i.e., causation running from economic growth to the demographic growth. But here we give importance to demographic system as it is human population which control the economy first of all.

converge to mean value in the long-run. After initial spurts, the system is likely to be stable and thus, initial perturbations are not forever sustained in the system. That is the system does not contain any memory of past shocks. However, demographic system, like any other system, evolves over time and there is clear possibility of a shock becoming persistent. It might also be the case that the shocks may not even converge in the long-run. This leads to the following proposition.

Proposition 1 *Assuming that the EDM relation holds and n_t possesses a long-memory under definition 1, then long memory in output growth, y_t, can be represented by the long memory in population growth.*

Proof

Basically, we show that the long-run conditional mean and variance of output is a function of long-memory in n_t. We begin by modeling output growth (y_t) and aggregate population growth (n_t) in an autoregressive (AR) fractionally moving average(MA) (ARFIMA(p,d,q)) framework, where the AR order is given by p and MA order by q. d is the fractional order of integration. The ARFIMA (p,d,q) has the advantage of endogenizing the effect of stochastic shocks in terms of past dependence as well as specifying the evolution of the system with a history dependent character. Degree of imperfection of the system or the interacting system is reflected by the corresponding degree of order of integration. For instance, modeling population growth, n_t in ARFIMA (p,d,q) setting implies that

$$\begin{aligned}&(1-\phi_1 L-\phi_2 L^2-\cdots-\phi_p L^p)(1-L)^d n_t\\&=(1+\theta_1 L+\theta_2 L^2+\cdots+\theta_q L^q)\epsilon_t\end{aligned} \tag{3}$$

with usual definitions: $\mathbb{E}[\eta_t\epsilon_s]=\sigma^2_{\tau\epsilon}$ if $t=s$, 0, otherwise. In the above, L is backward shift operator, with the usual property that $Ln_t=n_{t-1}, L^2n_t=n_{t-2}$, etc. Formally, $(1-L)^d$ can be expressed by power series expansion:

$$(1-L)^d=\sum_{j=0}^{\infty}(-1)^j\left(\frac{d(d-1)(d-2)\dots(d-j+1)}{j!}\right) \tag{4}$$

where $\frac{d(d-1)(d-2)\dots(d-j+1)}{j!}$ is the binomial coefficient defined for any real number d and non-negative integer j. The intuitive exposition of $(1-L)^d$ for a time series can be traced via their infinite order MA or AR representations. In this instance, expressing $MA(\infty)$ of $(1-L)^d$ for the time series would mean that we have an expression: $\sum_{j=0}^{\infty} h_j L^j$, where $h_0 = 1$ and

$$h_j = \frac{-d\Gamma(j-d)}{\Gamma(1-d)\Gamma(j+1)} = \frac{j-d-1}{j} h_{j-1}, j \geq 1. \tag{5}$$

Equation (5) is the impulse response function of the effect of a stochastic shock on n_t distributed over time. The interpretation of d with different range of real values are presented in Table 1.

Table 1: Fractional components and their interpretation

d	Interpretation
0	: Short-memory population growth, log population is $I(1)$
1	: Non-stationary population growth, log population is $I(2)$
$< 0, 0.5 >$	: Long-memory population growth, log population is I(d+1)

The next step is to show that a stochastic long-memory in demographic system may result in a long-memory stochasticity in economic growth. Granger (1980) in an important work showed that long-memory in aggregate variable might arise due to the aggregation of short-memory character of individual components. Since population growth is an aggregate time series, it might be possible to describe the process as a long memory because of possible short-memory features of its components, i.e., population age-structures. However, since demographic process is different from macroeconomic or financial time series, the individual components of the aggregate (population) do not follow independent distributions because of the existence of overlap of population generations and latter inclusion in other population groups.[4] We will not therefore not consider Granger's (1980) aggregation principle as a source of long-memory in aggregate population. However, economic growth may well be described by Granger's process. Again, since demography may affect different components of the economic system, we discuss, below for the sake of brevity, the long memory effect of aggregate population on aggregation economic activity. What we show below is that the conditional mean and variance of output growth is a function of stochastic long-memory parameter of population growth. This implies that if there is slow-converging stochastic shock in the population growth, the mean and variance of long-run output will be non-constant and will rather depend upon the degree of convergence of long-memory component of demographic shocks.

[4]For instance, young age population after say 15 years are in working age basket, similarly working age after say 20 years are in retired cohort basket. In this case, because of the explicit overlap, it is not possible to say that aggregation of different orders of integration from each component of the aggregate population gives rise to long memory.

To elucidate, in (3), we assume $\phi(L) \neq 0$ for $z \leq 1$. Re-writing (3) as $n_t = \phi(L)^{-1}(1-L)^{-d}\theta(L)\epsilon_t$ and denoting $\omega(L) = \phi(L)^{-1}$, where $\omega(L) = \sum_{i=0}^{\infty} \omega_i L^i$ we use the identity $\omega(L)\phi(L) = 1$ to find the unknown coefficients recursively:

$\omega_0 = 1,$
$\omega_1 = \phi_1\omega_0,$
$\omega_2 = \phi_1\omega_1 + \phi_2\omega_0$ and so,
$\omega_i = \phi_1\omega_{i-1} + \cdots + \phi_p\omega_{i-p}$ for $i = p, p+1, \cdots$.

Now utilizing $(1-L)^{-d} = \sum_{i=0}^{\infty} \frac{(d+j-1)\cdots(d+1)d}{i!} L^i$ and multiplying $\phi(L)^{-1}$, we get

$$(1-L)^{-d}\phi(L)^{-1} = \sum_{j=0}^{\infty} z_j L^j \tag{6}$$

where
$z_j = 1$ if $j = 0,$
$z_j = \omega_0 \frac{(d+j-1)\cdots(d+1)d}{j!} +$
$\omega_1 \frac{(d+j-2)\cdots(d+1)d}{(j-1)!} + \cdots + \omega_{j-1}d + \omega_j$, otherwise.

And finally, for $j \geq 0$, describe
$\psi_j = z_j + z_{j-1}\theta_1 + \cdots z_{j-q}\theta_q$
with $z_{-1} = \cdots = z_{-q} = 0$.
Denote by $Y_t^{(k)}$ the cumulative $k-$period output, y_t. Let's use the $MA(\infty)$ representation of y_t from above:

$$y_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}. \tag{7}$$

To know the effect of stochastic population shocks on aggregate output, we utilize EDM and $MA(\infty)$ representations such that:

$Y_t^{(k)} = \sum_{l=1}^{k} y_{t+l} = \gamma.\sum_{l=1}^{k}\sum_{j=0}^{\infty} \psi_j \epsilon_{t-1-j+l} + \sum_{l=1}^{k} \eta_{t+l}$
Representing $\zeta_i^{(k)} \equiv \psi_i + \psi_{i-1} + \cdots + \psi_{i-(k-1)}$,
we can write
$Y_t^{(k)} = \gamma.\sum_{i=0}^{\infty} \zeta_i^{(k)} \epsilon_{t-1-j+l} + \sum_{l=1}^{k} \eta_{t+l}$

The *conditional expectation* of $Y_t^{(k)}$ then equals:

$$\mathbb{E}\left[Y_t^{(k)}\right] = \gamma.\sum_{i=0}^{\infty} \zeta_i^{(k)} \epsilon_{t-1-j+l} \tag{8}$$

and the *conditional variance* of $k-$period cumulative output is:

$$\begin{aligned} & Var_t\left(Y_t^{(k)} - \mathbb{E}\left[Y_t^{(k)}\right]\right) \\ & = \gamma^2 . \sum_{l=1}^{k}\left(\zeta_{k-l}^{(k)}\right)^2 \sigma_\epsilon^2 + \gamma . \sum_{k-l}^{(k)} \sigma_{\epsilon\eta} + \sigma_\eta^2 \end{aligned} \tag{9}$$

Expressed in terms of ζ, aggregate output is now a function of long-memory in population growth, which completes the proof. □

3 Empirical example

Evidence on long-memory in n_t can be obtained by estimating d (the fractional integration parameter) using real world population data. For illustration, we have used a set developing countries' aggregate and age-structured population data (viz., population age 0-14, 15-64, and 65+) for the period 1950-2004 and have estimated d employing modified log periodogram regression (MLPR) of Kim and Phillips (2000).[5] All data have been obtained from the World Bank Development Indicators.

The MLPR method is a modified version of the following Geweke and Porter-Hudak (GPH, 1983) log periodogram regression:

$$ln[I_n(\lambda_\zeta)] = -2dln|1 - e^{i\lambda_\zeta}| + ln(f_u(\lambda_\zeta)) + \eta_j \tag{10}$$

where the periodogram ordinates of population growth (left hand side of the equation) are regressed over the spectral representation of the error term and the transformation of $(1-L)^d$ in the frequency domain. The ordinates are evaluated at the fundamental frequencies $\zeta = 1, ..., \nu$. Kim and Phillips (2000) note that (10) is a moment condition and not a data generating mechanism. The modified GPH, i.e., the MLPR is given as[6]:

$$ln(I_V(\lambda_\zeta)) = \alpha - dln|1 - e^{i\lambda_\zeta}|^2 + u(\lambda_\zeta) \tag{11}$$

in which the periodogram ordinates, $ln(I_n(\lambda_\zeta))$ are replaced by $ln(I_V(\lambda_\zeta)) = V_n(\lambda_\zeta)V_P(\lambda_\zeta)^*$ with $\alpha = ln(f_u(0))$ and $u(\lambda_\zeta) = ln[I_n(\lambda_\zeta)/f_n(\lambda_\zeta)] + ln(f_u(\lambda_\zeta)/f_u(0))$. Note that $V_n(\lambda_\zeta)V_P(\lambda_\zeta)^*$ is the discrete fourier transform and is to be used in the regression instead of $ln(I_V(\lambda_\zeta))$.

[5] The data covers 63 developing countries (the list of countries are available with the authors.

[6] For details refer to Kim and Phillips, 2000

A practical problem is the choice of ν, the number of periodogram ordinates to be used in the regression. Geweke and Porter-Hudak (GPH, 1983) suggests that the optimal $\nu = T^{\alpha}$ where $\alpha = 1/2$ and T is the sample size. The choice involves a tradeoff that may be described as follows. The smaller the bandwidth, the less likely the estimate of d is contaminated by higher frequency dynamics, i.e., the short-memory. However, at the same time smaller bandwidth leads to smaller sample size and less reliable estimates. As in the case of GPH method, the smaller value of α (as in $\nu = T^{\alpha}$) implies the smaller number of harmonic ordinates (i.e., the smaller bandwidth) will be used for the estimation of d. Generally, in empirical analysis, preference is given to increasing the value of α to check for the consistency of the estimate of d although simulation experiments can confirm the validity of the selection. For our purpose, we have used $\alpha = 0.60$ through $\alpha = 0.80$ to estimate d. We choose $\alpha = 0.7$ based on a Monte Carlo simulation experiment (see table below) where we have minimum bias for that bandwidth.[7]

Table 2: Monte Carlo simulation for choice of bandwidth

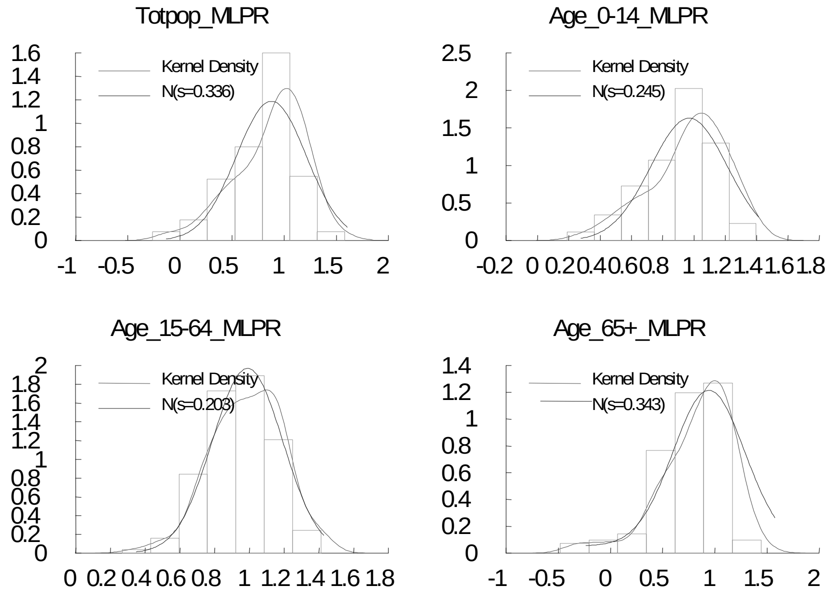
Bandwidth	Estimated bias	Significance	RMSE bias
τ=0.60	0.018	3.03	0.019
τ=0.65	0.021	2.86	0.023
τ=0.70	0.014	2.20	0.015
τ=0.75	0.015	2.47	0.016
τ=0.8	0.017	2.83	0.018

Figure 1 plots density functions of estimated d values[8] for 63 developing countries. The logarithm of total and age-structured population series have been first differenced to calculate growth rates. In the graph MLPR for total population is denoted by $MLPR_{totpop}$. Similar denotation are used for different age-structures. From Figure 1, notice that d values for all series exhibit clear long-memory patterns. The hypothesis we tested is whether there is short-memory against the alternative of long-memory. While tails of the density plots mark the presence of short memory and unit root non-stationary processes, most countries' population growth fall within the non-stationary range with convergent shocks. The age-structured population growth series also exhibit

[7]Davidson's (2007) Time Series Modelling software is used to carry out the simulation experiment which is built for the GPH model.

[8]The standard errors and d values are available from the authors upon request. The series have been tested and adjusted for outliers and structural breaks.

Figure 1: Modified log-periodogram estimation of long-memory for total and age-structured population growth for developing countries



similar patterns. The finding supports our conjecture that population growth might exhibit long-memory. Similar results were obtained by Gil-Alana (2003) for some OECD countries.

4 Conclusion

In this paper we introduced long-memory mechanism in demographic system to explain possible stochasticity in economic growth. The analytical results presented in the paper demonstrated how the long-run conditional mean and variance of aggregate economic growth would depend on the convergence pattern of shocks in demographic system. Empirical illustrations for a set of developing countries basically supported our proposition. A possible implication of our result is that long-run economic policy of a country needs to reconsider stabilization of demographic shocks at the first place before adopting control measures for other macroeconomic fundamentals. After all, a stable demography-economic co-evolution is necessary for a sustainable and stable growth.

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