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Further empirical evidence of nonlinearity in the us monetary policy rule

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Abstract

Given conflicting results on whether the US monetary policy rule exhibited nonlinearity in the post-war period we employ a new Granger non-causality nonlinearity test and non-parametric procedures to re-examine the issue. Both procedures suggest that the Fed followed a nonlinear Taylor rule with respect to expected inflation and expected output gap prior to 1979 but not post 1982.

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1. Introduction

Recently, Péguin-Feissolle, Strikholm and Teräsvirta (2008) (PST) have provided a new test for nonlinear causality. Our purpose in this paper is to employ their test in conjunction with others, particularly a non-parametric method, in a further analysis of whether the US Taylor rule exhibits nonlinearity. This seems worthwhile given that previous literature has reported contradictory results. Since Clarida *et al.* (2000) reported linear estimates of Taylor rules in the US, a number of papers, motivated by the research on asymmetric preferences or opportunistic behavior of policy makers (e.g. Cukierman 2002, Orphanides and Wieland 2000, and Orphanides 2003), have reported evidence of nonlinearity in the response of interest rates to its assumed determinants[1]. However, the reported results are contradictory. For example, Kim *et al.* (2005, 1960:I-2000:IV) found nonlinearity in both expected inflation and expected output prior to 1979 but linear thereafter. Surico (2007, 1960:I-2003:II) obtained evidence of nonlinearity only in expected output up to 1983 and linear thereafter. Cukierman and Muscatelli (2002, 1979:III-1999:IV, and 2008, 1960:I-2005:IV) reported nonlinearity with respect to expected inflation and expected output, except in the Paul Volcker period, whilst Dolado *et al.* (2004, 1970:M1-2000:M12) found nonlinearity in expected inflation only since 1983.

We consider the properties of the Taylor rule over a sample period longer that the ones employed by previous researchers, 1960:I to 1979:II and 1982:IV to 2008:IV. Furthermore, given the promising results of Kim *et al.* (2005), we employ non-parametric regressions in order to determine whether there is graphical evidence of non-linear (or linear) behavior consistent with the PST test. Essentially, results of both methods support significant nonlinearity in the Taylor Rule in the period up to 1979 but not in the period since 1983. This nonlinearity is apparently parsimoniously captured by the hyperbolic tangent smooth transition regression (HTSTR) model proposed by Cukierman and Muscatelli (2002 and 2008). The standard errors and critical values of the HTSTR estimated parameters are obtained employing a block bootstrap methodology in order to deal with possible serial correlation and identification issues.

The plan of the paper is as follows. In section 2, we discuss the Granger non-causality nonlinearity test and present their results. In section 3, we spell out our non-parametric procedures and show plots of the partial derivatives of interest rates to expected inflation and expected output. Section 4 reports estimates of the policy rule based on the HTSTR model proposed by Cukierman and Muscatelli (2002 and 2008). This appears to capture parsimoniously the nonlinearity in the first period of our sample. The paper concludes with a brief summary in section 5.

2. Granger Non-causality Nonlinearity Test Framework and Nonlinear Test

Our data is quarterly from 1960:I to 2008:IV and we consider two sample periods, 1960:I to 1979:II and 1982:IV to 2008:IV. We exclude the period between 1979:III and 1982:III because the monetary procedures were largely different from before and after that period. In particular, it is acknowledged that the Federal Reserve targeted non-borrowed reserves rather than short-term interest rates from 1979:IV to 1982:III. Inflation is measured

[1] We note the important contribution of Minford *et al.* (2002) which shows that estimates of the response of interest rates to expected inflation and output may not be the policy rule followed by the central bank but rather observationally equivalent to a money supply rule. Also the important contribution of Cochrane (2007) demonstrates that identification of a Taylor rule may not be feasible in certain model structures. From both perspectives our results could be interpreted as evidence of linearity or nonlinearity in the reduced form rather than the Taylor rule per-se.

as the annualised rate of change of the GDP deflator (p_t) between two subsequent quarters: $\pi_t = 400 * (\ln(p_t) - \ln(p_{t-1}))$. The output gap measure (x_t) is 100 times the difference between the logarithms of real GDP and the estimate for potential real GDP constructed by the Congressional Budget Office (CBO). The interest rate is the average Federal Funds rates in the first month of each quarter. All these series were downloaded from the Federal Reserve Bank of St. Louis. We obtain expected inflation and expected output gap using the same method and instruments as Kim *et al.* (2005).[2] In particular, $E_t\pi_{t+1} = \alpha + \beta_1\pi_{t-1} + \beta_2\pi_{t-2} + \beta_3\pi_{t-3} + \beta_4\pi_{t-4} + \gamma x_{t-1}$ and $E_t x_{t+1} = \tau + \delta_1 x_{t-1} + \delta_2 x_{t-2} + \delta_3 x_{t-3} + \varphi(\bar{\imath}_{t-1} - \bar{\pi}_{t-1})$ where $\bar{\imath}_t$ and $\bar{\pi}_t$ denote four quarter moving average of current and previous interest and inflation rates, and we assume horizons for inflation and output gap as $k = q = 1$ following Clarida *et al.* (2000) and Kim *et al.* (2005).

2.1 Granger Non-causality Nonlinearity Framework-PST TEST

We have two time series $\{x_t\}$ and $\{y_t\}$ between which the functional form of the relationship is unknown, but it is assumed that the possible causal relationship between them is adequately represented by the following equation

$$y_t = f_y\big(y_{t-1}, \cdots, y_{t-p}, x_{t-1} \cdots x_{t-q}; \theta\big) + \varepsilon_t, \tag{1}$$

Where θ is a parameter vector and $\varepsilon_t \sim nid(0, \sigma^2)$. In this framework, x does not Granger cause y if

$$f_y\big(y_{t-1}, \cdots, y_{t-p}, x_{t-1} \cdots x_{t-q}; \theta\big) = f^*(y_{t-1}, \cdots, y_{t-p}; \theta^*) \tag{2}$$

This means that the conditional mean of y_t is not a function of past values of x_t. Given that the functional form of f_y is unknown, by linearising f_y with a Taylor series approximation, we obtain the following form,

$$y_t = \beta_0 + \sum_{j=1}^{p_1} \beta_j\, y_{t-j} + \sum_{j=1}^{q_1} \gamma_j\, x_{t-j} + \sum_{j_1=1}^{p_1}\sum_{j_2=1}^{q_1} \delta_{j_1 j_2}\, y_{t-j_1} x_{t-j_2} + \sum_{j_1=1}^{q_1}\sum_{j_2=j_1}^{q_1} \tau_{j_1 j_2} x_{t-j_1} x_{t-j_2} + remains \tag{3}$$

Expansion (3) contains combinations of lagged values of $\{y_t\}$ and $\{x_t\}$. Rejection of the null hypothesis $\delta_j = \tau_j = 0$ implies x_t nonlinearly Granger causes y_t. We employ two lags of both expected inflation and expected output gap for testing the null hypothesis in our Granger non-causality nonlinearity test,

$$\begin{aligned} i_t = {} & \alpha + \beta E_t\pi_{(\{t+1\}-1)} + \gamma\, E_t x_{(\{t+1\}-1)} + \rho_1 i_{t-1} + \rho_2 i_{t-2} + \delta_1 i_{t-1} i_{t-2} + \delta_2 i_{t-1} i_{t-2} E_t\pi_{(\{t+1\}-1)} \\ & + \delta_3 i_{t-1} i_{t-2} E_t\, x_{(\{t+1\}-1)} + \delta_4 i_{t-1} i_{t-2} E_t\pi_{(\{t+1\}-1)} E_t\pi_{(\{t+1\}-2)} \\ & + \delta_5 i_{t-1} i_{t-2} E_t\, x_{(\{t+1\}-1)} E_t\, x_{(\{t+1\}-2)} + \tau_1 E_t\pi_{(\{t+1\}-1)} E_t\, \pi_{(\{t+1\}-2)} + \\ & + \tau_2 E_t\, x_{(\{t+1\}-1)} E_t\, x_{(\{t+1\}-2)} + \tau_3 E_t\pi_{(\{t+1\}-1)} E_t\pi_{(\{t+1\}-2)} E_t\, x_{(\{t+1\}-1)} E_t\, x_{(\{t+1\}-2)} + \varepsilon_t \end{aligned} \tag{4}$$

where i_t is Federal Funds rates, $E_t\pi_t$ and $E_t x_t$ are expected inflation and expected output gap.

[2] Kim *et al.* (2005) employed instruments dated at time t. We also obtained results using that information set which are quantitatively similar but prefer those dated t-1 to ensure orthogonality to the disturbance term.

The results of the PST test reported in Table 1 reject the null of a linear model in the period up to 1979 but not in the period since 1983. These results are validated by a number of alternative tests which provide further support for the nonlinearity in the first period but not in the second one. One is the non-parametric test based on Hsiao *et al*. (2007), which employs kernel functions and smoothers using least squares cross-validation. The other nonlinear tests are the residual-based test of Cukierman and Muscatelli (2002) and the Ramsey Reset test.

3. Non-parametric Analysis

3.1 Non-parametric Procedures

We complement the previous results of the nonlinearity tests employing a non-parametric regression to determine whether there is any evidence of an obvious parametric nonlinear shape. We employ the non-parametric package of Hayfield and Racine (2009) which enables us to plot the partial derivatives of interest rates with respect to expected inflation and expected output gap (including lagged interest rate terms) without establishing specific functional forms between parameters and variables. This software is easy to implement relative to the non-parametric random field estimator of Hamilton (2001) which was a novelty employed by Kim *et al.* (2005) in this setting.

A regression equation is expressed as,

$$Y_i = m(X_i) + \varepsilon_i, \tag{5}$$

we can specify the conditional mean of Y_i as,

$$m(x) = E[Y_i|X_i = x] = \frac{\int y \cdot f(x,y)dy}{\int f(x,y)dy} = \frac{\int y \cdot f(x,y)dy}{f(x)} \tag{6}$$

where $f(x, y)$ denotes the joint density function of X_i and Y_i and $f(x)$ denotes the marginal density function of X_i. The non-parametric kernel regression is based on locally weighted averages of the equation (6)'s numerator and denominator, and it can be formulated as

$$\widehat{m}_h(x) = \frac{n^{-1}\sum_{i=1}^{n} Y_i K_h(X_i - x)}{n^{-1}\sum_{i=1}^{n} K_h(X_i - x)} = n^{-1}\sum_{i=1}^{n} W_{hi}(x) \cdot Y_i \tag{7}$$

where $k_h(\cdot)$ denotes a kernel density with a bandwidth of h, and $W_{hi}(x) = \frac{n^{-1} \cdot K_h(X_i - x)}{\hat{f}_h(x)}$ are normalised weights for each Y_i value (Härdle, 1990). By employing cross validated bandwidth selection and second order Gaussian kernel we estimate the following model,

$$i_t = F(E_t\pi_{t+1},\ E_t x_{t+1},\ i_{t-1},\ i_{t-2}) + \varepsilon_t \tag{8}$$

where i_t is Federal Funds rates, $E_t\pi_{t+1}$ and $E_t x_{t+1}$ are expected inflation and expected output gap, respectively. After estimating the partial derivatives of interest rates with respect to expected inflation and expected output gap, we calculate error bounds by bootstrap with 399 replications.

3.2 Non-parametric Results: Plots of the Partial Derivatives

Figures 1, and 2 plot the gradients of the Fed's reaction function to expected inflation and expected output gap for the pre and post Volcker period; 1960:I-1979:II and 1982:IV-2008:IV. Figure 1(a) is suggestive that the response to expected inflation increased with expected inflation and is more marked when expected inflation exceeded 3%. However, since the reported bootstrap confidence gets wider as the rate of inflation increases we may not conclude that there is a nonlinear relationship between interest rates and expected inflation based solely on this graph. However, in conjunction with the formal nonlinearity tests, a parametric formulation which captures the plot form seems warranted. Figure 1(b) presents a more clear-cut suggestion. The response to the output gap was seemingly more pronounced for negative rather than positive gaps, which suggests that the Fed was more concerned about recessions than expansions in the 1960s and 1970s.

In the period since 1983 the gradient of the Fed's interest response to expected inflation shown in Figure 2(a) is essentially flat suggesting that, on average, the response to inflation in the second period is linear. The gradient of interest rates to expected output gap in Figure 2(b) decreases but the slope became relatively flatter after zero. However, as the confidence intervals of the output gap are much wider, until output gap of zero, the nonlinear pattern is clearly much less reliable and consistent with the statistical tests.

4. Nonlinear Taylor Rules

4.1 Nonlinear Taylor Rule Estimates

Based on our non-parametric figures, we formulate the US monetary policymakers' reaction function. We employ a smooth transition model to estimate the US monetary policy reaction function. In particular, we utilize the hyperbolic tangent smooth transition regression (HTSTR) model (Cukierman and Muscatelli, 2002 and 2008), namely,

$$i_t = \alpha + \beta_1 E_t\pi_{t+1} + \gamma_1 E_t x_{t+1} + \beta_2(E_t\pi_{t+1} - \pi^*)\tanh[\psi_\pi(E_t\pi_{t+1} - \pi^*)] + \gamma_2 E_t x_{t+1}\tanh[\psi_x(E_t x_{t+1})] + \rho_1 i_{t-1} + \rho_2 i_{t-2} + \varepsilon_t \tag{9}$$

where ψ denotes the degree of nonlinearity. The presence of the nonlinearity in the monetary policy can be verified by conducting the hypothesis test that β_2 or γ_2 is equal to zero. We assume ψ is 0.2, the same as in Cukierman and Muscatelli (2008), and the threshold value for inflation and output gap as 3% and zero, respectively, representing our non-parametric results. These numbers were obtained by grid search in which the model with the least information criterion, e.g. AIC, is selected. The t-statistics of the coefficients on the nonlinear variables; expected inflation (β_2) and expected output (γ_2), were obtained using a block bootstrap procedure with 999 repetitions since there is evidence of significant residual serial correlation. We choose a block size of 7 following Patton *et al.* (2009).[3]

Table 2 reports the results of the HTSTR model for the two sub periods; 1960:I-1979:II and 1982:IV-2008:IV[4,5]. We note that, in column 2, the coefficients of β_2 is well over the conventional 5% significant level based on standard t-statistics but is significant only at the 5% level based on the block bootstrap t-statistics. The other nonlinear parameter, γ_2, is

[3] We also tried the block bootstrap with block size of 4, and the results were qualitatively similar to the ones reported with block size 7.

[4] We also employed a GMM estimation, and the results were qualitatively similar to the nonlinear regression.

[5] We find only nonlinearity in expected output gap prior to 1979 using the quadratic reaction function (Surico, 2007).

strongly significant on both metrics. These results imply that the Fed's reaction function is nonlinear with respect to expected inflation and expected output in the former period. In particular, the positive sign on β_2 implies that the interest rate response to inflation gets stronger as inflation rises. On the other hand, the Fed response to a negative output gap is more aggressive than to a positive gap as evidenced by the negative coefficient of γ_2. The hyperbolic functions for these values are plotted in Figure 3. The changing slope is indicative of high inflation and recession avoidance preferences of the Fed (Cukierman and Muscatelli 2008, and Surico 2007). These findings are in line with the non-parametric evidence presented in Figure 1 above.

Finally, the results in the later period shown in column 3 indicate that the US monetary authority followed a linear Taylor rule, which is compatible with the findings of Kim *et al.* (2005) and Surico (2007).

5. Conclusions

Reported results on the relationship between interest rates and expected inflation and expected output gap in post-war US data are contradictory. Employing the recently proposed test for nonlinear causality of Péguin-Feissolle, Strikholm and Teräsvirta (2008) (PST) and also the relatively easy to implement non-parametric package of Hayfield and Racine (2009) we re-examine the nonlinearity in the US monetary policy rule over the periods 1960:I-1979:II and 1982:IV-2008:IV. Results of the PST and other linearity tests strongly suggest a nonlinear relationship in the period up to 1979 and a linear relationship in the period since 1982. The hyperbolic tangent smooth transition regression (HTSTR) model of Cukierman and Muscatelli (2002 and 2008) appears to capture the nonlinearity in the first period, where the Fed seems to react more aggressively the higher the inflation rate and has a more aggressive response to a recession rather than to a boom. Meanwhile, the promising result using the PST test in the US nonlinear monetary policy suggests further potential applications in areas such as the relationship between real exchange rates and its equilibrium determinants where this type of nonlinearity is theoretically plausible.

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Table 1. Test of Nonlinearity

	1960:I-1979:II		1982:IV-2008:IV	
Granger Non-causality Nonlinearity-PST test[a]	$\delta_i = \tau_j = 0$ 8.71(0.00)*		$\delta_i = \tau_j = 0$ 1.30(0.25)	
Non-parametric test (Hsiao *et al.*, 2007)	2.66(0.00)*		0.29(0.16)	
Cukierman and Muscatelli (2002)[b]	$\delta_1 = 0$	$\delta_2 = \delta_3 = 0$	$\delta_1 = 0$	$\delta_2 = \delta_3 = 0$
$z_{td} = inflation$	2.76(0.04)*	3.88(0.00)*	1.49(0.21)	1.81(0.09)
$z_{td} = output\ gap$	0.42(0.80)	3.94(0.00)*	0.25(0.91)	0.27(0.97)
Ramsey Reset(2)	10.04(0.00)*		2.39(0.10)	

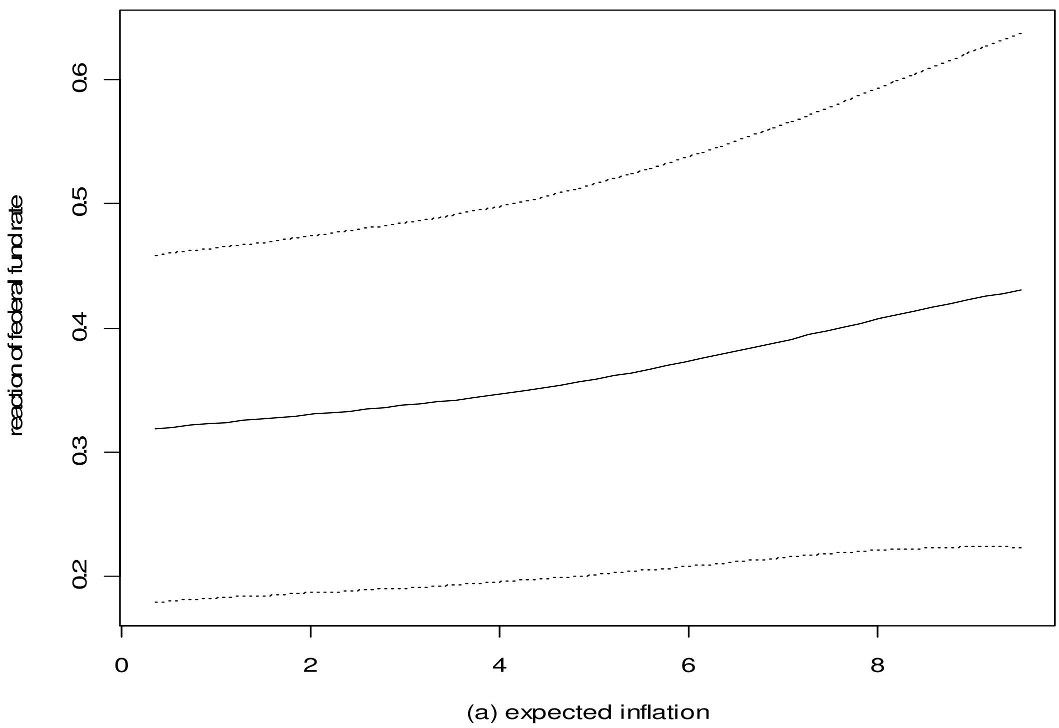
Notes: a. $i_t = \alpha + \beta E_t\pi_{(\{t+1\}-1)} + \gamma\, E_t x_{(\{t+1\}-1)} + \rho_1 i_{t-1} + \rho_2 i_{t-2} + \delta_1 i_{t-1} i_{t-2} + \delta_2 i_{t-1} i_{t-2} E_t\pi_{(\{t+1\}-1)}$
$+\delta_3 i_{t-1} i_{t-2} E_t\, x_{(\{t+1\}-1)} + \delta_4 i_{t-1} i_{t-2} E_t\pi_{(\{t+1\}-1)} E_t\pi_{(\{t+1\}-2)}$
$+ \delta_5 i_{t-1} i_{t-2} E_t\, x_{(\{t+1\}-1)} E_t\, x_{(\{t+1\}-2)} + \tau_1 E_t\pi_{(\{t+1\}-1)} E_t\, \pi_{(\{t+1\}-2)} +$
$+\tau_2 E_t\, x_{(\{t+1\}-1)} E_t\, x_{(\{t+1\}-2)} + \tau_3 E_t\pi_{(\{t+1\}-1)} E_t\pi_{(\{t+1\}-2)} E_t\, x_{(\{t+1\}-1)} E_t\, x_{(\{t+1\}-2)} + \varepsilon_t,$
$H_0 : \delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_5 = \tau_1 = \tau_2 = \tau_3 = 0.$
b. $\hat{\varepsilon}_t = \delta'_0 X_t + \delta'_1 X_t z_{td} + \delta'_2 X_t (z_{td})^2 + \delta'_3 X_t (z_{td})^3, \delta_1 = 0, \delta_2 = \delta_3 = 0$
where z_{td} is the transition variable.
P-values are in parenthesis. An asterisk (*) indicates that the null hypothesis is rejected at 5% level.

Table 2. Results of Hyperbolic Tangent Smooth Transition Regression

	1960:I-1979:II	1982:IV-2008:IV
α	0.806	-0.177
	(0.196)*	(0.335)
β_1	0.206	0.253
	(0.081)*	(0.103)*
γ_1	0.247	0.107
	(0.061)*	(0.041)*
β_2	0.272	-0.040
	(0.155)	(0.529)
	[0.049]*	[0.455]
γ_2	-0.231	-0.081
	(0.117)*	(0.054)
	[0.026]*	[0.106]
ρ_1	0.883	1.327
	(0.113)*	(0.138)*
ρ_2	-0.247	-0.406
	(0.110)*	(0.128)*
$\bar{R}^2$	0.903	0.965
BG(4)	4.45(0.00)	1.76(0.14)
ARCH(1)	17.32(0.00)	6.62(0.01)
JB	37.71(0.00)	8.09(0.02)

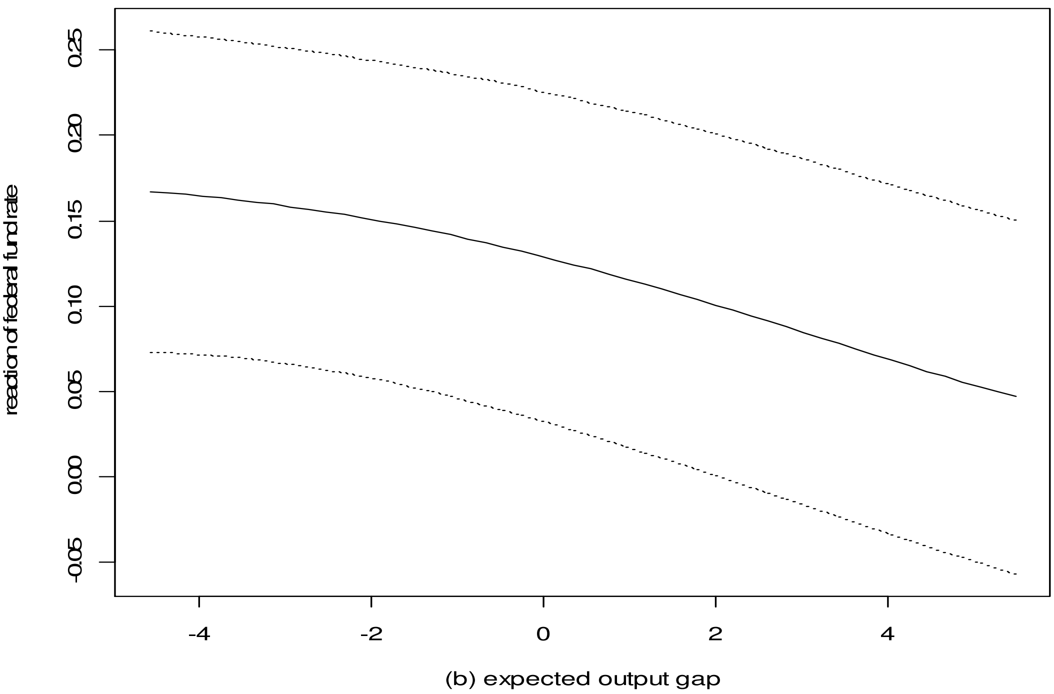
Note : Newey-West standard errors are in parenthesis and P-values for Block bootstrap errors are in brackets. An asterisk (*) denotes significance at 5%. BG(4) is the Breuch-Godfrey LM test for serial correlation up to order 4 and associated p-value. ARCH(1) is the F-statistics and associated P-value for Arch of order 1. JB is the Jarque-Bera test for normality and associated P-value.

Figure 1. The Gradients of the Fed's Reaction Function (1960:I-1979:II)



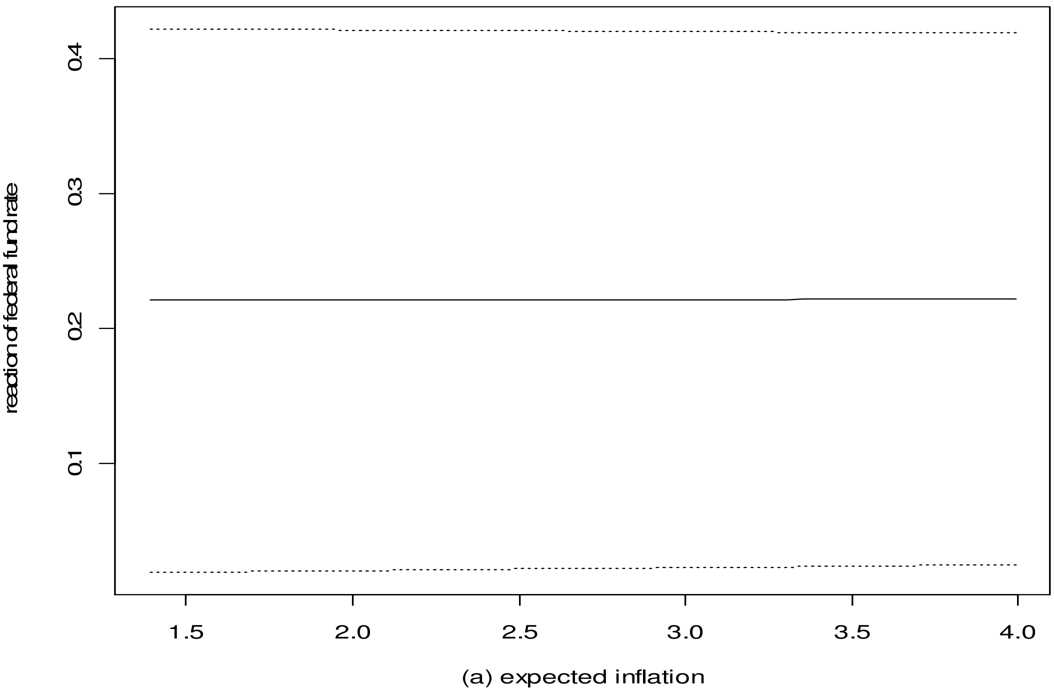
Note : The solid line represent the gradients of the Fed's reaction function and the dotted lines are error bounds calculated by bootstrap resampling 399 times.

Figure 1. The Gradients of the Fed's Reaction Function (1960:I-1979:II, Continued)



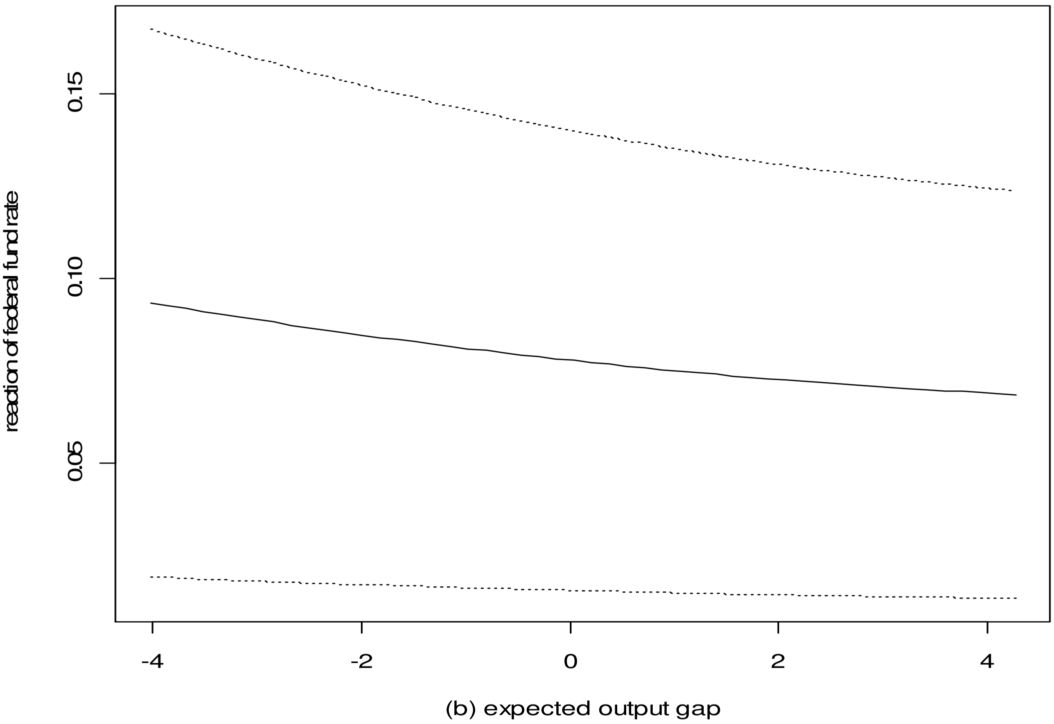
Note : The solid line represent the gradients of the Fed's reaction function and the dotted lines are error bounds calculated by bootstrap resampling 399 times.

Figure 2. The Gradients of the Fed's Reaction Function (1982:IV-2008: IV)



Note : The solid lines represent the gradients of the Fed's reaction function and the dotted lines are error bounds calculated by bootstrap resampling 399 times.

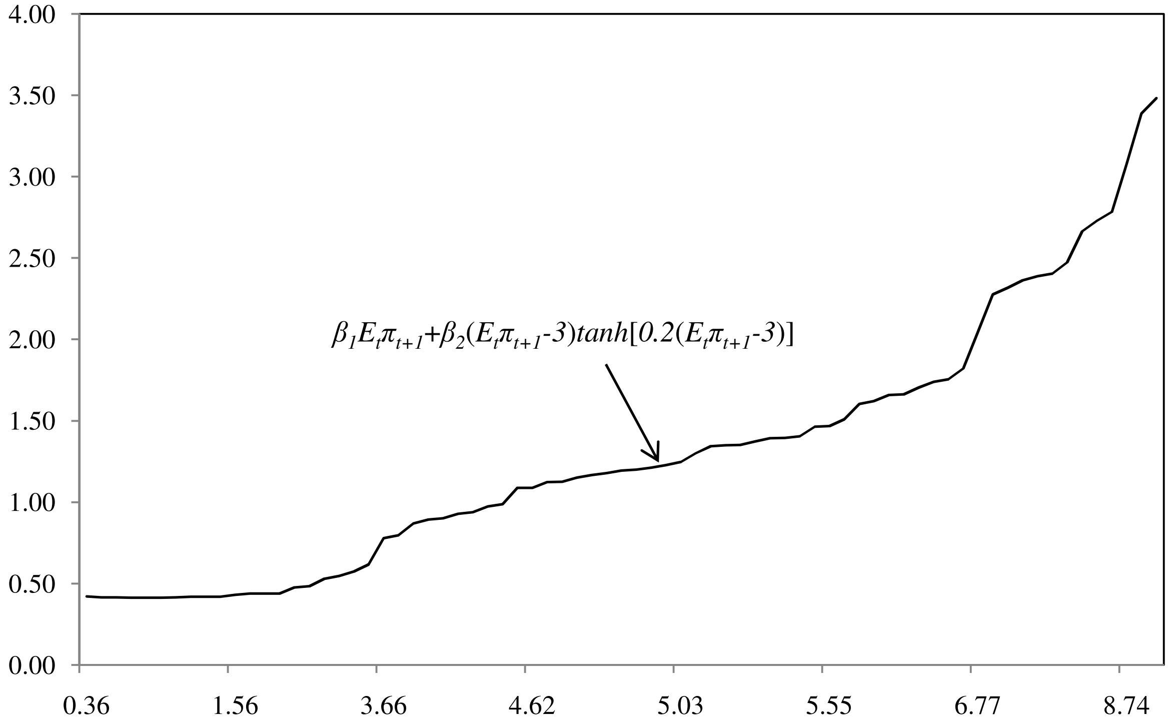
Figure 2. The Gradients of the Fed's Reaction Function (1982:IV-2008: IV, Continued)



Note : The solid lines represent the gradients of the Fed's reaction function and the dotted lines are error bounds calculated by bootstrap resampling 399 times.

Figure 3. The Hyperbolic Tangent Function (1960:I-1979:II)

(a) Expected inflation



(b) Expected output