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Large-scale portfolios using realized covariance matrix: evidence from the Japanese stock market

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Abstract

This paper examines effects of realized covariance matrix estimators based on high-frequency data on large-scale minimum-variance equity portfolio optimization. The main results are: (i) the realized covariance matrix estimators yield a lower standard deviation of large-scale portfolio returns than Bayesian shrinkage estimators based on monthly and daily historical returns; (ii) gains to switching to strategies using the realized covariance matrix estimators are higher for an investor with higher relative risk aversion; and (iii) the better portfolio performance of the realized covariance approach implied by ex-post return per unit of risk and switching fees seems to be robust to the level of transaction costs.

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1 Introduction

Measuring and controlling the risk of assets are important factors in portfolio management, together with option pricing and value-at-risk analysis. In the finance literature, risk is usually indicated by the variability of asset returns. Thus, estimating their covariance matrix plays a key role for determining the portfolio allocation, for example, using mean-variance portfolio optimization based on the investment theory of Markowitz (1952). Using high-frequency financial data, nonparametric variance and covariance measures have attracted the attention of financial econometricians. The measures are called realized variance and covariance, which are constructed by summing outer-products of intraday return data. They potentially provide very accurate estimates of the underlying quadratic variation and covariation.

There are some earlier studies for evaluating the economic benefit of the realized covariance approach in the context of investment decisions. Fleming et al. (2003) consider that a risk-averse investor uses conditional mean-variance analysis to allocate funds composed of three assets: S&P 500 futures, Treasury bond futures, and gold futures. They empirically show that the economic gains yielded by rolling covariance matrix estimators, based on intraday returns, are economically large. Bandi et al. (2008) and Pooter et al. (2008) examine the effect of the realized covariance estimators that are constructed under optimal sampling frequency on optimized portfolios using three stocks and 78 stocks in S&P 100 index constituents, respectively.

While these papers examine the performance of small- or medium-scale portfolios, this paper measures the economic benefit of large-scale minimum variance equity portfolio optimized using the realized covariance approach. For the analysis of a large-scale portfolio, Chan et al. (1999) construct a portfolio made up of 250 stocks randomly selected from domestic common stock issues on the New York Stock Exchange and the American Stock Exchange. They show that the large-scale portfolio optimization using a factor model with the monthly historical returns is helpful for risk control. Clarke et al. (2006) construct a minimum-variance portfolio consisting of the 1,000 largest market capitalization U.S. stocks using Bayesian shrinkage estimators proposed in Ledoit and Wolf (2003) with the past monthly and daily returns. Overall, there has been a shortage of empirical evidence evaluating portfolio performance using the intraday returns and different risk optimization methods. To our knowledge, there has been no research that applies the realized covariance approach to the large-scale portfolio optimization. It is important to compare the realized covariance approach with current monthly or daily return-based covariance matrix structuring methodology suited for the large-scale portfolio optimization.

We construct large-scale realized covariance matrix estimators based on the past intraday returns for 500 of the largest market capitalization JPN stocks and assess the out-of-sample performance of the minimum-variance portfolios by comparing with portfolios based on Bayesian shrinkage estimators using the past monthly and daily returns. The empirical results report the statistics, including annualized ex-post returns in excess of the risk-free rate, a standard deviation of the portfolio excess returns, the return per unit of risk (Sharpe ratio), and the net excess return with transaction costs. In addition, we estimate how much a risk-averse investor would be willing to pay to use the realized covariance estimators based on the intraday returns.

The remainder of the paper is organized as follows. Section 2 describes the data and our methodology, including the construction of the large-scale covariance matrix estimators, the minimum variance methodology, and the performance measurement criteria. Empirical results are discussed in Section 3. Section 4 concludes.

2 Data and Methodology

The data set was obtained from the Nikkei NEEDS-TICK data and the Nikkei NEEDS Financial Quest. The sample consists of intraday and daily returns from November 1999 to February 2007 and monthly returns from November 1995 to February 2007. The out-of-sample period for an examination of the portfolios is every month (76 months) from November 2000 to February 2007. Among the Tokyo Stock Exchange (TSE) 1st section-listed stocks, we pick the 500 largest market capitalization JPN stocks with no stock splits during the sample period.

2.1 Large-scale covariance matrix estimators

First, we define a monthly risk measure for period t computed with intraday data that has higher frequency than the forecasting horizon. The high-frequency data were obtained from the Nikkei NEEDS-TICK data. For each day τ, the total trading time on the TSE is 270 minutes, from 9:00 to 11:00 in the morning session and from 12:30 to 15:00 in the afternoon session. On the other hand, we cannot obtain the high-frequency data from 15:00 to 9:00 (overnight) and from 11:00 to 12:30 (lunchtime).

Let $\boldsymbol{p}$, $h^{(ON)}$, and $h^{(L)}$ denote the $(N \times 1)$ vector logarithmic prices and time intervals for overnight and lunchtime, where $N = 500$ is the number of stocks. Trading sessions on the TSE are divided into the time intervals of equal length h. Suppose that on day τ, we have the vector of overnight returns, $\boldsymbol{r}_{\tau-1+h^{(ON)}} := \boldsymbol{p}_{\tau-1+h^{(ON)}} - \boldsymbol{p}_{\tau-1}$, returns in the morning session, $\boldsymbol{r}_{\tau-1+h^{(ON)}+ih} := \boldsymbol{p}_{\tau-1+h^{(ON)}+ih} - \boldsymbol{p}_{\tau-1+h^{(ON)}+(i-1)h}$ for $i = 1, \ldots, I$, lunchtime returns $\boldsymbol{r}_{\tau-1+h^{(ON)}+Ih+h^{(L)}} := \boldsymbol{p}_{\tau-1+h^{(ON)}+Ih+h^{(L)}} - \boldsymbol{p}_{\tau-1+h^{(ON)}+Ih}$, and returns in the afternoon session, $\boldsymbol{r}_{\tau-1+h^{(ON)}+Ih+h^{(L)}+jh} := \boldsymbol{p}_{\tau-1+h^{(ON)}+Ih+h^{(L)}+jh} - \boldsymbol{p}_{\tau-1+h^{(ON)}+Ih+h^{(L)}+(j-1)h}$ for $j = 1, \ldots, J$, where $h^{(ON)} + h^{(L)} + (I + J)h = 1$. Then, the realized covariance matrix $\mathbf{V}_{t,h}$ in month t is defined as:

$$\begin{aligned}\mathbf{V}_{t,h} &= \sum_{\tau\in t} \boldsymbol{r}_{\tau-1+h^{(ON)}} \boldsymbol{r'}_{\tau-1+h^{(ON)}} + \sum_{\tau\in t}\sum_{i=1}^{I} \boldsymbol{r}_{\tau-1+h^{(ON)}+ih} \boldsymbol{r'}_{\tau-1+h^{(ON)}+ih} \\ &+ \sum_{\tau\in t} \boldsymbol{r}_{\tau-1+h^{(ON)}+Ih+h^{(L)}} \boldsymbol{r'}_{\tau-1+h^{(ON)}+Ih+h^{(L)}} \\ &+ \sum_{\tau\in t}\sum_{j=1}^{J} \boldsymbol{r}_{\tau-1+h^{(ON)}+Ih+h^{(L)}+jh} \boldsymbol{r'}_{\tau-1+h^{(ON)}+Ih+h^{(L)}+jh}.\end{aligned} \tag{1}$$

The realized covariance matrix estimator in (1) is based on the calendar time sampling, which is a well-used scheme sampled at equidistantly spaced intervals with the length of h over the trading session. However, the raw intraday data are usually unevenly spaced and non-synchronous. In this paper, the previous-tick interpolation method defined as the manipulation of taking the most recent price is applied to obtain a homogeneous time series, which is an artifact constructed from the raw intraday data. Although a linear interpolation exists (weighted average of the most recent price and the immediate price) as the alternative interpolation method, Dacorogna et al. (2001) show that the realized variance and covariance with the linear interpolation do not converge in probability to the quadratic variance and covariance, and Barucci and Renó (2002) also show the bias by the linear interpolation using Monte Carlo simulation.

The realized variance and covariance measures constructed in (1) have the potential to provide very accurate estimates of the underlying quadratic variations and covariations. However, these measures have been shown to be sensitive to market microstructure effects that can be induced by various market frictions such as the discreteness of price changes, bid-ask bounces, inter alia.[1] When we estimate the underlying quadratic variations and covariations of the stocks from high-frequency observations, it is necessary to use a sufficiently large sample size while we avoid the market microstructure effects because the realized covariance estimator with finer high-frequency data may have larger bias and variance. In our analysis, we calculate the realized covariance matrix based on five-minute returns ($I + J = 54$) because realized variance and covariance plots for every sampling frequency showed the estimates from under five minutes to thirty minutes were comparatively stable.[2] In addition, the average of realized covariance matrices using intraday returns from the preceding six and twelve months

[1]For example, the growing literature on market microstructure provides important insights from early studies including Roll (1984), who derives a simple estimator of the bid-ask spread based on the negative autocovariance of returns. Harris (1990) examines the rounding effects emanating from the discreteness of transaction prices. Ubukata and Oya (2009) analysis a dependence of the market microstructure effects.

[2]Recent literature on integrated variance and covariance estimation with market microstructure effects have been developed by Zhou (1996), Zhang et al. (2005), Zhang (2006), Hansen and Lunde (2006), Voev and Lunde (2007), and Barndorff-Nielsen et al. (2008), inter alia. Also, the optimal frequency based on the minimization of the mean squared errors has been proposed by Bandi and Russell (2008).

are constructed for the portfolio optimization as follows:

$$\mathbf{V}_{t,h}^{(6)} = \frac{1}{6}\sum_{k=0}^{5}\mathbf{V}_{t-k,h}, \quad \mathbf{V}_{t,h}^{(12)} = \frac{1}{12}\sum_{k=0}^{11}\mathbf{V}_{t-k,h}. \tag{2}$$

The simple average realized covariance matrix estimators $\mathbf{V}_{t,h}^{(6)}$ and $\mathbf{V}_{t,h}^{(12)}$ are input to the optimization routine.

Second, we construct the covariance matrix using monthly and daily closing prices obtained from the Nikkei NEEDS Financial Quest. The standard approach is, for example, to use the sample covariance matrix estimators with the past five years of monthly returns or the past year of daily returns. However, it is widely known that the approach for the large-scale optimized portfolio creates some problems in which the sample covariance matrix may be singular (noninvertible) and estimation outliers can dominate the optimized portfolio. Ledoit and Wolf (2003, 2004) show that their shrinkage estimators of the covariance matrix improve the portfolio performance over the sample covariance matrix estimator. Therefore, we compute the Bayesian shrinkage covariance matrix proposed in Ledoit and Wolf (2003). Let $\boldsymbol{r} = (\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N)'$ denote the $(N \times K)$ matrix of the historical returns. K represents 60 months or about 240 days for the use of the corresponding monthly and daily historical data, respectively. Then the sample covariance matrix, which is not divided by K, is calculated by $\mathbf{V} = \boldsymbol{r}\boldsymbol{r}'$. The Bayesian shrinkage covariance matrix (denote $\mathbf{V}_{BS}$) is defined as the weighted average of the two-parameter prior covariance matrix $\mathbf{V}_{prior}$ and the sample covariance matrix $\mathbf{V}$, as follows:

$$\mathbf{V}_{BS} = \lambda\mathbf{V}_{prior} + (1-\lambda)\mathbf{V}, \tag{3}$$

where a scalar shrinkage parameter λ bounded between zero and one is given by:

$$\lambda = \frac{\mathrm{SUM}[\boldsymbol{r}.^2\boldsymbol{r}'.^2] - \mathrm{SUM}[\mathbf{V}.^2]/K}{\mathrm{SUM}[(\mathbf{V}-\mathbf{V}_{prior}).^2]}, \tag{4}$$

where $.^2$ is the element-by-element squaring and $\mathrm{SUM}[\quad]$ is the sum of the matrix elements. The diagonal and off-diagonal elements in the prior covariance matrix $\mathbf{V}_{prior}$ are the average value of the diagonal elements and the $N(N-1)/2$ off-diagonal elements in the sample covariance matrix, respectively. Finally, we divide the Bayesian shrinkage covariance matrix $\mathbf{V}_{BS}$ by K and, if the calculation is based on daily historical returns, the daily Bayesian shrinkage covariance matrix is multiplied by the number of trading days per month. Thus, we input the Bayesian shrinkage covariance matrices based on monthly and daily historical returns to the optimization routine.

2.2 Minimum-variance portfolio

The portfolio optimization reduces to finding the asset weights that minimize the portfolio covolatility while aiming for a target expected return or maximize the portfolio return while targeting a certain covolatility. We conduct the different covolatility optimizations by constructing two scenarios of minimum-variance portfolios. Each month, we solve the following minimization problem:

$$\min_{\boldsymbol{w}} \left(\boldsymbol{w}'_{t+1} \boldsymbol{\Sigma}_{t+1} \boldsymbol{w}_{t+1} \right) \tag{5}$$
$$\begin{array}{lrl} \text{subject to :} & & \\ \text{Scenario 1 :} & & \boldsymbol{w}_{t+1} > \mathbf{0} \quad \text{and} \quad \boldsymbol{w}'_{t+1}\mathbf{1} = 1, \\ \text{Scenario 2 :} & \boldsymbol{w}'_{t+1}\boldsymbol{\mu}_{t+1} = \mu_p, & \boldsymbol{w}_{t+1} > \mathbf{0} \quad \text{and} \quad \boldsymbol{w}'_{t+1}\mathbf{1} = 1, \end{array}$$

where $\boldsymbol{w}_{t+1} = (w_1, w_2, \ldots, w_N)'$ is the $(N \times 1)$ vector of portfolio weights, $\mathbf{1}$ is an $(N \times 1)$ vector of ones, and $\boldsymbol{\Sigma}_{t+1}$ is the $(N \times N)$ conditional covariance matrix. We use the estimator of $\boldsymbol{\Sigma}_{t+1}$ as $\mathbf{V}_{t,h}^{(6)}$, $\mathbf{V}_{t,h}^{(12)}$, and the Bayesian shrinkage covariance estimators based on monthly and daily historical returns. For the constraints on the portfolio weights, the portfolio weights are required to be nonnegative because short selling is not generally a common practice for most investors. The total portfolio weight is assumed to be equal to one. In scenario 1, we construct the minimum-variance portfolio that minimizes risk without an expected return because we consider purely the strategy that the portfolio weights are determined by the estimators of the conditional covariance matrix. The scenario is associated with the previous study of Clark et al. (2006), which also examined large-scale minimum-variance equity portfolios that do not rely on any specific expected return. They show that the minimum-variance portfolios based on the past monthly and daily returns give higher realized returns and lower realized standard deviations than a market-capitalization weighted portfolio. We also conduct scenario 2 where the minimum-variance portfolio is determined by minimizing covolatility given a target expected return. $\boldsymbol{\mu}_{t+1}$ is set as the average monthly returns in the complete out-of-sample period from November 2000 to February 2007 and the target expected return μ_p is set to 10% (annualized).

2.3 Assessing portfolio performance

We assess the empirical out-of-sample performance of portfolios based on the different covariance matrix estimators on several grounds. The performance of the portfolios is evaluated using the ex-post realized portfolio returns over 76 months from November 2000 to February 2007. First, we compare the return per unit of risk, that is, measure the corresponding Sharpe ratios given by:

$$SR = \frac{\bar{r}_P - r_f}{\hat{\sigma}_{r_P}}, \tag{6}$$

where r_f is a risk-free rate and we use the unsecured one-month call rate to proxy the risk-free rate. $\bar{r}_P - r_f$ represents the mean of the ex-post realized portfolio returns in excess of the risk-free rate. $\hat{\sigma}_{r_P}$ is the standard deviation of the realized excess returns. The Sharpe ratio is used to characterize how well the realized return of the portfolio compensates the investor for the risk. A higher Sharpe ratio implies that a portfolio's risk-adjusted performance is better.

Second, we evaluate the economic benefit of the different covariance matrix estimators following Fleming et al. (2001, 2003). On a utility-based approach to measure the value of the portfolio's performance gains, we assume a risk-averse investor with the following quadratic utility:

$$U(r_{Pt+1}) = W_0\Big((1 + r_f + r_{Pt+1}) - \frac{\gamma}{2(1+\gamma)}(1 + r_f + r_{Pt+1})^2\Big), \tag{7}$$

where r_{Pt+1} is the portfolio return and γ is the investor's relative risk aversion. W_0 is initial wealth and is set equal to one for simplicity. Let $r_{P_1,t+1}$ and $r_{P_2,t+1}$ be the portfolio returns on the strategies using the two different covariance matrix estimators. The maximum amount Δ_γ that the investor would be willing to pay to switch from the first strategy to the second is then determined by:

$$\sum_{t=1}^{T} U(r_{P_1,t+1}) = \sum_{t=1}^{T} U(r_{P_2,t+1} - \Delta_\gamma), \tag{8}$$

where $T = 76$ is the out-of-sample period for the portfolio performance. Comparing the realized performance fees for $\mathbf{V}_{t,h}^{(6)}$ and $\mathbf{V}_{t,h}^{(12)}$ over Bayesian shrinkage covariance estimators measures the improvement due to the use of intraday data. We report the value of the switching fee Δ_γ as the annualized percentage at the relative risk aversion parameters of $\gamma = 1$ and 10.

Third, we assess the different portfolio performances that incorporate transaction costs. The transaction costs play a nontrivial role for portfolio selections because the higher turnover implies that the investor has to pay a higher cost by more active trading and, then, the net returns of the portfolio decrease. However, it is not generally easy to compute the total transaction cost, including stock trading commissions, the bid–ask spread, and the account management fee, inter alia. Following Pooter et al. (2008), we assume that transaction costs amount to the sum of absolute changes in the portfolio weights multiplied by a fixed percentage cost c as follows:

$$c_{t+1} = c \sum_{i=1}^{N} \mid w_{i,t+1} - w_{i,t} \mid, \tag{9}$$

where c_{t+1}, intuitively, represents a cost to reallocate the portfolio at the point of rebalancing. c is set to 2% and 4%, expressed in annualized percentage. Then, the net portfolio return is given by $r_{Pt+1} - r_f - c_{t+1}$. We also report a portfolio turnover as the total amount of purchases and sales over the same month divided by the total net asset value of the portfolio.

3 Empirical Results

Table 1 summarizes statistics of minimum-variance portfolios optimized under the conditions in scenario 1 (without expected return input). The panels A, B and C show the statistics where the level of annualized transaction costs c take 0%, 2% and 4%, respectively. For each panel, the first row contains the performance for a simple diversification strategy that involves no optimization; namely, the 500 stocks value-weighted portfolio composed of the same stocks as the others. The 500 stocks value-weighted portfolio might be regarded as the market portfolio because its realized returns are highly correlated with those of the Tokyo stock price index (TOPIX), which is a stock market index based on the total number of shares, tracking the TSE 1st section-listed stocks. The Sharpe ratio of the market portfolio is the lowest of all due to the lowest mean return in excess of the risk-free rate and the highest standard deviation of the portfolio excess returns. This result implies that to conduct optimized procedures is very helpful to the improvement of portfolio performance although the value-weighted portfolio may be often considered as a passive benchmark portfolio.

For panel A in Table 1, we find that using two realized covariance matrix estimators $\mathbf{V}_{t,h}^{(6)}$ and $\mathbf{V}_{t,h}^{(12)}$ yields higher Sharpe ratios of 0.270 and 0.259 than the Bayesian shrinkage covariance estimators based on the past monthly and daily returns. This is because of the high reduction of the standard deviation for the estimators $\mathbf{V}_{t,h}^{(6)}$ and $\mathbf{V}_{t,h}^{(12)}$. Figure 1 plots the cumulative portfolio returns over 76 months by the estimators with monthly, daily, and intraday returns. Although the three cumulative portfolio returns tend to move similarly through the whole period, we can see that the starting cumulative returns for $\mathbf{V}_{t,h}^{(6)}$ have less volatility and their large rise and drop are relatively not seen so much. In Table 1, the returns of the realized covariance matrix estimators have a lower correlation with TOPIX and the tracking error takes a higher value than the Bayesian shrinkage covariance estimators. This means that the intraday returns-based strategy takes a more different investment style from the value-weighted strategy than the monthly or daily returns-based strategies.

In addition, the turnover of $\mathbf{V}_{t,h}^{(6)}$ and $\mathbf{V}_{t,h}^{(12)}$ takes annualized 27.699% and 19.578% values that are higher than those of the 500 stocks value-weighted portfolio and the portfolio using the Bayesian shrinkage estimator based on the monthly returns. In order to evaluate the effect of the transaction costs, on panels B and C in Table 1, we describe the statistics of minimum-variance portfolios when the two levels of transaction cost ($c = 2\%, 4\%$) defined as (9) are imposed on every monthly rebalancing. Even in the cases of $c = 2\%$ and 4%, the $\mathbf{V}_{t,h}^{(6)}$ and $\mathbf{V}_{t,h}^{(12)}$ still yield higher Sharpe ratios and lower standard deviations than the portfolio using the Bayesian shrinkage estimator based on the monthly returns. The Sharpe ratios for $\mathbf{V}_{t,h}^{(12)}$ are 0.238 and 0.218, which exceed those for $\mathbf{V}_{t,h}^{(6)}$ although $\mathbf{V}_{t,h}^{(6)}$ earns the highest Sharpe ratio in the case without transaction cost, as in panel A. It is noted that $\mathbf{V}_{t,h}^{(12)}$

Table 1: Performance of minimum-variance portfolios (scenario 1)

	Annualized Mean	Std Dev	Sharpe Ratio	Correlation with TOPIX	Turnover	Tracking Error
Panel A: $c = 0$						
500 stocks value-weighted	2.502	49.315	0.051	0.984	2.851	
Monthly	6.670	31.045	0.215	0.734	9.107	34.051
Daily	7.220	29.575	0.244	0.695	27.386	35.013
$\mathbf{V}_{t,h}^{(6)}$	7.340	27.147	0.270	0.675	27.699	36.543
$\mathbf{V}_{t,h}^{(12)}$	7.273	28.086	0.259	0.688	19.578	35.535
Panel B: $c = 2$						
500 stocks value-weighted	2.386	49.318	0.048			
Monthly	6.274	31.038	0.202			
Daily	6.485	29.636	0.219			
$\mathbf{V}_{t,h}^{(6)}$	6.427	27.089	0.237			
$\mathbf{V}_{t,h}^{(12)}$	6.692	28.067	0.238			
Panel C: $c = 4$						
500 stocks value-weighted	2.270	49.322	0.046			
Monthly	5.879	31.032	0.189			
Daily	5.751	29.703	0.194			
$\mathbf{V}_{t,h}^{(6)}$	5.513	27.034	0.204			
$\mathbf{V}_{t,h}^{(12)}$	6.110	28.050	0.218			

Note: We compute the realized returns yielded by the different strategies using the 500 stocks value-weighted rate, using the Bayesian shrinkage estimators based on monthly and daily returns “Monthly” and “Daily” and realized covariance matrix estimators based on the past six- and twelve-month intraday returns “$\mathbf{V}_{t,h}^{(6)}$” and “$\mathbf{V}_{t,h}^{(12)}$”. The table reports the average annualized return in excess of the risk-free rate, annualized standard deviation of the excess returns, Sharpe ratio, correlation with TOPIX, turnover for each strategy and tracking error from the 500 stocks value-weighted portfolio. Panels A, B and C show the statistics where the levels of transaction cost c are 0%, 2% and 4%, respectively.

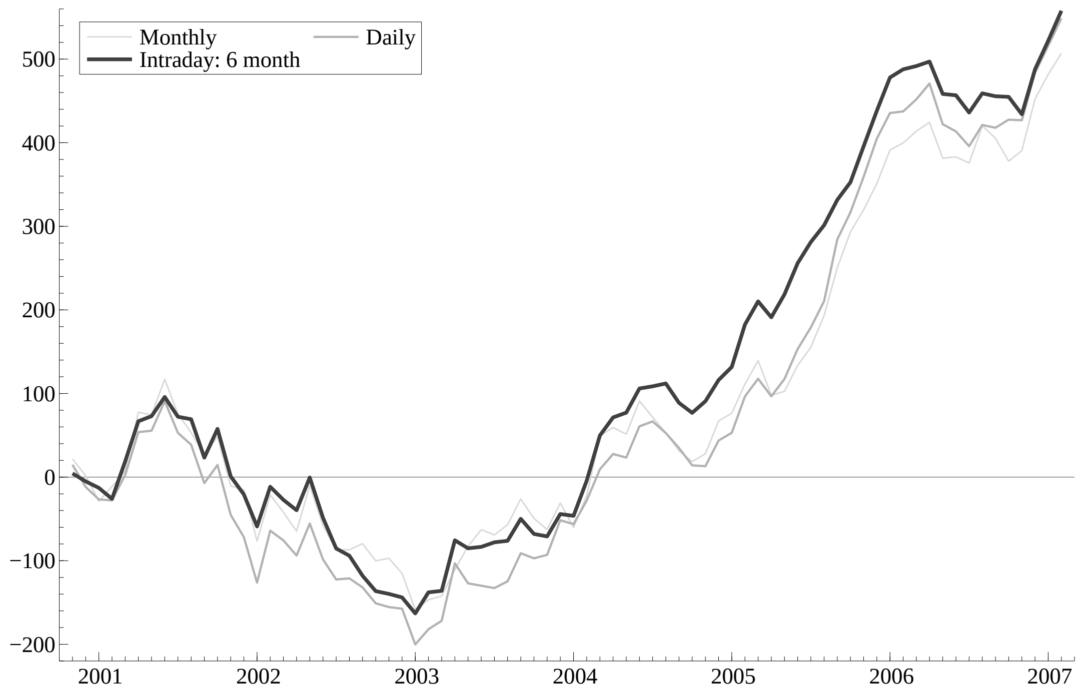

Figure 1: Cumulative portfolio returns

is the realized covariance estimator using more historical intraday data than $\mathbf{V}_{t,h}^{(6)}$. The less effect of the portfolio for $\mathbf{V}_{t,h}^{(12)}$ on the transaction costs implies that the portfolio determined by the realized covariance estimator with the longer past intraday returns would be characterized as the active portfolio with a lower turnover. We think that, for the construction of the portfolio based on intraday data, the investor can use the length of the past intraday returns according to their preference to degrees of the activity of buying and selling.

Table 2 shows the annualized fees Δ_γ that the investor with relative risk aversion parameter $\gamma = 1$ and 10 switches from the strategies using the Bayesian shrinkage estimators based on monthly and daily returns to using the realized covariance matrix estimators based on the intraday returns. A risk-averse investor pays the positive switching fees Δ_γ with $\gamma = 1$ and 10. We also find that an investor with high relative risk aversion $\gamma = 10$ would be willing to pay larger fees than an investor with low relative risk aversion $\gamma = 1$. The increase of the fees from $\gamma = 1$ to $\gamma = 10$ consistent with the investor with high relative risk aversion is preferable to switching to the portfolios using realized covariance matrix estimators that yield lower standard deviations. Therefore, the strategies of using intraday returns also make more economic gains than those based on the monthly and daily returns.

So far, we have discussed the performance of the minimum-variance portfolios optimized under no constraint for the target expected return in scenario 1. It is also important to consider the case where

Table 2: The economic gains of strategies using intraday returns (scenario 1)

		$c=0$		$c=2$		$c=4$	
		$\Delta_{\gamma=1}$	$\Delta_{\gamma=10}$	$\Delta_{\gamma=1}$	$\Delta_{\gamma=10}$	$\Delta_{\gamma=1}$	$\Delta_{\gamma=10}$
$\mathbf{V}_{t,h}^{(6)}$	vs monthly	1.863	12.734	1.355	11.998	0.846	11.264
	vs daily	0.849	9.380	0.701	8.927	0.553	8.506
$\mathbf{V}_{t,h}^{(12)}$	vs monthly	1.526	10.898	1.340	10.439	1.153	9.988
	vs daily	0.509	6.943	0.683	6.826	0.858	6.752

Note: The table represents the annualized fees Δ_γ that the investor with relative risk aversion parameter $\gamma = 1$ and 10 switches from the strategies using the Bayesian shrinkage estimators based on monthly and daily returns "Monthly" and "Daily" to using the realized covariance matrix estimators based on the past six- and twelve-month intraday returns "$\mathbf{V}_{t,h}^{(6)}$" and "$\mathbf{V}_{t,h}^{(12)}$".

Table 3: Performance of minimum-variance portfolios (scenario 2)

	Annualized Mean	Std Dev	Sharpe Ratio	Turnover
Panel A: $c = 0$				
Monthly	10.701	32.791	0.326	10.043
Daily	9.512	31.226	0.305	20.837
$\mathbf{V}_{t,h}^{(6)}$	10.577	28.495	0.371	25.053
$\mathbf{V}_{t,h}^{(12)}$	9.766	29.492	0.331	18.324
Panel B: $c = 2$				
Monthly	10.217	33.000	0.310	
Daily	8.798	31.447	0.280	
$\mathbf{V}_{t,h}^{(6)}$	9.741	28.606	0.341	
$\mathbf{V}_{t,h}^{(12)}$	9.190	29.665	0.310	
Panel C: $c = 4$				
Monthly	9.796	32.993	0.297	
Daily	8.089	31.460	0.257	
$\mathbf{V}_{t,h}^{(6)}$	8.813	28.539	0.309	
$\mathbf{V}_{t,h}^{(12)}$	8.589	29.642	0.290	

Note: The target expected return is set to annualized 10%. We compute the realized returns yielded by the different strategies using the Bayesian shrinkage estimators based on monthly and daily returns "Monthly" and "Daily" and using the realized covariance matrix estimators based on the past six- and twelve-month intraday returns "$\mathbf{V}_{t,h}^{(6)}$" and "$\mathbf{V}_{t,h}^{(12)}$". The table reports the average annualized return in excess of the risk-free rate, annualized standard deviation of the excess returns, Sharpe ratio, correlation with TOPIX and turnover for each strategy. Panels A, B and C show the statistics where the levels of transaction cost c are 0%, 2% and 4%, respectively.

Table 4: The economic gains of strategies using intraday returns (scenario 2)

		$c = 0$		$c = 2$		$c = 4$	
		$\Delta_{\gamma=1}$	$\Delta_{\gamma=10}$	$\Delta_{\gamma=1}$	$\Delta_{\gamma=10}$	$\Delta_{\gamma=1}$	$\Delta_{\gamma=10}$
$\mathbf{V}_{t,h}^{(6)}$	vs monthly	1.322	16.875	1.000	16.152	0.505	15.324
	vs daily	1.952	13.426	1.862	12.833	1.660	12.160
$\mathbf{V}_{t,h}^{(12)}$	vs monthly	0.196	14.194	0.115	13.623	-0.065	13.052
	vs daily	0.829	10.132	0.981	9.693	1.095	9.295

Note: The table represents the annualized fees Δ_γ that the investor with relative risk aversion parameter $\gamma = 1$ and 10 switches from the strategies using the Bayesian shrinkage estimators based on monthly and daily returns “Monthly” and “Daily” to using the realized covariance matrix estimators based on the past six- and twelve-month intraday returns “$\mathbf{V}_{t,h}^{(6)}$” and “$\mathbf{V}_{t,h}^{(12)}$”.

the portfolio weight is determined by minimizing a variance under a given target expected return. Table 3 reports the statistics of minimum-variance portfolios with annualized target expected return equal to 10% in scenario 2. For panel A, all the means of the annualized excess returns are around the target expected return of 10%, but the standard deviations of the portfolios based on the intraday returns are lower than those using the monthly and daily returns. In consequence, we obtain the higher Sharpe ratios of 0.371 and 0.331 for $\mathbf{V}_{t,h}^{(6)}$ and $\mathbf{V}_{t,h}^{(12)}$. For panels B and C where the transaction costs of $c = 2\%$ and 4% are imposed, the portfolio for the realized covariance estimator $\mathbf{V}_{t,h}^{(6)}$ is the most efficient in the sense of the trade-off return and risk, although the Sharp ratio for the Bayesian shrinkage estimator based on the monthly returns exceeds that for $\mathbf{V}_{t,h}^{(12)}$ because of the lowest turnover of the portfolio using the monthly returns. Table 4 also represents annualized performance fees Δ_γ to switch from the monthly or daily returns for the Bayesian shrinkage covariance estimator to the average realized covariance matrix with six and twelve months, $\mathbf{V}_{t,h}^{(6)}$ and $\mathbf{V}_{t,h}^{(12)}$. In cases without transaction costs and with $c = 2\%$, all of the performance fees take positive values. For the transaction cost of c equal to 4%, the fees from using the monthly or daily returns to using $\mathbf{V}_{t,h}^{(6)}$ are unalterably positive in cases with $\gamma = 1$ and 10. By contrast, the performance fee for $\mathbf{V}_{t,h}^{(12)}$ versus the use of monthly returns falls slightly below zero, -0.065, at the low relative risk aversion $\gamma = 1$, but the fee at the high relative risk aversion $\gamma = 10$ considerably exceeds zero, 13.052. The empirical results show that the large-scale portfolio optimization based on the realized covariance matrix estimators using the past intraday returns can yield substantial benefits in terms of risk reduction. The results are also suggestive of the better performance of the large-scale portfolio constructed by the realized covariance approach.

4 Conclusion

The objective of this paper is to examine effects of the realized covariance matrix estimators based on intraday returns on large-scale minimum-variance equity portfolio optimization. We empirically assess out-of-sample performance of portfolios with different covariance matrix estimators: the realized covariance matrix estimators and the Bayesian shrinkage estimators based on the past monthly and daily returns. The main results are: (i) the realized covariance matrix estimators using the past intraday returns yield a lower standard deviation of the large-scale portfolio returns than the Bayesian shrinkage estimators based on the monthly and daily historical returns; (ii) gains to switching to strategies using the realized covariance matrix estimators are higher for an investor with higher relative risk aversion; and (iii) the better portfolio performance of the realized covariance approach implied by ex-post returns in excess of the risk-free rate, the standard deviations of the excess returns, the return per unit of risk (Sharpe ratio) and the switching fees seems to be robust to the level of transaction costs.

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