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The construction of choice. A computational voting model.

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Abstract

Social choice models usually assume that choice is among exogenously given and non decomposable alternatives. Often, on the contrary, choice is among objects that are constructed by individuals or institutions as complex bundles made of many interdependent components. In this paper we present a model of object construction in majority voting and show that, in general, by appropriate changes of such bundles, different social outcomes may be obtained, depending upon initial conditions and agenda, intransitive cycles and median voter dominance may be made appear or disappear, and that, finally, decidability may be ensured by increasing manipulability or viceversa.

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1. The problem

In social choice theory agents choose from a set of alternatives which are both “simple” and exogenously given. Alternatives are “simple” in that they are one-dimensional points lacking an internal structure and they are exogenously given in that the pre-choice process through which they are constructed is not analyzed.

We try and model a world in which objects are first constructed as bundles of features and then submitted to voting according to a majority rule.

For instance, when called to make a decision on “what shall we do tonight”, the process of building the two elements of the choice set “going to the movies” and “going to a restaurant” might well be considered. Under this perspective, the two objects are underdetermined labels for bundles of elements (e.g. with whom, where, when, movie genre, director, type of food, etc.) that are to be bundled together in possibly different ways. It then follows that objects’ internal structure is likely to interact in non trivial ways with interdependencies and non-separabilities in individual preferences. In the “what shall we do tonight” choice setting, for instance, my preference on the “with whom” element is likely to be highly interdependent with other elements, as I may well find a given person a perfect companion for an evening at the movies but dislike his company if we finally decide to go to a restaurant. We study how building an object by bundling its features in different ways affects the selection process of a social outcome.

We show that, in general, by appropriate manipulations of the set of objects, almost every social outcome may be obtained from a given social choice rule. Such “object construction power”, that is, the power of bundling objects according to different compositions of their features appears to be stronger than agenda power in determining social outcomes.

We then show that this power of manipulation also includes the possibility of breaking or creating intransitive cycles and of overturning the median voter effect.

Finally we discuss an emerging trade-off between non manipulability and decidability. While some object constructions make social decisions less manipulable but increase the likelihood of intransitive cycles and the time required to reach a socially optimal outcome (if any), others make cycles less likely and reach a social outcome faster, but the number of locally optimal social outcomes greatly increases thus making the scope for manipulability broader.

Our work is close in spirit with context-dependent voting. In Callander and Wilson (2006) or Kahneman and Tversky (2006) context-dependency refers to the violation of the axiom of Independence of Irrelevant Alternatives. In our model we assume a different form of context dependency: preferences between two instantiations of an element (“feature” in our terminology) in general depend on the value taken by other traits. We argue why this form of non-separability is very likely to happen in our context of objects made up of interdependent features. Our work is also clearly similar to the literature on agenda power – McKelvey (1976) and Plott and Levine (1978) – but we generalize some of those results in showing that even agenda power is subject to manipulation through object construction.

2. The model

Choices are made over bundles of features. We call $F = \{f_1, f_2, \ldots, f_n\}$ the finite set of such features. Each feature takes a value out of a finite set of possible values. We assume that each feature can take the same number of values $\ell \geq 2$. Thus the set of of social outcomes is thus given by the set $X = \{x_1, x_2, \ldots, x_{\ell^n}\}$.

Objects are defined as bundles of features. Let $I = \{1, 2, \ldots, n\}$ be an index set and let an object $C_i \subseteq I$ be a non empty subset thereof. We call the size of object C_i , its cardinality $|C_i|$. We define an object scheme as a set of objects $C = \{C_1, C_2, \ldots, C_k\}$ such that $\bigcup_{i=1}^{k} C_i = I$. The size of an object scheme is defined by its largest component: $|C| = \max \{|C_1|, |C_2|, \ldots |C_k|\}$. An objects-scheme does not necessarily have to be a partition as features may belong to more than one object.

There exist h individual agents $A = \{a_1, a_2, \ldots, a_h\}$, each characterized by a transitive and complete preference relation $\succeq_i$ over the set of outcomes X.

Individual preferences are aggregated through some social decision rule $\Re$. We assume that individual preferences are expressed sincerely through majority voting (even if any social decision rule might be adopted). We write $x_j \geq^{\Re} x_i$ if x_j defeats x_i according to $\Re$.

Given an outcome $x_i \in X$ and an objects-scheme C, we call instantiation of an object $C_j \in C$, that we denote by $x_i(C_j)$, the substring of length $|C_j|$ containing the components in x_i belonging to object C_j: $x_i(C_j) = f^i_{j_1} f^i_{j_2} \ldots f^i_{j_{|C_j|}}$ for all $j_h \in C_j$

Two object instantiations can be united by means of the non commutative $\vee$ operator which produces the union of the two instantiations with the first instantiation's components where the two intersect: $x(C_j) \vee y(C_h) = z(C_j \cup C_h)$ where $z_\nu = x_\nu$ if $\nu \in C_j$ and $z_\nu = y_\nu$ otherwise. We can therefore write $x_i = x_i(C_j) \vee x_i(C_{-j})$ for any C_j.

An agenda $\alpha = C_{\alpha_1} C_{\alpha_2} \ldots C_{\alpha_k}$ over the objects scheme C is a permutation of the set of objects which states the order in which the objects are examined.

We suppose that if an initial social outcome is (randomly) given then the first object of the agenda is considered and all object instantiations are generated. At every step agents vote the status quo against a new outcome in which the components of the object under consideration are replaced by new object instantiations, whereas all other objects are kept unchanged in their initial values. The outcome obtaining the majority becomes the (new) status quo.

When all instantiations have been examined for the first object in the agenda, the same procedure is repeated for the second, third, ..., $k-th$ object in the agenda. As to the stopping rule we adopt the following: objects which have already been settled can be re-examined if new social improvements have become possible.

Given an objects-scheme $C = \{C_1, C_2, \ldots, C_k\}$, we say that an outcome x_i is a preferred neighbour of outcome x_j with respect to an object $C_h \in C$ if the following three conditions hold: 1) $x_i \geq x_j$; 2) $x_i^\nu = x_j^\nu \; \forall \nu \notin C_h$; 3) $x_i \neq x_j$.

Conditions 2 and 3 require that the two outcomes differ only in components belonging to object C_h. According to the definition, a neighbour can be reached from a given outcome by voting on a single object.

We call $H_i(x, C_i)$ the set of preferred neighbours of an outcome x for object C_i.

A path $P(x_i, C)$ from an outcome x_i and for an objects-scheme C is a sequence, starting from x_i, of preferred neighbours: $P(x_i, C) = x_i, x_{i+1}, x_{i+2}, \ldots$ with $x_{i+m+1} \in H(x_{i+m}, C)$.

An outcome x_j is reachable from another outcome x_i and for objects-scheme C if there exists a path $P(x_i, C)$ such that $x_j \in P(x_i, C)$.

A path can end up either in a social (local) optimum, i.e. an outcome which does not have any preferred neighbour, or in a cycle among a set of outcomes which are preferred neighbours to each other. The latter is the well-known case of intransitive social preferences.

The set of best neighbours $B_i(x, C_i) \subseteq H_i(x, C_i)$ of an outcome x for object C_i is the set of the socially most preferred outcomes in the set of neighbours: $B_i(x, C_i) = \{y \in H_i(x, C_i) \text{ such that } y \succ^{\Re} z \ \forall z \in H_i(x, C_i)\}$ By extension from a single object to the entire objects-scheme, we can define the set of neighbours for an objects-scheme as: $H(x, C) = \bigcup_{i=1}^{k} H_i(x, C_i)$

An outcome x is a local optimum for the objects-scheme C if there does not exist an outcome y such that $y \in H(x, C)$ and $y \succ^{\Re} x$.

Suppose outcome x_j is a local optimum for objects-scheme C, we call basin of attraction of x_j for objects-scheme C the set of all outcomes from which x_j is reachable: $\Psi(x_j, C) = \{y, \text{ such that } \exists P(y, C) \text{ with } x_j \in P(y, C)\}$.

A cycle is a set $X^0 = \{x_1^0, x_2^0, \ldots, x_j^0\}$ of outcomes such that $x_1^0 \succ^{\Re} x_2^0 \succ^{\Re} \ldots \succ^{\Re} x_j^0 \succ^{\Re} x_1^0$ and that for all $x \in X^0$, if x has a preferred neighbour $y \in H(x, C)$ then necessarily $y \in X^0$.

3. Objects, local optima and cycles

We now discuss the fundamental properties of paths in the set of outcomes which are generated by voting processes. Our algorithmic approach allows us to trace all the possible paths and characterize all possible outcomes for every initial condition.

We will only discuss only the more general case in which all objects can be always re-examined until no further social improvement becomes possible.

We show that, in general, social outcomes depend upon the adopted objects scheme and that by appropriately modifying it one can obtain different social outcomes or even the appearance or disappearance of intransitive cycles.

We first show that, in general, different objects-schemes can produce different social outcomes.

Consider first a very simple example in which 5 agents have a common most preferred choice. Table 1 presents their individual preferences, ranked from the most to the least preferred outcome:

Order	Agent1	Agent2	Agent3	Agent4	Agent5
1st	011	011	011	011	011
2nd	111	000	010	101	111
3rd	000	001	001	111	000
4th	010	110	101	110	010
5th	100	010	000	100	001
6th	110	111	110	001	101
7th	101	101	111	010	110
8th	001	100	100	000	100

Table 1: Objects and social outcomes

It is easy to show that if voting is based upon the objects-scheme $C = \{\{f_1, f_2, f_3\}\}$ the only local optimum is the global one 011 whose basin of attraction is the entire set X. If instead voting is based upon the objects-scheme $C = \{\{f_1\}, \{f_2\}, \{f_3\}\}$ we have the appearance of multiple local optima and agenda-dependence. If for instance the agenda is the sequence $\{f_1\}, \{f_2\}, \{f_3\}$ then 000 is the local optimum whose basin of attraction contains half the possible initial outcomes. For instance, if we start from 110, three out of five agents will vote for changing the first component into a 0: 010 is in fact the best neighbour of 110 for object $\{f_1\}$. Then object $\{f_2\}$ is considered and again the majority (3 out of 5) decide to move to 000. Then no other change can get a majority consensus. If instead the agenda is the sequence $\{f_3\}, \{f_2\}, \{f_1\}$ it is easy to check that the same initial condition 110 will lead to the global optimum 011.

Even stronger cases may be generated where different objects-schemes produce different global optima. Table 2 presents one such example.

Rank	Agent1	Agent2	Agent3
1st	011	010	000
2nd	000	100	110
3rd	010	101	101
4th	110	011	011
5th	100	000	010
6th	101	110	100
7th	001	001	001
8th	111	111	111

Table 2: Different objects induce different global optima

In this three agents case it is easy to verify that with the objects scheme $C = \{\{f_1, f_2\}, \{f_3\}\}$ 000 is the (unique) global optimum, while with the different scheme $C = \{\{f_1\}, \{f_2, f_3\}\}$ 011 is instead the (unique) global optimum.

Both multiplicity of social outcomes and agenda-dependence appear to be linked to the specific set of objects which voting is based upon.

Another property of social decision rules is the well-known voting paradox: even in the presence of transitive individual preferences, social preferences expressed through some voting rule may be cyclical and therefore social outcomes are indeterminate. In our model this property turns out to be dependent upon the specific scheme of objects through which voting takes place. By appropriately modifying objects, cycles may in fact appear or disappear, holding the set of social outcomes and agents' preferences constant. This "possibility" result may be illustrated by means of an example which is a translation in our formalism of the standard textbook case. Consider the case of three agents and three objects with individual preferences expressed by the following Table 3:

Order	Agent 1	Agent 2	Agent 3
1st	x	y	z
2nd	y	z	x
3rd	z	x	y

Table 3: Cycles in social preferences

It is easy to verify that with these individual preferences, social preferences expressed through majority rule are intransitive and cycle among the three objects: $x \succ^{\Re} y$ and $y \succ^{\Re} z$, but $z \succ^{\Re} x$.

Suppose now that x,y,z are three-components objects which we encode according to the following mapping: $x \mapsto 000, y \mapsto 100, z \mapsto 010$. All other combinations of components are dominated by x,y and z for all agents and we suppose, for simplicity, that preferences among them are identical across agents. All in all, individual preferences are given in Table 4.

Order	Agent 1	Agent 2	Agent 3
1st	000	100	010
2nd	100	010	000
3th	010	000	100
4th	110	110	110
5th	001	001	001
6th	101	101	101
7th	011	011	011
8th	111	111	111

Table 4: Objects and intransitivity I

It is easy to verify that if voting is based upon the unique object $C = \{\{f_1, f_2, f_3\}\}$ the voting process always ends up in the cycle among x,y and z. The same happens if each component is a separate object: $C_a = \{\{f_1\}, \{f_2\}, \{f_3\}\}$.

However, if schemes $C_b = \{\{f_1\}, \{f_2, f_3\}\}$ or $C_d = \{\{f_1, f_3\}, \{f_2\}\}$ are employed, voting always produces the unique global social optimum 010 in both cases. The latter outcome is the most preferred one by agent 3, who can therefore try to have one of these schemes adopted. All other objects-schemes always determine cycles: the social outcomes 000 and 100 which are the ones most preferred by, respectively, agents 1 and 2 cannot be obtained as social optima by any set of objects with this encoding. They could however be obtained with a different encoding.

Consider the following encoding for x, y, z: $x \mapsto 100, y \mapsto 010, z \mapsto 001$ and individual preferences of table 5:

Order	Agent 1	Agent 2	Agent 3
1st	100	010	001
2nd	010	001	100
3th	001	100	010
4th	000	000	000
5th	110	110	110
6th	101	101	101
7th	011	011	011
8th	111	111	111

Table 5: Objects and intransitivity II

Once again we obtain cycles when voting is based upon the unique object $C = \{\{f_1, f_2, f_3\}\}$, if instead each component is voted as a separate object: $C = \{\{f_1\}, \{f_2\}, \{f_3\}\}$ we have three local optima: $100, 010, 001$ whose basins of attraction depend, both in size and location, upon the agenda. With the objects-scheme $C = \{\{f_1\}, \{f_2, f_3\}\}$ we have only the two local optima 100 and 010, while $C = \{\{f_1, f_3\}, \{f_2\}\}$ produces the two local optima 010 and 001 and $C = \{\{f_1, f_3\}, \{f_2\}\}$ produces the two local optima 100 and 001.

Let us recall also that if $|C|$ is the size of the objects-scheme, the number of pairwise votes needed to find an optimum or a cycle is proportional to $\ell^{|C|}$. Thus small size objects render decidability more likely not only in the sense that cycles are less likely, but also in the sense that a choice may be made in a reasonable time. However, decidability may be obtained only by increasing manipulability because smaller size objects highly increase the number of locally optimal outcomes.

4. Objects and the median voter theorem

A relatively trivial consequence of the framework outlined so far is that also the median voter theorem is weakened in a more general setting in which objects can be modified by aggregating or disaggregating basic components.

By applying the framework developed so far we can easily design examples in which we do not have cycles and the median voter's most preferred policy does indeed win a pairwise majority contest for some objects-schemes but not for others, where, on the contrary, he or she might lose on all the objects.

Let us provide a simple example in which this happens. Let us suppose that some overall policy can be implemented with 8 possible levels of strength ranked from 0 (the null level) to 7 (the strongest implementation level). There are seven voters, each of whom preferring a different level, with the exception of level 0, [1] which nobody prefers. For all voters the remaining levels are ranked according to their distance from the most preferred one and in case of equal distance, the higher level is preferred to the lower. Individual preferences are summarized in Table 6:

Order	Ag1	Ag2	Ag3	Ag4	Ag5	Ag6	Ag7
1st	1	2	3	4	5	6	7
2nd	2	3	4	5	6	7	6
3rd	0	1	2	3	4	5	5
4th	3	4	5	6	7	4	4
5th	4	0	1	2	3	3	3
6th	5	5	6	7	2	2	2
7th	6	6	0	1	1	1	1
8th	7	7	7	0	0	0	0

Table 6: Median voter theorem, an example: part I

Agent 4 is the median voter, every agent has single peaked preferences and therefore level 4 is the unique social outcome of pairwise voting.

However, let us now suppose that policy levels are codified by 3 digits binary numbers:

Order	Ag1	Ag2	Ag3	Ag4	Ag5	Ag6	Ag7
1st	001	010	011	100	101	110	111
2nd	010	011	100	101	110	111	110
3rd	000	001	010	011	100	101	101
4th	011	100	101	110	111	100	100
5th	100	000	001	010	011	011	011
6th	101	101	110	111	010	010	010
7th	110	110	000	001	001	001	001
8th	111	111	111	000	000	000	000

Table 7: Median voter theorem, an example: part II

If voting is based upon the largest object $C = \{\{f_1, f_2, f_3\}\}$ the unique social optimum 100, corresponding to level 4, is again always achieved. However if each component is

[1] We omit an agent preferring level 0 in order to have an odd number of agents and a well-defined median voter.

voted as a separate object, i.e. $C = \{\{f_1\}, \{f_2\}, \{f_3\}\}$ we have two local optima: one that corresponds to the median voter's most preferred policy, i.e. 100 and the other that is exactly the opposite of the median voter's most preferred combination of components, i.e. 011. No cycles appear. Thus, with an appropriate combination of objects-schemes and initial conditions, the median voter's inexorable "democratic dictatorship" can be overturned and the median voter transformed into an outright loser of majority vote, also in the absence of any cycle.

Notice that if the number of components increases we can obtain once again an increasing number of local optima. For instance, if we build an analogous binary encoding example with 8 components, 256 possible social outcomes and 255 agents, we obtain a unique social optimum 10000000, corresponding to the median voter's most preferred outcome, if voting is based upon the objects-scheme $C = \{\{f_1, f_2, \ldots, f_8\}\}$; two opposite local optima *10000000* and 01111111 if the two objects $\{f_1, f_2, f_3, f_4\}$ and $\{f_5, f_6, f_7, f_8\}$ are used; and two additional specular local optima, 01111011 and 10000100, if every component is voted separately.

5. Objects and outcomes with random agents

An interesting is to try and measure how likely or plausible such phenomena are, that is to ask questions like: a) how many local optima are we likely to encounter? b) how different and/or distant from each other are such local optima? c) how does the number and location of local optima change with a modification of objects? d) how likely are cycles?

We simulate in fact the above described voting model for populations of randomly generated agents, i.e. agents whose order relation over the elements of the set X is totally random but always derived from transitive preferences.

In the first benchmark simulation we consider a set of 8 binary components and therefore a space of 256 outcomes, on which a population of 99 random agents vote following the majority rule. All the results we present here and below – unless otherwise specified – are averages over 1,000 repetitions of a simulation all with the same parameters but a different randomly generated population.

We have tested the following agendas:

- $\alpha_1 = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- $\alpha_2 = \{1, 2, 3, 4\}, \{5, 6, 7, 8\}$
- $\alpha_4 = \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}$
- $\alpha_8 = \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}$

Table 8 presents a summary of results:

Agenda	**No. of cases with optima**	**Average no. of social optima**	**No. of cases with cycles**	**Average cycle length**
α_1	47	1 (0)	953	39.61 (13.88)
α_2	940	3.93 (1.45)	1000*	4.67 (1.38)
α_4	1000	9.19 (2.33)	1000**	4.03 (1.09)
α_8	1000	15.66 (3.05)	318**	3.11 (0.48)

Table 8: Objects, local optima and cycles (n=8, No. agents=99, 1000 repetitions)

(* indicates that *some* cases present cycles for some initial conditions and local optima for others; ** indicates that *all* cases present cycles for some initial conditions and local optima for others; standard deviations in brackets)

The table shows that for agenda α_1, that is a single object containing all the components, we almost always have intransitive cycles and that these cycles are rather long (almost 40 different social outcomes on average). Only in about 5% of the randomly generated populations do we obtain a social optimum, which is obviously always achieved by voting based on α_1. All in all, intransitive social cycles are the rule in all but a small number of cases.

If instead we take the other extreme, i.e. agenda α_8 based on the set of finest objects, in 682 out of 1000 populations we do not observe cycles, but voting ends in a local optimum. On average there are 15.66 local optima (with standard deviation 3.05).

In the remaining 318 cases we observe that voting can end up either on a local optimum or in a cycle, depending upon the initial condition. In particular, in those cases in which we observe cycles, the latter are the outcome in – on average – 42.83 (with a large standard deviation of 32.58) out of the 256 possible starting conditions. When they appear, cycles are short, consisting on average in about 3 outcomes. Thus, cycles are not very frequent, but on the other hand, we have a considerable number of local optima, whose selection depends upon the initial condition.

With agenda α_4 we always (all 1000 repetitions) observe the coexistence of cycles and local optima in the same social decision problem, depending upon the initial condition. On average, out of the 256 initial conditions, 128.85 (standard deviation 28.26) lead to a cycle and the remaining to a local optimum. In the latter event, the average number of local optima is 9.19.

Finally, with agenda α_2 we observe 60 repetitions in which we observe only cycles for all 256 initial conditions, whereas in the 940 remaining cases cycles appear on average for 206.53 (standard deviation 28.61) initial conditions. The other initial conditions lead to one out of about 4 local optima. Also in this case cycles tend to be short, as they are made up of on average 4.67 outcomes.

To summarize, we observe a very clear trade-off between the presence of cycles and the number of local optima. When large objects are employed, cycles are very likely to occur. The likelihood rapidly drops when increasingly fine objects are employed, but at the same time the number of local optima increases. This implies that a social outcome is determined (and as already mentioned can be reached in a shorter time) but which specific social outcome strongly depends upon the specific objects-scheme employed, the agenda and the initial condition, i.e. the social outcome becomes easily manipulable by an authority with object construction power.

We have also checked whether local optima tend to concentrate in particular parts of the space, that is if, for a single repetition of the simulation, local optima are somehow similar, in the sense that they display at least for some components the same value. All tests reject this hypothesis: the distribution of local optima in the outcome space appears indistinguishable from a randomly generated one.

If we decrease the number of agents we do not observe any difference for the case of one object agenda α_1, while for finer objects we observe a slow increase in the number of local optima and a decrease in the frequency of cycles. For instance, with 9 agents and the eight finest objects (α_8), the number of local optima increases on average to 16.89 and cycles appear in 284 repetitions, and in those cases on average only 34 initial conditions lead to a cycle. With only three agents the average number of local optima is 20.01 (st. dev. 3.15) and cycles appear in 176 out of 1000 repetitions, and in the latter only for 30.52 out of 256 initial conditions. A smaller number of agents seems therefore to reduce the likelihood of cycles.

Finally we can test what happens if we decrease the number of components. Table 9 presents the results of an analogous simulations with 99 agents on a "simpler" decision problem with only four components and the three agendas: 1) $\alpha_1 = \{\{1, 2, 3, 4\}\}$; 2) $\alpha_2 = \{\{1, 2\}, \{5, 6\}\}$ and 3) $\alpha_4 = \{\{1\}, \{2\}, \{3\}, \{4\}\}$.

Agenda	**No. of cases with optima**	**Average no. of social optima**	**No. of cases with cycles**	**Average cycle length**
$\boldsymbol{\alpha_1}$	369	1 (0)	631	5.02 (1.78)
$\boldsymbol{\alpha_2}$	932	1.64 (0.69)	702*	3.87 (1.41)
$\boldsymbol{\alpha_4}$	988	9.19 (2.33)	75*	3.23 (0.79)

Table 9: Objects, local optima and cycles
(n=4, No. agents=99, 1000 repetitions)

(* indicates that *some* cases present cycles for some initial conditions and local optima for others;)

Results are in line with those of the previous table. Of course we observe a considerable decrease in the number of local optima and length of cycles due to the vast decrease of the size of the combinatorial search space. We also observe an overall decrease in the occurrence of cycles for all sets of objects.

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