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Licensing Schemes in Endogenous Entry

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Abstract

This paper examines the optimal licensing scheme when the number of licensees is determined endogenously. We demonstrate that a license holder obtains monopoly profit even if the license holder uses only a fixed fee as long as the marginal cost is constant. Furthermore, we show that under free entry of licensees, a license holder can obtain monopoly profit with any combination of a positive fixed fee and a unit royalty that satisfies a certain condition. Even if the fixed fee is regulated to be a certain level, a license holder can achieve monopoly profit by means of a unit royalty. This result is in contrast with that of a case where the number of licensees is exogenously determined.

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1 Introduction

The optimal licensing scheme has been a matter of great concern for economic researchers. A pioneering work in this regard is Kamien and Tauman (1986), who compare a unit royalty scheme and a fixed fee scheme. They conclude that a fixed fee scheme is optimal for license holders with Cournot competition of licensees. Furthermore, under a fixed fee scheme, a license holder earns monopoly profit, with only one licensee. The literature that extends Kamien and Tauman (1986) primarily focuses on the case wherein the number of licensees is given exogenously.

This paper considers a licensing scheme wherein the number of licensees is endogenously determined. A two-part tariff scheme, i.e., a combination of fixed fee and unit royalty schemes and a general demand function is considered. We show that if the marginal cost of production is constant, then a fixed fee continues to be sufficient for license holders to earn monopoly profit. More precisely, under free entry of licensees, a license holder can obtain monopoly profit with any combination of a positive fixed fee and a unit royalty that satisfies a certain condition. Our result shows that even if the demand function is in a general form and the number of licensees is endogenously determined, Kamien and Tauman's main theorem is robust, i.e., a fixed fee scheme is sufficient for a license holder to earn monopoly profit. This is in a sharp contrast with the result of de Meza (1986), who shows that a combination of fixed fee and positive unit royalty schemes is necessary for earning monopoly profit if the marginal cost of production is increasing, not constant.[1] Furthermore, as an extension of our basic model, we consider the case where the fixed fee is regulated to be a certain level; we show that the optimal unit royalty with a regulated fixed fee enables license holders to obtain monopoly profit even in this case.

In this paper, we extend a standard patent licensing model to the endogenous entry environment. In the model, a license holder offers a licensing scheme for downstream firms (e.g., producers). If downstream firms buy licenses, then they employ the technology in their production processes. Downstream firms engage in quantity-setting competition. The license holder's technology is essential for the operation of each of the downstream firm's businesses in the industry. Our model generalizes Kamien and Tauman (1986) in the following two aspects.[2] Kamien and Tauman (1986) assume that the demand function is linear and that the license scheme is a fixed fee or a unit royalty. We consider a model with a general demand function and a general license scheme that combines the fixed fee as well as unit royalty.

[1]de Meza (1986) considers endogenous entry of licensees. See Table 1 for main assumptions made in de Meza (1986).

[2]The situation wherein it is essential for all downstream firms to possess a license holder's technology is considered. These downstream firms do not possess any alternative (or old) technology for manufacturing products. This represents a simplified situation, which is termed as "drastic innovation" by Arrow (1962). He defines that an innovation is drastic if the monopoly price under the new technology does not exceed the price in perfect competition under the alternative technology. Even if alternative technology with a modestly higher unit production cost is incorporated in our model for replicating the non-drastic innovation model, our qualitative analysis remains unchanged.

Table 1: Main Assumptions

	Kamien and Tauman (1986)	de Meza (1986)	Sen and Tauman (2007)	This Paper
Number of Licensees	exogenous	endogenous	exogenous	endogenous
Licensing Scheme	fixed fee or unit royalty	two-part tariff (unit royalty and fixed fee)	two-part tariff (unit royalty and auction)	two-part tariff (unit royalty and fixed fee)
Cost Function	constant marginal cost	increasing marginal cost	constant marginal cost	constant marginal cost
Demand Function	linear	linear	linear	general

Following Kamien and Tauman (1986), the optimal license scheme for a license holder has been considered in numerous papers. Kamien (1992) provides a useful survey on this topic. The existing literature primarily focuses on comparing licensing by means of a unit royalty and a fixed fee.[3] Moreover, Sen and Tauman (2007) explore the optimal combination of a fixed fee and a unit royalty. In their model, a fixed fee is determined by auction; thus, the fixed fee is the winning bid of licensees. They show that as claimed by Kamien and Tauman (1986), a fixed fee scheme is sufficient for monopoly profit and that the optimal number of licensees is exactly one under a linear demand function.[4] Table 1 is provided in order to elucidate the differences among the main assumptions of this paper and those of Kamien and Tauman (1986), de Meza (1986), and Sen and Tauman (2007).[5]

The rest of this paper is organized as follows: Section 2 introduces our model and Section 3 presents the result. Section 4 discusses the case wherein the fixed fee is regulated. Section 5 provides concluding remarks.

[3] Recently, Ino (2010) extends Kamien and Tauman (1986) by considering the general cost function and the general demand function. He compares the unit royalty scheme and the fixed fee scheme with a fixed number of licensees. He shows that a unit royalty scheme is superior to a fixed fee scheme when the downstream market is sufficiently competitive.

[4] Sen and Tauman (2007) also consider a case wherein the license holders may produce goods themselves. See also Wang (1998) for further discussion on so-called incumbent license holders, who sell a license as well as produce goods.

[5] Note that since de Meza (1986) proves his proposition using only a figure, the assumptions of his model are deduced from the figure.

2 The Model

Consider a license holder who has innovative technology and n of identical licensees that produce and sell goods in the retail market. The license holder offers a licensing scheme (w, f) to all licensees where $w \in \mathbb{R}$ is a unit royalty, and $f \in \mathbb{R}_+$ is a fixed fee. Only a positive fixed fee is assumed in this study. If $w = 0$ in an optimal contract, a license holder selects a fixed fee scheme. On the other hand, if in an optimal contract contains both w and f, a license holder selects a two-part tariff scheme.[6] Licensee i $(= 1, ..., n)$ requires the technology of a license holder to produce its product. We assume that all licensees are identical and produce homogenous goods. If a licensee buys a license, then it can produce a product with marginal cost c. These identical licensees compete in quantity.

Given a total output Q, let $P(Q)$ denote market price. We assume that the function P is differentiable and that $P'(Q) < 0$ and $P'(Q) + QP''(Q) < 0$ for all Q. These are standard assumptions and the latter one guarantees the stability of equilibrium. The cost function of licensee i is given by

$$(c + w)q_i + f.$$

where $q_i \in \mathbb{R}_+$ is the output level of licensee i.

The profit of the license holder is given by

$$\pi^L = nf + wQ.$$

The profit of licensee i $(= 1, ..., n)$ is given by:

$$\pi_i = q_i(P(Q) - w - c) - f.$$

As a benchmark, consider a case where a license holder does not sell its technology and produces goods with his own technology. Obviously, such a license holder will obtain monopoly profit, which is a solution to:

$$\max_{q_L} \quad q_L(P(q_L) - c),$$

where $q_L \in \mathbb{R}_+$ is the license holder's output. The first-order condition is $P(q_L) - c + P'q_L = 0$ and the license holder obtains a profit that is denoted by π_L^m. Hereafter, the profit equal to π_L^m is termed monopoly profit.

This game runs as follows. In the first stage, the license holder offers terms (w, f) to all its potential licensees. We assume that (w, f) is identical for all i. In the second stage, each licensee decides whether to buy a license and to enter the market. In the third stage, licensees compete in quantities.

[6]Note that a negative unit royalty is allowed in this model. A negative unit royalty implies a subsidy for production. See Liao and Sen (2005) for further discussion on negative unit royalty.

3 Result

We solve this game by backward induction. Given a license scheme (w, f), the licensees compete in quantities in the third stage. The profit maximization problem of each licensee i $(= 1, ..., n)$ is as follows:

$$\max_{q_i} q_i(P(Q) - w - c) - f. \tag{1}$$

The first-order condition of each licensees i $(= 1, ..., n)$ is given by

$$P(Q) - c - w + P'q_i = 0. \tag{2}$$

Note that since we assume that $P'(Q) + QP''(Q) < 0$, the second-order condition is satisfied.

In the second stage, licensees enter the market as long as they can obtain positive profits. Thus, we have the following zero-profit condition for each licensee i $(= 1, ..., n)$:

$$q_i(P(Q) - w - c) - f = 0. \tag{3}$$

In what follows, we focus on the symmetric equilibrium where all the licensees choose the same strategies, i.e., $q_i = q$ for all i $(= 1, ..., n)$. Now we have the equilibrium conditions in the second and third stages as follows:

$$\begin{aligned} P(Q) - c - w + P'q &= 0, & (4)\\ q(P(Q) - c - w) - f &= 0. & (5)\end{aligned}$$

Given w and f, let $q(w, f)$ and $n(w, f)$ be the solutions to (4) and (5). Therefore, from equations (4) and (5), the implicit function theorem implies the following[7]:

$$\begin{bmatrix} P'q + q^2P'' & P' + P'n + qP''n \\ q^2P' & (P - w - c) + nqP' \end{bmatrix} \begin{bmatrix} \frac{dn}{dw} \\ \frac{dq}{dw} \end{bmatrix} = \begin{bmatrix} 1 \\ q \end{bmatrix},$$

and

$$\begin{bmatrix} P'q + q^2P'' & P'n + P' + qP''n \\ q^2P' & (P - w - c) + nqP' \end{bmatrix} \begin{bmatrix} \frac{dn}{df} \\ \frac{dq}{df} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

This yields:

$$\frac{dn}{dw} = \frac{1}{q} \frac{(P - c - w - qP') - nq^2P''}{(P - c - w)(P' + qP'') - q(P')^2}, \tag{6}$$

$$\frac{dq}{dw} = \frac{-q^3P''}{(-(P - c - w)q(P' + P''q)) + q^2(P')^2}, \tag{7}$$

$$\frac{dn}{df} = \frac{P' + nqP'' + nP'}{(-(P - c - w)q(P' + P''q)) + q^2(P')^2}, \tag{8}$$

$$\frac{dq}{df} = -\frac{q(P''q + P')}{(-(P - c - w)q(P' + P''q)) + q^2(P')^2}. \tag{9}$$

[7] Since the determinant is $(P - c - w)(P' + wP'') - q(P')^2 < 0 (\neq 0)$, the implicit function theorem can be applied.

The assumptions on the demand function and the first- and second-order conditions indicate that $dn/dw < 0$, $dn/df < 0$ and $dq/df > 0$. The sign of dq/dw is indeterminate and depends on the sign of P''. Moreover, the signs of dQ/dw and dQ/df are obtained as follows:

$$\frac{dQ}{dw} = n\frac{dq}{dw} + q\frac{dn}{dw} < 0,$$
$$\frac{dQ}{df} = n\frac{dq}{df} + q\frac{dn}{df} < 0.$$

Thus, the total output is decreasing in both w and f. Let q^*, Q^*, n^*, w^* and f^* denote the equilibrium output level of each licensee, equilibrium total output, equilibrium number of licensees, equilibrium unit royalty and equilibrium fixed fee, respectively.

In the first stage, the license holder chooses a (w, f) that maximizes its own profit. Note that given (w, f, n, q), the license holder's profit is $\pi^L = n(qw + f)$. Substituting the equilibrium condition (5), we can rewrite π^L as follows:

$$\pi^L = nq(P(nq) - c).$$

Therefore, the license holder's profit maximization problem, denoted by (LP), is as follows:

(LP) $$\max_{\{w,f\}} nq(P(nq)-c).$$

Suppose that the unit royalty is given as $w = \bar{w}$. As an auxiliary step, consider a reduced problem, denoted by (LP'), as follows:

(LP') $$\max_{f} nq(P(nq)-c).$$

Using equations (8) and (9), the first-order condition of (LP') is:

$$\frac{d\pi^L}{df} = -\frac{P'(P - c + n^*q^*P')}{P'(P - c - \bar{w} - P'q^*) + P''q^*(P - c - \bar{w})} = 0.$$

Since it is assumed that $P' < 0$, we have $P - c - \bar{w} - qP' > 0$. In addition, according to the first- and second-order conditions in the third stage, the denominator, $P'(P - c - \bar{w} - q^*P') + P''q^*(P - c - \bar{w})$, is negative. Thus, we have the following equilibrium condition:

$$P(n^*q^*) - c + n^*q^*P' = 0. \tag{10}$$

Given any $\bar{w}$, the system of equations (4), (5) and (10) determines q^*, Q^*, n^*, and f^*.[8]

[8] Note that the second-order condition in the first stage is satisfied. The second-order condition with respect to f is as follows:

$$(2P' + QP'')(\frac{dQ}{df})^2 + (P - c + QP')\frac{d^2Q}{df^2} < 0,$$

where $Q = nq$. Since the first-order condition in the first stage implies $P - c + QP' = 0$, and $(dQ/df)^2 > 0$, it can be rewritten as $2P' + QP'' < 0$. Thus, since we assume $P' + QP'' < 0$ and $P' < 0$, the second-order condition is satisfied.

Because $n^* q^* = Q^*$, we can rewrite the condition (10) as follows:

$$P(Q^*) - c + Q^* P' = 0. \tag{11}$$

Equation (11) implies that the license holder obtains monopoly profit given $\bar{w}$. Monopoly profit is the maximum profit for the license holder to obtain under the consumer demand function P. Since monopoly profit is achieved when $\bar{w}$ is given, even if the license holder chooses w in addition to f, the license holder cannot increase its profit more than monopoly profit. In other words, the following equation holds in this case:

$$\max_{f} nq(P(nq) - c) = \max_{\{w,f\}} nq(P(nq) - c).$$

Therefore, two problems, (LP) and (LP'), yield the same profit. Moreover, since we take $\bar{w}$ arbitrarily, a pair of w and f which satisfies equation (11) must be a solution to (LP). Now, we have the following proposition.

Proposition 1 *When the number of licensees is determined endogenously, a license holder can earn monopoly profit by offering any combination of a unit royalty and a positive fixed fee that satisfies equation (11). Moreover, even if the license holder uses only a fixed fee, it can earn monopoly profit.*

Proposition 1 implies that the result of Kamien and Tauman (1986) is robust when the demand function is general. They show that if the demand function is linear, then a license holder can earn monopoly profit by setting the fixed fee, f, equal to the monopoly profit, and only one licensee buys the license. Here, we show that even if we consider general forms of the demand function, the license holder obtains monopoly profit. In particular, there is an equilibrium wherein the license holder only employs a fixed fee as in the case in Kamien and Tauman (1986). (Note that this is the case where the license holder chooses $w = 0$.)

de Meza (1986) considers a two-part tariff scheme and incorporates the endogenous entry of licensees. He shows that the license holder cannot achieve monopoly profit using a fixed fee scheme if the demand function is linear and the marginal cost is increasing. He argues that if the number of licensees is endogenously determined, a positive unit royalty must be combined with a fixed fee to earn monopoly profit. However, our result implies that when the marginal cost for production is constant, a fixed fee is sufficient for monopoly profit even if the demand function is in general form.[9]

Note that when alternative inferior technology is freely available, especially when the unit production cost with this inferior technology is lower than monopoly price, i.e., the non-drastic innovation case in Kamien and Tauman (1986) and in other standard literature, a similar result may be derived. In the non-drastic innovation case, the equilibrium market price is determined by the unit production cost of the alternative

[9]Note that in the analysis of de Meza (1986), the equilibrium number of licensees is implicitly assumed to be greater than one. This may be another reason why a positive unit royalty is necessary.

technology when the unit production cost is constant. Thus, as in our model above, the levels of w and f do not affect the equilibrium market price and total output. Therefore, the license holder can extract the entire industrial surplus with any combination of w and f that ensures that the equilibrium market price is equal to the unit production cost of the alternative technology.

4 Discussion

Proposition 1 implies that when a unit royalty and a fixed fee are both available, the license holder always earns monopoly profit. In what follows, we demonstrate that monopoly profit is achieved even if the level of fixed fee is regulated.

Consider a licensing scheme $(w, \bar{f})$ where w denote the unit royalty, and $\bar{f}$ denote the regulated fixed fee.[10] From the perspective of the competition policy, excessively high prices of licenses are occasionally regulated in several countries e.g., Korea. The policymaker establishes a particular level for $\bar{f}$. Subsequently, given $\bar{f}$, the equilibrium conditions in the second and third stages are derived as in Section 3. Since the fixed fee is determined by the policy, the equilibrium output level and the equilibrium number of licensees are functions of w rather than functions of (w, f). Let q^{**}, Q^{**}, n^{**}, and w^{**} denote the equilibrium output level of each licensee, equilibrium total output, equilibrium number of licensees, and equilibrium unit royalty, respectively.

In the first stage, the license holder chooses w in order to maximize its profit $\pi^L = n(qw + \bar{f})$. Substituting the zero-profit condition, π^L can be rewritten as follows:

$$\pi^L = nq(P(nq) - c). \tag{12}$$

This implies that despite the fixed fee being determined by regulation, the license holder's profit is equal to the total industrial profit as in the case of the two-part tariff scheme. Then, the license holder maximizes π^L with a unit royalty w. Using equations (6) and (7), the first-order condition is:

$$\frac{d\pi^L}{dw} = \frac{(P - c + n^{**}P'q^{**})\,(P - c - w^{**} - P'q^{**})}{P'(P - c - w^{**} - P'q^{**}) + P''q^{**}(P - c - w^{**})} = 0.$$

Thus, we have the equilibrium condition as follows:

$$P(Q^{**}) - c + Q^{**}P' = 0. \tag{13}$$

The equation (13) yields the following proposition.[11]

[10] We assume $\bar{f} > 0$.

[11] The second-order condition with respect to w is as follows:

$$(2P' + QP'')(\frac{dQ}{dw})^2 + (P - c + QP')\frac{d^2Q}{dw^2} < 0,$$

where $Q = nq$. Since the first-order condition implies that $P - c + QP' = 0$, and $(dQ/dw)^2 > 0$, it can be rewritten as $2P' + QP'' < 0$. Since we assume that $P' + QP'' < 0$ and $P' < 0$, the second-order condition is satisfied.

Proposition 2 *When the number of licensees is determined endogenously, a license holder can achieve monopoly profit with a unit royalty scheme that satisfies equation (13) despite the level of fixed fee being regulated.*

This proposition asserts that a policymaker's decision never affects the equilibrium outcome. This result contrasts with the case where the number of licensees is exogenously determined. If n is determined as a fixed number $\bar{n}$, then the profit of a license holder is $\pi_L = \bar{n}\bar{f} + \bar{n}qw$, and each licensee may obtain positive profit. Therefore, the zero-profit condition does not hold and the license holder cannot extract the entire profit from the market.[12]

Kamien and Tauman (1986) study the case of a unit royalty scheme as well; however, they assume that the number of licensees is exogenously given. In the discussion here, a unit royalty scheme is considered under endogenous entry of licensees. We show that if any fixed cost other than the fixed fee, e.g., a set-up cost, does not exist, then the license holder earns the monopoly profit.

Sen and Tauman (2007) examine the two-part tariff scheme, and show that it is optimal for a license holder to offer a fixed fee that is equal to monopoly profit and to make only one licensee accept this offer. In this discussion, even if the level of fixed fee is regulated to make it lower than monopoly profit, the license holder can achieve monopoly profit by offering an appropriate unit royalty. We can easily understand that a lower fixed fee increases the number of licensees from one. However, the equilibrium output and equilibrium market price remain unchanged and are determined as the monopoly level. A license holder obtains the monopoly profit, irrespective of the level of competition in the downstream market.

5 Concluding Remarks

We considered the licensing scheme of cost-reducing innovation where the number of licensees is endogenously determined. First, it was shown that as long as the marginal cost is constant, a fixed fee scheme is sufficient for a license holder to earn monopoly profit. Thus, with a constant marginal cost, the result of Kamien and Tauman (1986) was found to be robust. Second, we analyzed a case with a regulated fixed fee. Even if the license holders cannot choose their fixed fees, they can obtain monopoly profit with the optimal unit royalty wherein the number of licensees is endogenously determined.

This result implies that license holders do not have to determine the number of licensees that they sell to when a sufficient number of potential licensees exist. License holders are only required to determine the price of their license, and to take open access policy for their technologies. Any discrimination in the license schemes among licensees

[12] Note that the zero-profit conditions of licensees are also satisfied in the fixed fee case of Kamien and Tauman (1986). A license holder can impose the zero-profit condition on licensees by setting f equal to monopoly profit.

or the exclusivity of licenses, which are usually discussed as the anti-competitive aspects of license contracts, are not crucial contract conditions for license holders to earn monopoly profit.

Furthermore, the number of licensees, i.e., the competitiveness of the downstream market, does not affect a license holder's profit. Thus, when there is sufficient number of potential licensees under free entry, any regulations that affect the pricing of licenses cannot increase or decrease the social welfare. Policymakers must pay attention to the competitive environment of potential licensees, especially whether or not entry is free, when they consider interventions.

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