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The Cournot-Bertrand profit differential in a differentiated duopoly with unions and labour decreasing returns

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Abstract

This paper compares Cournot and Bertrand equilibria in a differentiated duopoly, total wage bill maximizing unions and labour decreasing returns. It is shown that the standard result, that equilibrium profits are always higher under Cournot, may be reversed even for a fairly low degree of product differentiation. Moreover, the presence of diminishing returns to labour tends to reinforce the mechanisms that contribute to the reversal result, making this event possible for a wider range of situations, with respect to those identified by the earlier literature.

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1 Introduction

A cornerstone result in duopoly theory is that, when goods are imperfect substitutes, firms' profits are higher under competition *à la* Cournot than *à la* Bertrand. Singh and Vives (1984) first showed such result by developing the Dixit's (1979) differentiated duopoly model with linear demand structure and exogenous (constant) marginal costs.[1] More recently, its robustness has been investigated by introducing, in the same framework of Singh and Vives (1984), a two-stage game. While in the second stage firms compete in the product market, in the first stage either sole duopolists or duopolists together with an upstream agent make choices that affect their production costs. In particular, Qiu (1997) analyzes the case in which, prior to the standard product market game, each duopolist chooses a level of cost-reducing research and development (R&D) investment and shows that the relative efficiency of Cournot and Bertrand competition depends on three factors: R&D productivity, the extent of spillovers and the degree of product market differentiation. Correa-López and Naylor (2004), instead, introduce upstream "suppliers" in the form of unions and consider a decentralized wage-bargaining game played between each firm and a firm-specific labour union. In this context, they find that, if unions are sufficiently powerful and care enough about wages, the standard result (i.e. firms' profits are higher under Cournot competition) may be reversed.

This paper strongly relates to Correa-López and Naylor (2004) (CL&N, from here onwards), but with an important departure. Whilst, following the previous literature on differentiated duopoly, they assume the presence of labour constant returns (or, in other words, constant marginal costs), we introduce labour decreasing returns, which also imply increasing marginal costs, into the analysis.[2] In particular, we show that, with labour decreasing returns, the argument by Correa-López and Naylor (2004), that the standard profit-ranking can never be reversed if unions attach equal weight to wages and employment, no longer holds true, since the "reversal result" may also

[1]With Singh and Vives's (1984, p. 456) words, "[...] profits are larger, equal, or smaller in Cournot than in Bertrand competition, according to whether the goods are substitutes, independent, or complements" (see also, among others, Okuguchi 1987, Cheng 1985, and Vives 1985).

[2]Indeed, although diminishing returns to labour feature as the most common hypothesis in microeconomic modelling (at least, with reference to the short-run), the effects that they produce in a unionised duopoly have not yet been investigated.

apply in the presence of “total wage bill-maximizing” unions.

The rest of this paper is organized as follows. In Section 2, we present the basic model, in which two firms compete in the product market by producing differentiated goods. Under Cournot and Bertrand competition, we derive equilibrium values for the key variables of interest. In Section 3, we compare Cournot and Bertrand equilibrium profits. Finally, Section 4 concludes, while in the Appendix the case with an exogenous wage is provided as a benchmark.

2 Model

Following Singh and Vives (1984), we consider a model of differentiated product market duopoly, in which each firm sets its output, given pre-determined wages, to maximize profits. The product market demand for the representative firm i is linear and given by:

$$p_i(q_i, q_j) = \alpha - \gamma q_j - q_i \tag{1}$$

where q_i and q_j are outputs by firm i and j, respectively, $\alpha > 0$ and $\gamma \in (0, 1)$ denotes the extent of product differentiation, with goods assumed to be imperfect substitutes.

Let assume that only labour input is used for production. As already discussed in the Introduction, another literature's standard assumption is that labour exhibits constant returns, which implies firms face constant marginal costs. In this paper, instead, we modify such hypothesis by introducing diminishing returns to labour. In particular, we assume the following production technology:

$$q_i = \sqrt{l_i} \tag{2}$$

where $l_i = q_i^2$ represents the number of workers employed by the firm i to produce q_i output units of variety i. The choice of such specific technology, described by the functional form of (2), allows for analytical results and also implies that firms have quadratic costs, which is a typical example of increasing costs in the literature.

Hence, the firm i's profit can be written as:

$$\pi_i = p_i q_i - w_i l_i = p_i q_i - w_i q_i^2 \tag{3}$$

where w_i is the per-worker wage paid by firm i, with $w_i < \alpha$.

Following the established literature on unionised oligopolies (e.g. CL&N, Naylor 1999, Dowrick 1989, and Horn and Wolinsky 1988), production costs (i.e. wages) are no longer assumed to be as exogenously given for firms, but they are the outcome of a strategic game previously played between each firm and a labour union.

In this paper, we consider the case in which firms' wages are fixed by (firm-specific) "monopolistic" unions, which are total wage bill-maximizing (e.g. Dowrick and Spencer 1994, Oswald 1985, and Pencavel 1985). Technically speaking, each union's utility function is given by $V_i = w_i l_i$, hence unions attach equal weight to wages and employment.[3] In particular, taking also (2) into account, the union i's utility function can be written as:

$$V_i = w_i q_i^2. \tag{4}$$

In what follows, we will study, according to the different types of product market competition, two different two-stage games. In stage 1, since both firms are unionised, unions' choices take place simultaneously across firms, with each union taking the wage of the other firm as given. In stage 2, by playing a non-cooperative oligopolistic game (which can be either Cournot-type or Bertrand-type), firms choose their levels of output and (given the technology) factor input, taking wages as determined in the prior stage. We proceed by backward induction beginning with the Cournot case.

2.1 Cournot equilibrium under labour decreasing returns

Taking (1) and (3) into account, profit-maximization under Cournot competition leads to the following firm i's best-reply function:

$$q_i(q_j) = \frac{\alpha - \gamma q_j}{2(w_i + 1)}. \tag{5}$$

As $\gamma > 0$, the best-reply functions are downward-sloping, that is, under the Cournot assumption, the product market game is played in strategic

[3]As well known, wage and employment choices in the presence of unionisation may be modelled according to different ways. In this regard, we have chosen to adopt a relatively simple structure because our aim is to provide a first analysis of the effects that labour decreasing returns produce in the study framework. Extensions to other hypotheses are left for future research.

substitutes. From (5), and its equivalent for firm j, we can obtain, for given w_i and w_j, the firm i's output as:

$$q_i(w_i, w_j) = \frac{\alpha\left[2(w_j+1)-\gamma\right]}{4(w_i+1)(w_j+1)-\gamma^2} \tag{6}$$

and, by substituting (6) in (3), the firm i's profit as:

$$\pi_i(w_i, w_j) = \frac{\alpha^2(w_i+1)[\gamma-2(w_j+1)]^2}{[4(1+w_i)(1+w_j)-\gamma^2]^2}. \tag{7}$$

By substituting (6) in (4) and maximizing with respect to w_i, we get also the following expression, which, under the assumption of a non-cooperative Cournot-Nash equilibrium in the product market, defines (for the union-firm pair i) the sub-game perfect best-reply function in relation to the wage:

$$w_i(w_j) = \frac{4(w_j+1)-\gamma^2}{4(w_j+1)}. \tag{8}$$

In symmetric sub-game perfect equilibrium $w_i = w_j = w$, hence, from (8), the equilibrium wage is given by:

$$w_C = \frac{\sqrt{4-\gamma^2}}{2} \tag{9}$$

where the subscript C recalls that it is obtained under Cournot competition in the product market.

Finally, the sub-game perfect equilibrium profit (after substitution of (9) in (7)) under Cournot competition is given by:

$$\pi_i = \pi_j = \pi_C = \frac{\alpha^2\left(2+\sqrt{4-\gamma^2}\right)}{2\left(2+\gamma+\sqrt{4-\gamma^2}\right)^2}. \tag{10}$$

2.2 Bertrand equilibrium under labour decreasing returns

We consider now the case in which the product market game is characterized by price-setting behaviour by firms, i.e. competition occurs *à la* Bertrand. From (1) and its counterpart for the firm j, we can write product demand for the firm i as:

$$q_i(p_i, p_j) = \frac{\alpha(1-\gamma) - p_i + \gamma p_j}{1-\gamma^2} \tag{11}$$

hence, using (3), the firm i's profit is given by:

$$\pi_i(p_i, p_j) = p_i \left[\frac{\alpha(1-\gamma) - p_i + \gamma p_j}{1-\gamma^2}\right] - w_i \left[\frac{\alpha(1-\gamma) - p_i + \gamma p_j}{1-\gamma^2}\right]^2. \tag{12}$$

From (12), the first-order condition for profit-maximization gives the firm's i price choice, as a function of the price chosen by firm j, as:

$$p_i(p_j) = \frac{[\alpha(1-\gamma) + \gamma p_j](2w_i + 1 - \gamma^2)}{2(w_i + 1 - \gamma^2)} \tag{13}$$

thus, for $\gamma > 0$, the Bertrand product market game is played in strategic complements. By substituting in (13) the corresponding equation for the firm j and solving for p_i, we get the Bertrand equilibrium price for given wages, w_i and w_j:

$$p_i(w_i, w_j) = \frac{\alpha(2w_i + 1 - \gamma^2)[2(w_j + 1) - \gamma(1+\gamma)]}{4(w_i+1)(w_j+1) - \gamma^2[2(w_i + w_j) + 5 - \gamma^2]}. \tag{14}$$

Hence, by substituting in (11), we get the sub-game perfect output as a function of wages, which are fixed by unions in the first stage of the game, as:

$$q_i(w_i, w_j) = \frac{\alpha[2(w_j + 1) - \gamma(1+\gamma)]}{4(w_i+1)(w_j+1) - \gamma^2[2(w_i + w_j) + 5 - \gamma^2]} \tag{15}$$

and, by using (14) and (12), the firm i's profit as:

$$\pi_i(w_i, w_j) = \frac{\alpha^2[2(w_j + 1) - \gamma(1+\gamma)]^2(w_i + 1 - \gamma^2)}{[4(w_i+1)(w_j+1) - \gamma^2[2(w_i + w_j) + 5 - \gamma^2]]^2}. \tag{16}$$

Also in the Bertrand competition case, the union's utility function is given by (4). Hence, by substituting (15) in (4), and maximizing with respect to w_i, we get the following expression:

$$w_i(w_j) = \frac{4(w_j + 1) + \gamma^2[\gamma^2 - (2w_j + 5)]}{2[2(w_j + 1) - \gamma^2]} \tag{17}$$

which defines, analogously to (8) for the Cournot case, the best-reply function in relation to the wage of the union-firm pair i. Solving for the symmetric equilibrium ($w_i = w_j = w$), from (17), we get:

$$w_B = \frac{\sqrt{4 - \gamma^2 \left(5 - \gamma^2\right)}}{2} \tag{18}$$

where the subscript B recalls that the equilibrium wage defined by (18) is obtained under Bertrand competition in the product market.

Finally, the sub-game perfect equilibrium profit (after substitution of (18) in (16)) under Bertrand competition is given by:

$$\pi_i = \pi_j = \pi_B = \frac{\alpha^2 \left[2\left(1-\gamma^2\right) + \sqrt{4 - \gamma^2 \left(5 - \gamma^2\right)}\right]}{2\left[2 + \gamma(1-\gamma) + \sqrt{4 - \gamma^2 \left(5 - \gamma^2\right)}\right]^2}. \tag{19}$$

3 Cournot-Bertrand profit differential under labour decreasing returns

In this section, we investigate if the conventional wisdom, according to which Bertrand competition yields, in equilibrium, lower profits with respect to Cournot competition, still holds in the presence of labour decreasing returns and total wage bill maximizing unions.

In particular, the Cournot-Bertrand profit differential (based on the comparison between (10) and (19)) is given by:

$$\begin{aligned}\Delta\pi =& \pi_C - \pi_B = \\ & \frac{\alpha^2\gamma^2\left(2 + \sqrt{4-\gamma^2}\right)\left[\gamma(1+\gamma) - \sqrt{4 - \gamma^2\left(5-\gamma^2\right)}\right]}{\left(2+\gamma+\sqrt{4-\gamma^2}\right)^2\left[2+\gamma(1-\gamma)+\sqrt{4-\gamma^2\left(5-\gamma^2\right)}\right]^2}\end{aligned} \tag{20}$$

from which, the following result derives.

Result 1 *In a context with labour decreasing returns (increasing quadratic costs), total wage bill maximizing unions and (imperfect) substitutes goods, profits are greater under Bertrand than under Cournot competition if, and only if, the degree of product differentiation is sufficiently low. In particular, we have that* $\Delta\pi \gtreqless 0 \Leftrightarrow \gamma \gtreqless 0.732 \equiv \overline{\gamma}$.

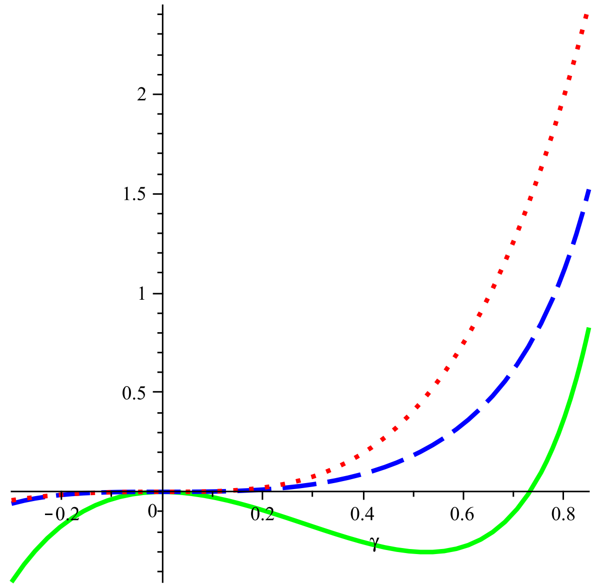

Figure 1: Cournot-Bertrand profit differentials according to γ[a]

[a]Solid green line: case i) with total wage bill maximizing unions and labour decreasing returns. Dashed blue line: case ii) with total wage bill maximizing unions and labour constant returns. Dotted red line: case iii) with exogenous wage and labour decreasing returns. Parameters: $\alpha = 1$, exogenous wage $\omega = 0.1$. For graphical reasons, profit differentials of solid green and dashed blue lines have been multiplied by 100.

Result 1 straightforwardly derives from the observation that the sign of $\Delta\pi$ depends only on the last term in squared brackets of the r.h.s. numerator. In particular, we have that:

$$\Delta\pi \gtreqless 0 \Leftrightarrow \gamma(1+\gamma) \gtreqless \sqrt{4-\gamma^2\left(5-\gamma^2\right)} \tag{21}$$

which, solving last inequality for the γ's values of interest, gives Result 1.

A graphic demonstration of Result 1 is provided in Figure 1, where the behaviour of the Cournot-Bertrand profit differential is plotted according to the degree of substitutability between goods (i.e. γ) and in relation to three different cases: i) the case formally studied in this paper, with total wage bill maximizing unions and labour decreasing returns (the green solid line); ii) the case with total wage bill maximizing unions and labour *constant* returns, studied, as special case, by CL&N[4] (the dashed blue line); and iii) the case with an *exogenous* (i.e. not fixed by unions) wage and labour decreasing

[4]In particular, in CL&N this case applies when the unions' relative bargaining power $\beta = 1$, the weight unions place on the wage $\theta = \frac{1}{2}$ and the reservation wage $\overline{w} = 0$. In particular, the dashed blue line of Figure 1 plots equation (24) at page 687 of CL&N, with β, θ and $\overline{w}$ parameters set as specified above.

returns (the dotted red line). In particular, the cases ii) and iii) are useful for comparisons with the one of interest in this paper, that is, the case i).

Figure 1 neatly illustrates that, with total wage bill maximizing unions and labour decreasing returns, it does exist a threshold value $\overline{\gamma}$, which is invariant with respect to the other economic parameters of the model, according to which profits can be lower, equal or higher with Bertrand competition according to $\gamma \lesseqgtr \overline{\gamma}$. Furthermore, since the figure also clearly shows that (for $\gamma \in (0,1)$) the "reversal result" never applies in both case ii) and case iii), we can infer that the role unions play in determining wages and the presence of labour diminishing returns are both necessary to get such a result.[5]

Although a full understanding of our finding deserves a deeper investigation, a first tentative explanation can be provide referring to the CL&N's results. In particular, CL&N establish that the possibility of the reversal result rests on two facts: a) under Cournot competition unions bargain a higher wage than under Bertrand competition, because an increase in the wage rate determines a greater decrease in employment under the latter than under the former, and this reduces the unions' incentives to settle for a higher wage when facing a Bertrand-type competitor in the product market; and b) equilibrium Cournot profits are more sensitive to the level of wages than are Bertrand profits. However, CL&N also stress that "[T]he force of these arguments is strong enough to overturn the standard result – that profits are higher under Cournot – only if unions have sufficient influence over wages and are sufficiently wage-oriented. If unions do not exert a strong influence over wages, then the standard result obtains" (CL&N, p. 692). In our case, however, the presence of diminishing returns to labour reinforces the facts a) and b), independently by the degree of unions' wage-orientation. This is because, when wages increase, *ceteris paribus*, the employment reduction

[5]In particular, the behaviour of the dashed blue line confirms, accordingly with CL&N's results, that, in the presence of constant marginal costs and total wage bill maximizing unions, the weight the latter place on wages in their utility functions is not sufficiently high to get the reversal result. On the other hand, as graphically displayed by the dotted red line, labour decreasing returns alone (without unions) are not enough for equilibrium profits to be higher under Bertrand-type competition (see also the final Appendix for a formal proof). Finally, also notice that, although the case of interest here has been restricted to substitutes goods ($\gamma > 0$), from Figure 1 (partly) emerges that, if goods are complements ($\gamma < 0$), the standard Singh and Vives's (1984) result (see fn 1) applies. This confirms also in this framework that, as emphasized by CL&N, the unionized oligopoly is not symmetric with respect to the effects of product differentiation.

is more severe under decreasing returns. Furthermore, also strategic effects, which imply Cournot equilibrium profits decrease more steeply in wages than do Bertrand equilibrium profits,[6] are magnified by the presence of diminishing returns. This produces the reversal result notwithstanding that unions are not distinctly wage-oriented.

4 Conclusion

In this paper we have investigated whether the conventional wisdom, according to which (with imperfect substitutes goods) the equilibrium profits under Cournot competition are higher than under Bertrand competition, still holds true when there are decreasing returns to labour and wages are unilaterally fixed by a total wage bill maximizing union.

It has been shown that the presence of labour decreasing returns tends to reinforce the mechanisms that contribute to the reversal result, making this event possible for a wider range of situations, with respect to those identified by the earlier literature.

Our result calls for further analyses that are deferred to future research. In particular, extensions to other hypotheses concerning wage and employment determination in the presence of unions (i.e. "right-to-manage" or efficient bargaining) deserve to be considered. Furthermore, we have not dealt with social welfare issues, which, nevertheless, may conduct to important results. Finally, whilst in this paper we have only concentrated on symmetric equilibrium, it would be particularly important and interesting to extend the analysis to asymmetric outcomes by introducing some source of heterogeneity between firms, such as different production technologies (cost functions) and/or different parameters in product market demands.

Appendix

Bertrand-Cournot profit differentials with labour decreasing returns and an exogenous wage

In this Appendix we show that, when the wage is exogenously given for firms, the reversal result *never* applies, i.e. profits are always greater under

[6] See CL&N, p. 691.

Cournot than under Bertrand competition, even if production technology exhibits decreasing returns to labour.

Let define with ω the exogenous wage rate. Firstly, consider that (in contrast with the analysis with unions in the main text) ω does not depend on: a) the type of product market competition (i.e. it is the same under both Cournot-type and Bertrand-type competition); and b) the product market parameter γ. Taking (7) and (16) into account and exploiting the symmetry hypothesis, we get the equilibrium profits under Cournot-type and Bertrand-type competition (with an exogenous wage and labour decreasing returns) as, respectively:

$$\pi_i = \pi_j = \overline{\pi}_C = \frac{\alpha^2(\omega+1)}{[(2(\omega+1)+\gamma]^2} \tag{A1}$$

$$\pi_i = \pi_j = \overline{\pi}_B = \frac{\alpha^2(\omega+1-\gamma^2)}{[(2(\omega+1)+\gamma(1-\gamma)]^2}. \tag{A2}$$

Hence, by using (A1) and (A2), we get that:

$$\Delta\overline{\pi} = \overline{\pi}_C - \overline{\pi}_B = \frac{\alpha^2\gamma^3\left[\gamma(\omega+2)+2(\omega+1)\right]}{[(2(\omega+1)+\gamma]^2\left[(2(\omega+1)+\gamma(1-\gamma)\right]^2} > 0 \tag{A3}$$

for any $\gamma > 0$.

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