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Anticipating correlations between EUAs and CERs: a Dynamic Conditional Correlation GARCH model

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Abstract

Previous literature has studied the empirical characteristics of European Union Allowances (EUAs) and Certified Emissions Reductions (CERs) time series by using vector autoregression, impulse response function, and cointegration analysis (Chevallier (2010)). This paper extends the analysis by modelling the inter-relationships between EUAs and CERs in a multivariate GARCH econometric framework, so as to reflect the dynamics of the correlations between the variables overtime. Using the DCC MGARCH model by Engle and Sheppard (2001) and Engle (2002) on daily data from March 09, 2007 to January 26, 2010, we confirm the presence of strong ARCH and GARCH effects. Besides, we provide strong empirical evidence of time-varying correlations in the range of [0.01;0.90] between EUAs and CERs that have not been considered by previous studies. Thus, our study shows that the correlations between EUAs and CERs extracted from the DCC MGARCH model appear as a useful tool to comprehend the nature of the inter-relationships between these two markets, and to reach optimal risk management, portfolio selection, and hedging as called by Engle (2009).

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1 Introduction

In his much acclaimed book *Anticipating Correlations: A New Paradigm for Risk Management*, Engle (2009) states that we must 'anticipate correlations' if we want to have optimal risk management, portfolio selection, and hedging[1]. Indeed, in the present global financial world, it appears imperative both for asset management and for risk analysis to gain a better understanding of the changing correlations between a large number of assets and even between different financial markets.

In this paper, we focus on the inter-relationships between European Union Allowances (EUAs) traded on the EU Emissions Trading Scheme (EU ETS), and Certified Emissions Reductions (CERs) arising from the Clean Development Mechanism under the Kyoto Protocol. These emissions assets have first been studied by Chevallier (2010) in a cointegrating and vector autoregressive framework, along with impulse response function analysis. The author showed that EUAs and CERs affect each other significantly through the vector autoregression model, and react quite rapidly to shocks on each other through the impulse response function analysis. Most importantly, both price series are found to be cointegrated, with EUAs leading the price discovery process in the long-term through the vector error correction mechanism.

We take this analysis one step further by modelling the daily returns of the two historical futures price series from March 09, 2007 to January 26, 2010 in the Multivariate GARCH (MGARCH) framework with dynamic conditional correlations (DCC) developed by Engle and Sheppard (2001) and Engle (2002). In our view, it appears important to document empirically the correlations between EUAs and CERs, since they represent the best proxies of the inter-relationships between respectively the European carbon market (the most developed scheme to date, see Ellerman et al. (2010)) and the 'world' price for carbon (as the CDM is the fastest growing device under the Kyoto Protocol with respect to the size of other burgeoning domestic and regional schemes, see World Bank (2010)). Therefore, this study is of direct interest for academics and professionals in the field of carbon markets.

Our study shows that the DCC MGARCH model fits well the contemporaneous relationships between the EUA ECX Futures and CER ECX Futures time series.

[1] Such forward-looking correlations are very important in risk management because the risk of a portfolio depends not on what the correlations were in the past, but on what they will be in the future. Similarly, portfolio choice depends on forecasts of asset dependence structure. Many aspects of financial planning involve hedging one asset with a collection of others. The optimal hedge will also depend upon the correlations and volatilities to be expected over the future holding period.

We conclude that there is strong evidence of time-varying correlations among the selected EUA and CER variables. Thus, the correlations between EUAs and CERs extracted from the DCC MGARCH model appear as a useful tool to comprehend the nature of the inter-relationships between these two markets, and to reach optimal risk management, portfolio selection, and hedging as called by Engle (2009).

The remainder of the paper is organized as follows. Section 2 reviews the econometric framework of the DCC MGARCH model. Section 3 presents the data used. Section 4 contains the estimations results. Section 5 concludes.

2 Review of the Dynamic Conditional Correlation Multivariate GARCH model

The main difficulty encountered with Multivariate GARCH modeling lies in finding a suitable system that describes the dynamics of the conditional variance-covariance matrix parsimoniously. Besides, the multiple GARCH equation needs to satisfy the positive definiteness of the conditional variance-covariance matrix, which is a numerically difficult problem. Finally, the number of parameters to be estimated increases very rapidly as the dimension of the time-series increases, which can take a very long time during the numerical implementation. To address these questions, we detail below one parametric formulation for the structure of the conditional covariance matrices (see Francq and Zakoian (2010) and Franses and Van Dijk (2000) for a review and some alternative models).

In this paper, the class of multivariate GARCH models examined is based on the decomposition of the conditional covariance matrix into conditional standard deviations and correlations. In such Dynamic Conditional Correlation MGARCH models (Engle and Sheppard (2001), Engle (2002)), the conditional correlation matrix is time-varying and the conditional covariance matrix may be written as follows:

$$H_t = D_t P_t D_t \tag{1}$$

where $D_t = diag(h_{1t}^{1/2}, \ldots, h_{Nt}^{1/2})$ and $P_t = [\rho_{ij,t}]$ is positive definite with $\rho_{ii} = 1, i = 1, \ldots, N$. Off-diagonal elements of the conditional covariance matrix are computed as:

$$[H_t]_{ij} = h_{it}^{1/2} h_{jt}^{1/2} \rho_{ij}, i \neq j \tag{2}$$

where $1 \leq i, j \leq N$. The conditional variances of r_{it} processes are similar to univariate GARCH(p,q) models:

$$h_t = \omega + \sum_{j=1}^{q} A_j r_{t-j}^2 + \sum_{j=1}^{p} B_j h_{t-j} \tag{3}$$

with ω a $N \times 1$ vector, A_j and B_j diagonal $N \times N$ matrices, and $r_t^2 = r_t \odot r_t$. When the conditional correlation matrix P is positive definite and the elements of ω and the diagonal elements of A_j and B_j positive, the conditional covariance H_t is positive definite.

According to Engle (2002), we introduce the following dynamic matrix process:

$$Q_t = (1 - a - b)S + a\epsilon_{t-1}\epsilon_{t-1}' + bQ_{t-1} \tag{4}$$

with a and b respectively positive and non-negative scalar parameters such that $a + b < 1$, S the unconditional correlation matrix of the standardized errors ϵ_t, and Q_0 is positive definite. To produce valid correlation matrices, Q_t needs to be re-scaled as follows:

$$P_t = (I \odot Q_t)^{-1/2} Q_t (I \odot Q_t)^{-1/2} \tag{5}$$

Having detailed the DCC MGARCH modeling, we present next the data used.

3 Data

We study the time-series of EUA and CER daily closing prices. Our study period goes from March 09, 2007 to January 26, 2010 which corresponds to a sample of 737 observations. The source of the data is the European Climate Exchange (ECX).

Figure 1 presents the daily time series of EUA Futures traded in €/ton of CO_2

on ECX. Visual inspection and standard unit root tests (ADF, PP, KPSS) suggest taking first differences to obtain a stationary time series[2]. Because the time series are non stationary, we consider the returns by taking first differences after taking logarithms. The ECX Futures are therefore presented in logreturn transformation in the bottom panel of Figure 1. Figure 2 presents the daily time series of CER Futures, also traded in €/ton of CO_2 on ECX. The same comments as above apply concerning the stationarity of the time series.

Descriptive statistics for the raw price series and logreturns may be found in Table 1. According to the Jarque-Bera test statistic, the distributional properties of the EUA and CER futures raw price series appear as non-normal. In logreturn transformation, the carbon futures are negatively skewed and since the kurtosis exceeds three, a leptokurtic distribution is indicated.

To sum up, none of the raw time-series under consideration may be approximated by the normal distribution. Besides, EUAs and CERs are found to be integrated of order 1 (*I(1)*). Therefore, both EUA and CER log-returns are considered in the econometric analysis. In the next section, we present our estimation results.

4 Estimation results

This section contains the estimation results for our modeling strategy of EUAs and CERs. We discuss first some issues concerning the estimation of the DCC MGARCH model presented in Section 2, and second we present the results obtained.

4.1 Estimation practicalities

In the DCC MGARCH model, positive definiteness of H_t in eq(1) is ensured if the conditional correlation matrix P_t is positive definite at each point in time, in addition to having well-defined conditional variances $h_{it,i=1,...,N}$. This leads to computationally demanding estimation procedures, as the correlation matrix has to be inverted for each t during every iteration.

As shown by Chevallier (2010), when selecting the adequate number of lags for vector-autoregressive modeling of EUAs and CERs, all criteria unambiguously

[2]See Chevallier (2010) for standard unit root test results applied to the time series of EUAs and CERs. These results are not reported here to conserve space and may be obtained upon request to the authors. This comment applies in the remainder of the paper.

point out an optimal lag of order 1. We follow thoroughly this approach here and choose the most parsimonious specification by setting $m = 1$ and $n = 1$ for the DCC(m,n) MGARCH model.

The BHHH algorithm (Berndt et al. (1974)) is used to produce quasi maximum likelihood parameter estimates and their corresponding asymptotic robust standard errors.

4.2 DCC MGARCH results

For the ease of presentation, we state here a simplified version[3] of the DCC(m,n) MGARCH model by Engle (2002) where it is set $m = n = 1$:

$$h_{i,t} = \omega_i + \alpha_i r_{i,t-1}^2 + \beta_i h_{i,t-1} \text{ for } i = 1, 2 \tag{6}$$

$$\begin{aligned} Q_t &= (1 - \alpha_1^* - \beta_1^*) \overline{Q} + \alpha_1^* \epsilon_{t-1} \epsilon_{t-1}' + \beta_1^* Q_{t-1} \\ R_t &= \tilde{Q}_t^{-1} Q_t \tilde{Q}_t^{-1} \end{aligned} \tag{7}$$

with $\epsilon_t = D_t^{-1} r_t$, $\epsilon_t \sim N(0, R_t)$, $\tilde{Q}_t$ a diagonal matrix containing the square root of the diagonal entries of Q_t, and $\overline{Q}_t$ the matrix of unconditional covariances. Eq(6) is a standard univariate GARCH model, and eq(7) is referred to as a DCC(1,1) model. We fit eq(6) and (7) to the time series of EUAs and CERs in logreturn transformation.

The standardized residuals of the DCC(1,1) MGARCH for EUA Futures and CER Futures logreturns are shown in Figure 3. It may be concluded that the residuals of the DCC(1,1) MGARCH model satisfy the necessary white-noise properties. As confirmed by the visual inspection of Figure 4, the autocorrelation functions of residuals and squared residuals do not exhibit autocorrelation. Results from the Ljung-Box-Pierce test confirm this first diagnostic: the p-values of the test are equal to, respectively, 0.191 and 0.674 for EUA and CER residuals. We can also look at the normal $Q - Q$ plots[4] of the standardized residuals in Figure 5. We observe only tiny deviations from the normal distribution, as for the DCC model

[3]Useful comments from an anonymous referee are gratefully acknowledged for pointing out this specification.

[4]Normal Q-Q plot stands for the quantiles of the standardized residuals plotted against the quantiles of the normal distribution.

the $Q-Q$ plots for EUAs and CERs almost lie on a straight line.

DCC(1,1) MGARCH estimates are reported in Table 2. The DCC(1,1) MGARCH model has 8 parameters, which are all statistically significant at the 1% level.

For both variables, we may remark that the level of the ARCH coefficient is quite low. The ARCH coefficient being an indicator of how news are impacting the volatility, a low value for the ARCH coefficient indicates that the variance adjustment following the arrival of new information is slow. In other words, we highlight that on carbon markets the GARCH coefficient is dominating, which means that these emissions markets exhibit high autocorrelation in the volatility process.

Next, the correlation structure of the DCC(1,1) MGARCH model has a clear interpretation: there is a non-constant interaction of the two time-series with respect to conditional correlation, and this correlation impacts current correlation with a lag of 1. This interaction effect would be neglected if EUAs and CERs were modeled in isolation, each with a univariate GARCH model.

Next, we reproduce Engle and Sheppard's (2001) test for the presence of dynamic correlation in the residuals of the DCC(1,1) MGARCH model[5]:

$$\begin{aligned} H_0 &: R_t = \overline{R} \ \ \forall t \in T \\ H_a &: vech(R_t) = vech(\overline{R}) + \beta_1 vech(R_{t-1}) + \beta_2 vech(R_{t-1}) + \ldots + \beta_p vech(R_{t-1}) \end{aligned} \tag{8}$$

The p value and χ^2 statistic testing for the dynamic correlation between EUAs and CERs are presented in the last two rows of Table 2. Under the null the constant and all of the lagged parameters in the model should be zero. Thus, we reject the null of a constant correlation in favor of a dynamic structure.

In Figure 6, we provide a visual representation of the dynamic correlations between EUAs and CERs estimated from the DCC(1,1) MGARCH model. The DCC MGARCH model thus offers an accurate description of the dynamics of the correlations between the two variables overtime. The values observed for $\rho_{EUA,CER}$ are comprised between 0.01 and 0.90. Significant peaks in the dynamics of $\rho_{EUA,CER}$ may be found during the period going from March 2007 to January 2010, which may be related to institutional developments in the respective emissions markets (see Mansanet et al. (2011) for a thorough analysis). Besides, it is worthy to re-

[5]The interested reader may refer to Engle and Sheppard's (2001) paper for a detailed description of the testing procedure.

mark that both emissions assets seem to de-correlate at certain points in time. This latter result shows that EUAs and CERs are not completely fungible to date (mainly due to their respective geographical scope and to the limitation on the import of CERs for compliance within the EU ETS). Compared to Figures 1 and 2, Figure 6 therefore offers us another view of the inter-relationships between EUAs and CERs.

5 Conclusion

During our study period, we document that the dynamic conditional correlations are quite high between EUA ECX Futures and CER ECX Futures logreturns (comprised between 0.01 and 0.90). As pointed out by Chevallier (2010), EUAs and CERs are subject to simultaneous price changes based on their respective fundamentals (supply of EUAs and import limit of CERs within the EU ETS, demand based on growth forecasts, weather forecasts and other energy markets (see Alberola et al. (2008), Chevallier (2009) and Hintermann (2010) for more details)).

Therefore, the correlation between EUAs and CERs could be used by analysts and industrial operators in order to track how these price series diverge or converge with respect to each other, in a moving institutional context (the status of the CDM post-2012 and their inclusion within the EU ETS during Phase III are still unclear at the time of writing). Hence, it seems that the dynamic conditional correlations extracted from the DCC MGARCH model could be used to reflect market participants' heterogeneous anticipations of the future evolution of the European and CDM schemes.

Finally, let us note that sometimes the correlation between EUAs and CERs converges to zero. This behavior suggests that the two time series decorrelate sometimes during our study period, maybe due to specific shocks on the EUA and CER markets. The EUA market for instance was attacked by VAT frauds from Eastern European countries towards the end of 2009 which obviously should not have any impact on the CER market (see European Commission (2009) for more details on this topic).

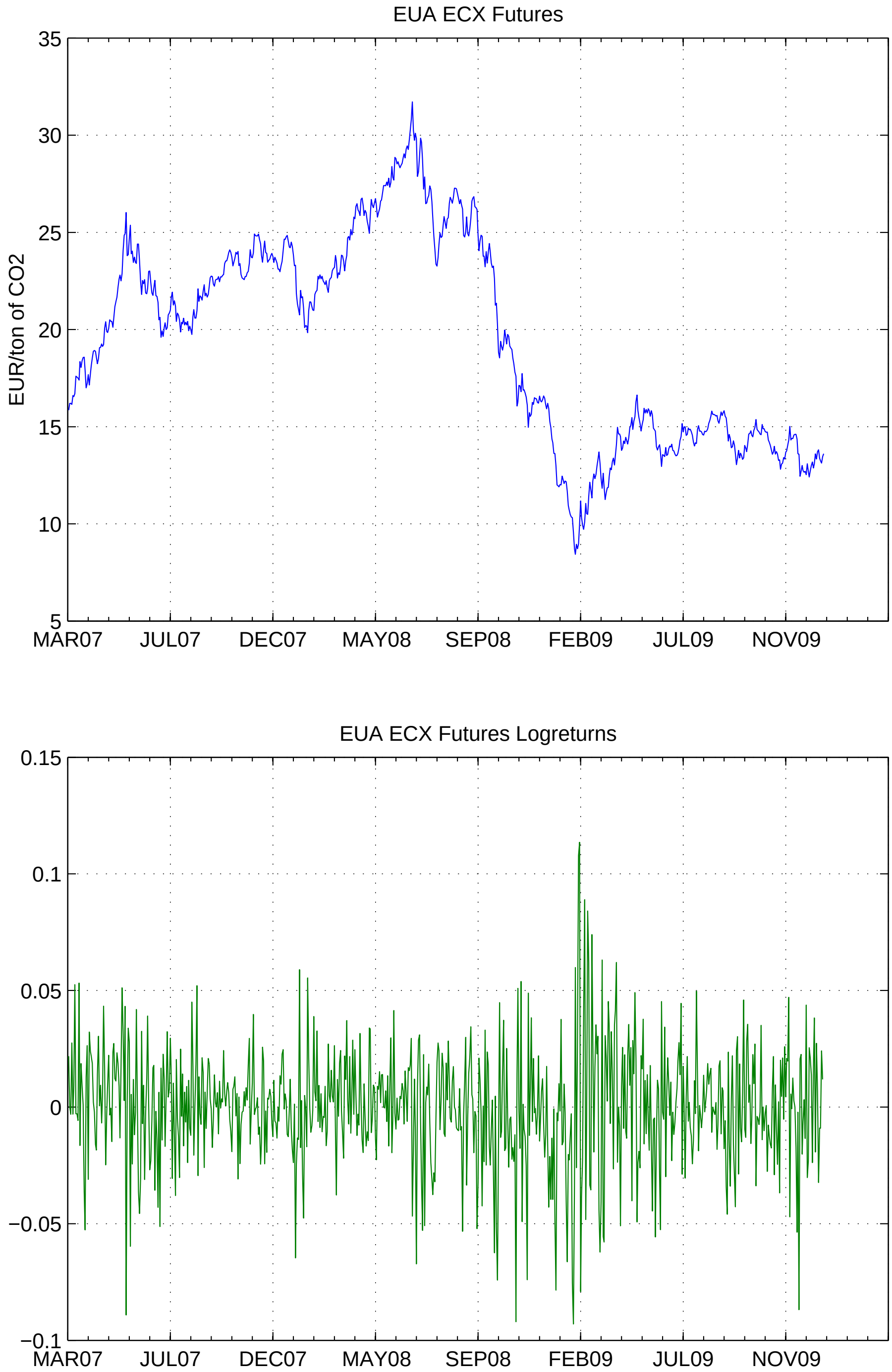


Figure 1
Time series of EUA Futures daily closing prices in raw form (top) and logreturn transformation (bottom) from March 09, 2007 to January 26, 2010
<u>Source:</u> European Climate Exchange

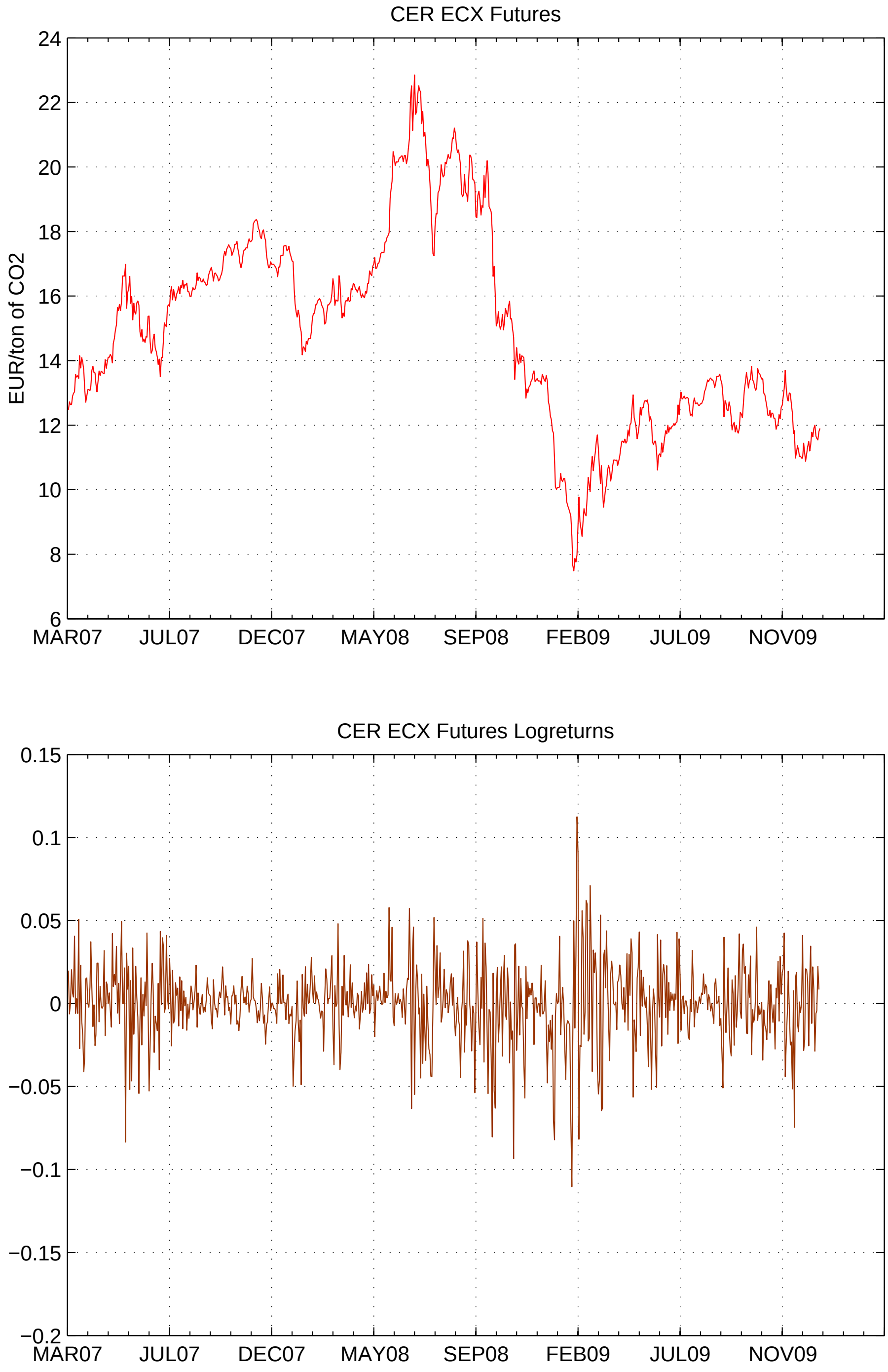
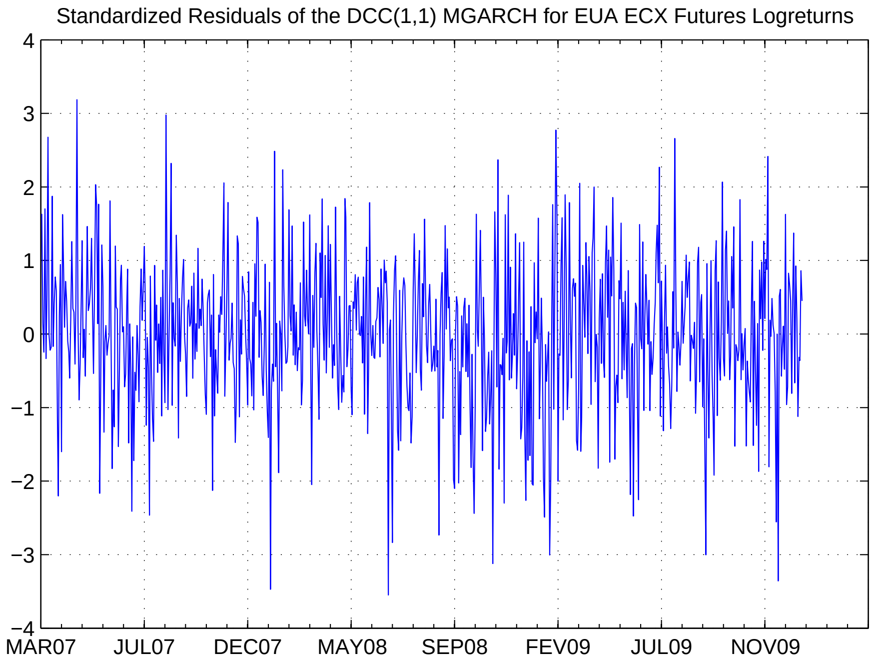


Figure 2
Time series of CER Futures daily closing prices in raw form (top) and logreturn transformation (bottom) from March 09, 2007 to January 26, 2010
<u>Source:</u> European Climate Exchange



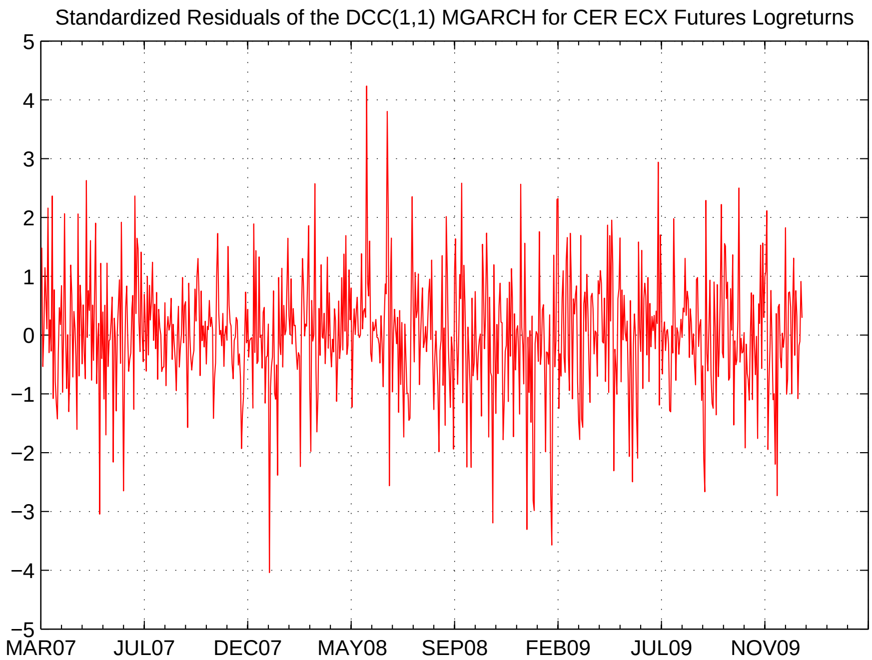


Figure 3
Standardized residuals of the DCC(1,1) MGARCH for EUA Futures (top) and CER Futures (bottom) logreturns

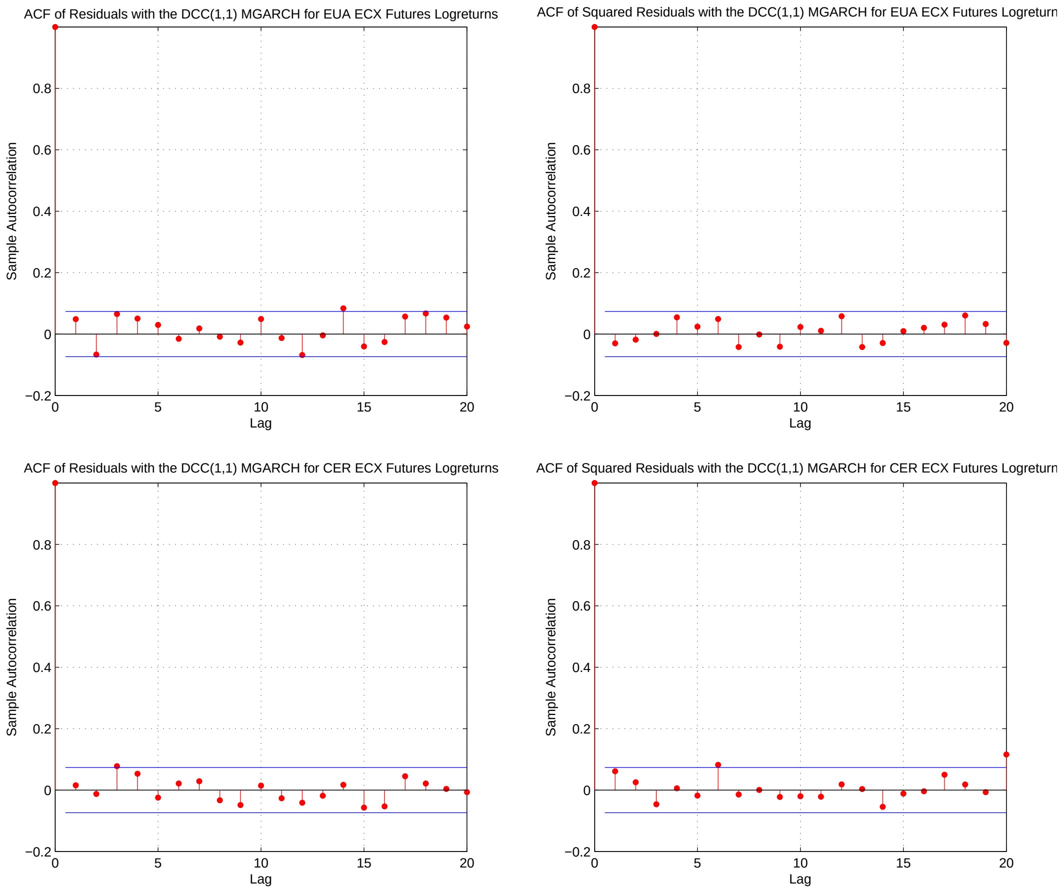


Figure 4
Autocorrelation function of residuals (left panel) and squared residuals (right panel) with the DCC(1,1) MGARCH for EUA ECX Futures (top) and CER ECX Futures (bottom) logreturns

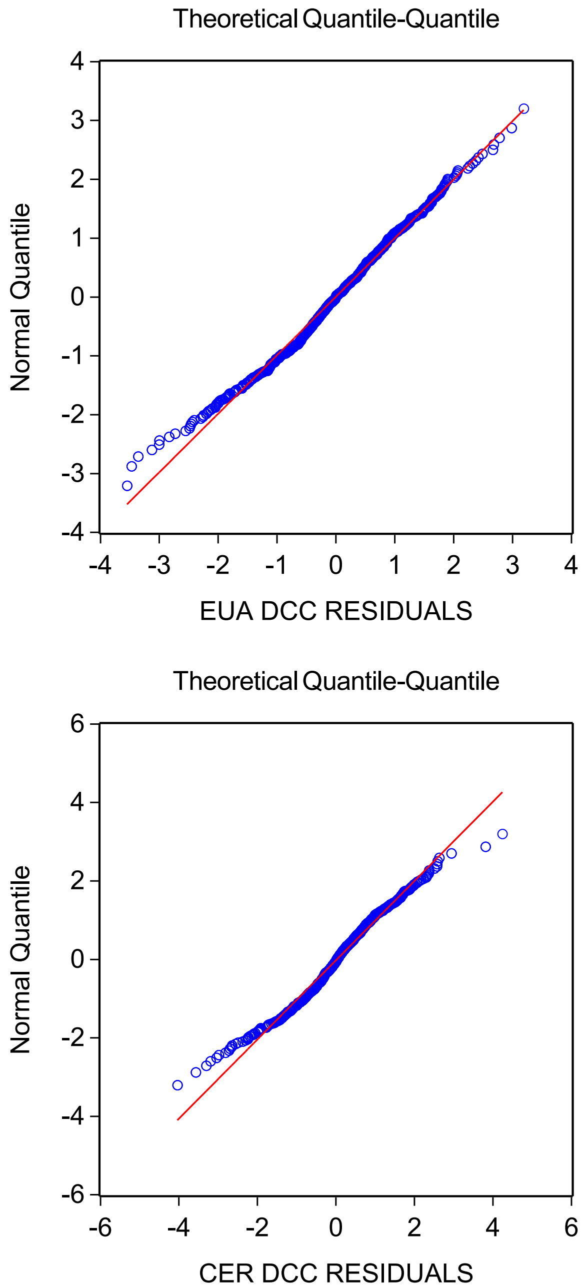


Figure 5
Normal Q-Q plots of the standardized residuals for the DCC(1,1) MGARCH with EUA ECX Futures (top) and CER ECX Futures (bottom) logreturns

Note: Normal Q-Q plot stands for the quantiles of the standardized residuals plotted against the quantiles of the normal distribution.

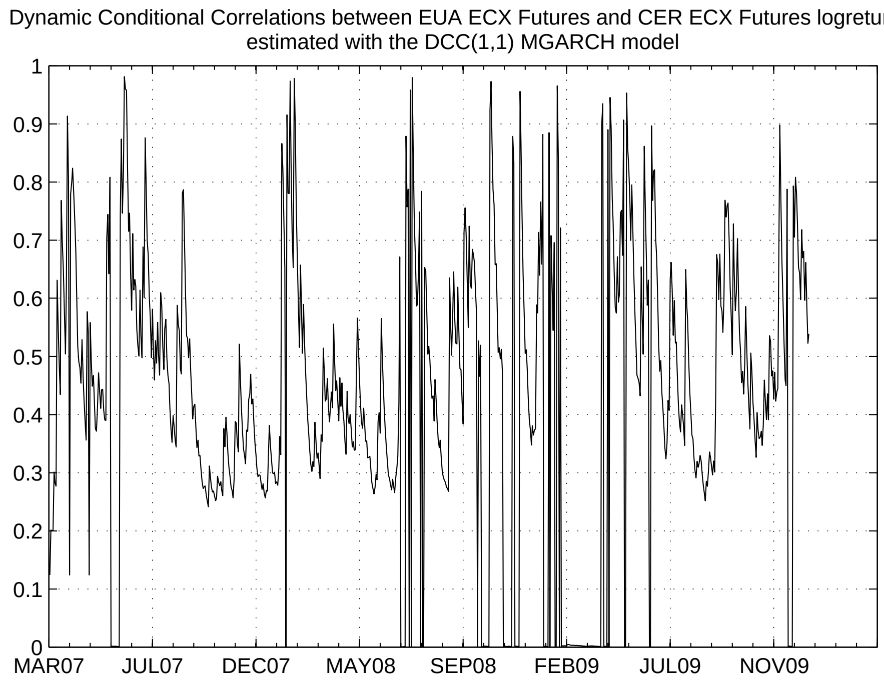


Figure 6
Dynamic Conditional Correlations between EUA ECX Futures and CER ECX Futures estimated with the DCC(1,1) MGARCH model

	$EUAECXFUT$	$EUAECXFUTRET$	$CERECXFUT$	$CERECXFUTRET$
Mean	19.48989	-0.000207	14.84186	-6.36E-05
Median	20.10000	0.000000	14.68000	0.000000
Maximum	31.71000	0.113543	22.85000	0.112545
Minimum	8.430000	-0.093014	7.484615	-0.110409
Std. Dev.	5.222809	0.025489	3.035417	0.023438
Skewness	0.047457	-0.117416	0.257790	-0.372537
Kurtosis	1.813932	4.851319	2.504244	5.677438
Jarque-Bera (JB)	43.47578	106.7975	15.71028	236.8635
Probability JB	0.000000	0.000000	0.000388	0.000000
Obs.	737	736	737	736

Table 1
Descriptive statistics

Note: $EUAECXFUT$ refers to the EUA Futures time series in raw form, $EUAECXFUTRET$ to the EUA Futures time series in logreturn transformation, $CERECXFUT$ to the CER Futures time series in raw form, $CERECXFUTRET$ to the CER Futures time series in logreturn transformation, Std. Dev. to standard deviation, and Obs. to the number of observations.

Parameter	Estimate
GARCH parameters	
ω_{EUA}	0.0001***
	(0.0001)
α_{EUA}	0.1340***
	(0.0002)
β_{EUA}	0.8118***
	(0.0005)
ω_{CER}	0.0001***
	(0.0001)
α_{CER}	0.1590***
	(0.0002)
β_{CER}	0.8038***
	(0.0003)
Correlation Parameters	
α_1^*	0.0525***
	(0.0004)
β_1^*	0.9442***
	(0.0005)
$Log - Lik.$	3682.1393
$ES\ p\ value$	0.0027
$ES\ \chi^2\ stat$	11.8259

Table 2
DCC(1,1) MGARCH estimates for EUA ECX Futures and CER ECX Futures logreturns

Note: EUA refers to EUA ECX Futures logreturns, and EUA refers to EUA ECX Futures logreturns. Robust standard errors in parentheses. *** indicates 1% significance level. The number of observations is 736. $ES\ p\ value$ and $ES\ \chi^2\ stat$ are Engle and Sheppard's (2001) dynamic correlation tests statistics for a maximum lag of order 1.

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