Public debt accumulation and institutional quality: can corruption improve welfare?

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Abstract
Corruption can enhance welfare in two complementary ways in a dynamic time inconsistency model: first, by mitigating the inflation bias of discretionary monetary policy; second, by reducing the loss due to the suboptimal distribution of distortions associated with public debt accumulation. The note thus proposes an original explanation of the existence of weak public governance in countries whose monetary regime lacks credibility.

1 Introduction

Are developing or transition economies suffering from bad governance and corruption always ready to devote efforts to strengthen their institutions? On the basis of a game-theoretic approach in the tradition of Barro and Gordon (1983) in which weak public governance is modeled as an erosion of the ability to collect revenue through regular tax channels, Huang and Wei (2006) argue that the majority of prescribed solutions to the monetary policy credibility problem are likely to fail in countries with low institutional capacity. The optimal inflation rate in such countries is actually higher than that normally required by a monetary arrangement like a fixed exchange rate or a currency board. These two authors show that some nations might fall into what they call a “poor-institution trap”: authorities faced with severe governance problems would no longer be willing to improve fiscal capacity and institutional quality from the moment that the cost of the reforms needed to fight corruption exceeds a certain threshold. By making use of a similar model, Hefeker (2010) highlights the existence of a trade-off between higher institutional quality with lower inflation versus higher taxes with lower output.

The present note provides a new insight into the lack of incentive from authorities to curtail corruption. I consider a two-period extension of the framework used by the aforementioned authors for investigating the consequences of institutional deficiencies regarding public debt. The main point is that corruption can theoretically enhance economic performance. This stems from the fact that the “monetary distortion” due to the commitment problem may be offset by the “institutional distortion” associated with corruption.

The rest of the note is as follows. Section 2 presents the model and calculates the debt level and welfare losses under discretion. Section 3 explores the impact of a change in institutional quality on output. Then, Section 4 turns to the consequences of corruption for debt accumulation. Section 5 finally concludes.

2 A Simple Dynamic Model

The model is standard and describes a game between a representative worker, who sets the nominal wage rate, and a central policymaker, who controls both inflation and taxes and chooses the amount of public debt in the first period. It is assumed that the policymaker is never able to commit, which is more realistic to represent a country facing institutional and political failures.

At any period $t$ ($t = 1, 2$), output, $x_t$, is given by a modified Lucas supply curve allowing the adverse impact of tax distortions to be taken into account:

$$x_t = \pi_t - \pi^e_t - \tau_t$$

where $\pi_t$ and $\pi^e_t$, respectively, denote the actual and expected inflation rates, and where $\tau_t$ is the tax rate on total output. For the sake of simplicity, there is no shock and the natural level of output is normalized to zero.
The budget constraint of the government is:

\[ g_t + (1 + R) d_{t-1} = \pi_t + \beta \tau_t + d_t \tag{2} \]

where \( g_t \) is the public spending level, \( d_{t-1} \) and \( d_t \) are, respectively, the amount of public debt carried over from the previous period and the amount of newly issued debt, and \( R \) is the (constant) real interest rate. The left-hand side of (2) represents the government’s total outlay, while the right-hand side indicates its various sources of receipts (seigniorage revenue, output tax revenue and new debt issue). Contrary to advanced economies featuring low holdings of base money on account of efficient financial systems, seigniorage remains an important source of government revenue for developing countries. Without loss of generality, \( d_0 = 0 \) and all debt must be repaid at the end of the second period (i.e. \( d_2 = 0 \)).

As in Huang and Wei (2006), institutional failure and corruption are supposed to lessen the government’s ability to collect revenue through regular tax channels and are modeled as a decrease in the value of \( \beta \) (0 \( \leq \beta \leq 1 \)) in (2). Thus, the lower \( \beta \), the greater will be the leakage of tax revenue. There is no corruption at all for \( \beta = 1 \); in contrast, if \( \beta = 0 \), so serious is the public governance problem that the regular tax collection system collapses completely.

The policymaker’s quadratic loss function is increasing in the deviations of inflation, output and public spending from their targets:

\[ V = \frac{1}{2} \sum_{t=1}^{2} \rho^{t-1} \left[ s_{\pi} \pi_t^2 + s_x x_t^2 + s_g (g_t - g_t^*)^2 \right] \tag{3} \]

The targeted inflation rate corresponds to price stability. The output target is set equal to zero as well: this is the natural output level reached in the absence of tax distortions whenever the price level is correctly anticipated by the private sector. The need to provide public goods, as \( g_t^* > 0 \), and the absence of lump-sum taxes are enough to generate the standard time inconsistency problem and so an inflation bias under discretion. \( s_{\pi} \), \( s_x \) and \( s_g \) denote the weights placed on the price stability, output and public spending objectives, respectively (\( s_{\pi} + s_x + s_g = 1 \)). \( \rho \) is the authorities’ subjective discount factor (0 \( < \rho \leq 1 \)).

Within this dynamic framework, the decision regarding how much to borrow is made in the first period while taking into account its consequences in the next period. The policymaker equates the marginal benefit from issuing more debt (i.e. smaller losses in \( t = 1 \) owing to lower tax distortions) to the (discounted) marginal cost (i.e. larger losses in \( t = 2 \) because of a higher debt service burden). Equation (4) below illustrates this intertemporal trade-off (see Appendix for all calculation details):

\[ g_t^* - d_t = \frac{(1 + R) \rho \Xi [(1 + R) d_1 + g_2^*]}{\Omega} \tag{4} \]

where \( \Xi \equiv s_{\pi} (s_x + \beta^2 s_g) + (1 + \beta)^2 s_x s_g \) and \( \Omega \equiv s_{\pi} (s_x + \beta^2 s_g) + (1 + \beta) s_x s_g \).
The left-hand side of (4) corresponds to the gain in the first period resulting from debt issue. A higher debt stock allows the decisionmaker to lower both inflation and corporate taxes in \( t = 1 \), hence a rise in output (see (A12)-(A13) in Appendix). Furthermore, public borrowing does more than compensate for the decrease in seigniorage and taxation, so government expenditure in period one goes up with the debt stock (see (A15)). The right-hand side of (4) represents the cost of debt accumulation in period two. The term \( \frac{1+R}{2} \) is referred as the authorities’ effective discount factor, in the sense that it varies with the commitment technology available to them.\(^1\) The higher the public debt amount, the higher future inflation and taxes, since a larger financing requirement will compel the policymaker to raise tax and seigniorage revenues, hence a fall in the second-period activity level (see (A4)-(A5) in Appendix).

The equilibrium debt stock under discretion follows from (4):

\[
d^D_1 = \frac{g^1_1 - \rho^D g^2_2}{1 + (1 + R) \rho^D}
\]  

where \( \rho^D \equiv \frac{(1+R)g^D}{\Psi} \) (the superscript \( D \) denoting discretion).

As in Beetsma and Bovenberg (1997), it is convenient to split the expression for welfare losses for both periods into two parts, so as to distinguish the intratemporal from the intertemporal component:

\[
V^D = L^D_{\text{intra}} \times L^D_{\text{inter}} \times \Psi^2
\]  

where \( \Psi \equiv (1 + R) g^1_1 + g^2_2 \).

The intratemporal loss factor, \( L^D_{\text{intra}} \), represents the distribution of distortions under discretion over the various available instruments within each period and therefore corresponds to the result that would be obtained in a simple one-shot game. The intertemporal loss factor, \( L^D_{\text{inter}} \), stems from the distribution of distortions across both periods and thus depends on the rate of time preference. These losses can be written as:

\[
L^D_{\text{intra}} = \frac{s_x s \sigma y \sigma \Xi}{2\Omega^2}
\]

\[
L^D_{\text{inter}} = \frac{\rho + \rho^D^2}{[1 + (1 + R) \rho^D]^2}
\]

### 3 Can Corruption Boost Activity?

The impact of a change in \( \beta \) on intratemporal losses is given by:

\[
\frac{\partial L^D_{\text{intra}}}{\partial \beta} = \frac{\beta s^2 \sigma^2 x^2 \sigma y^2 \left[(s_x - s_x)(s_x + \beta^2 s_y) - (1 + \beta)(1 + 2\beta) s_x s_y\right]}{\Omega^3}
\]

\(^1\)The terms \( \Xi \) and \( \Omega \) used in the expression for the effective discount factor are therefore specific to the discretionary regime. It can be demonstrated that the effective discount factor under commitment is simply equal to \( (1 + R) \rho \).
Furthermore, as (7) corresponds to the loss in a game without public debt, the effects of corruption on inflation and taxes at any period $t$ are given by the following partial derivatives (see Appendix):

\[
\frac{\partial \pi_t}{\partial \beta} = \frac{s_x s_x s_g g_t^* [s_x - \beta (2 + \beta) s_g]}{\Omega^2} \tag{10}
\]

\[
\frac{\partial \tau_t}{\partial \beta} = \frac{s_x s_g g_t^* [s_x (s_x + s_g) - \beta^2 s_x s_g]}{\Omega^2} \tag{11}
\]

These results make it possible to formulate the first proposition below:

**Proposition 1** If the policymaker puts a large weight on output but attaches little importance to the price stability and public expenditure objectives, more corruption leads to a decrease in intratemporal welfare losses.

**Proof.** According to (9), $\frac{\partial L}{\partial \beta} > 0$ when $s_x \rightarrow 1$ and $s_x, s_g \rightarrow 0$. This first result derives from the fact that corruption raises the cost of collecting revenue. All other things being equal, the corporate tax rate needed to supply a given amount of public goods goes up with the degree of leakage of public funds, hence a rise in the cost sustained by society in terms of foregone output and higher unemployment. If the objective of stabilizing output around its natural level prevails over price stability and public goods provision, the policymaker’s optimal reaction consists in cutting distortionary taxes (see (11): $\frac{\partial \tau_t}{\partial \beta} > 0$ if $s_x \rightarrow 1$ and $s_x, s_g \rightarrow 0$). So the effect of an escalation of corruption on activity turns out to be positive in that case. But this does not necessarily mean a shift of the revenue collection from regular tax to inflation tax. Actually, as can be seen from (10), if the policymaker primarily penalizes output deviations, the discretionary equilibrium inflation rate falls as well ($\frac{\partial \pi_t}{\partial \beta} > 0$ if $s_x \rightarrow 1$ and $s_x, s_g \rightarrow 0$), because the higher output level lessens the temptation to generate unexpected monetary shocks, and thereby alleviates the credibility problem. Thus, a rise in corruption, although implying a fall in public expenditure, can eventually improve the intratemporal distribution of losses through its impact on both monetary and fiscal policy choices.

The impact of corruption, however, appears to be very dependent on the values taken by the various weight parameters in (3). The above result no longer holds in general with a government that does not heavily penalize output deviations. To see this, let us examine the polynomial of degree two in $\beta$ in square brackets in the numerator of the partial derivative (9). Leaving aside the trivial case $\beta = 0$, the first-order condition $\frac{\partial L}{\partial \beta} = 0$ is satisfied if:

\[- (s_x + s_g) s_g \beta^2 - 3s_x s_g \beta + s_x (s_x - s_x - s_g) = 0 \tag{12}\]

The discriminant $\Delta$ equals $9s_x^2 s_g^2 + 4s_x s_g (s_x + s_g) (s_x - s_x - s_g)$. A sufficient condition for having $\Delta > 0$ is $s_x > s_x + s_g$. In that case, the two real roots are $\beta_1 = \frac{\sqrt{3 - 3s_x s_g}}{2(s_x + s_g)s_g}$ and $\beta_2 = -\frac{\sqrt{3 + 3s_x s_g}}{2(s_x + s_g)s_g}$; the latter must be ignored.
for $\sqrt{\Delta} > 0$ since $\beta \geq 0$ by assumption, whereas the former lies within the range $[0, 1]$ if $3sxs_g \leq \sqrt{\Delta} \leq 2sxs_g + 5sxs_g$. Accordingly, a rise in the corruption level exerts damaging effects if $\frac{\sqrt{\Delta} - 3sxs_g}{2(s_x + s_x)s_g} < \beta \leq 1$ but improves welfare if $0 \leq \beta < \frac{\sqrt{\Delta} - 3sxs_g}{2(s_x + s_x)s_g}$. The intratemporal loss, however, is continuously increasing in $\beta$ over the entire interval $[0, 1]$ for sufficiently large values of $s_x$ such that $\beta_1 > 1$, implying that more corruption, then, is always beneficial.$^2$

But a deterioration in institutional quality is likely to make a country worse off if the government does not place as large a weight on output.$^3$ In particular, it is worth considering the case $0 < \beta_1 < 1$, since corruption, when starting from a low level, initially harms welfare, but exerts a positive effect afterwards, once the leakage of tax revenue passes a certain threshold. As an illustration, the intratemporal loss admits a maximum at $\beta \approx 0.53$ if $s_x = 0.3$, $s_x = 0.6$ and $s_g = 0.1$: for a high initial quality level of institutions, a rise in corruption at first causes additional welfare losses on account of the drop in public spending; if the decline in $\beta$ continues, corruption will become beneficial once the leakage of tax revenue roughly exceeds 50%. The model thus suggests that the incentive to promote better governance may depend on the scale of the problem.

4 Does Corruption Involve Excessive Debt?

I now turn to the consequences of malfunctioning institutions for the intertemporal distribution of distortions across the two periods of the game and the conditions under which corruption might again exert a positive effect on welfare. The first step consists in examining the impact of a change in $\beta$ on the equilibrium amount of debt. It follows from the results of Section 2 that:

$$\frac{\partial d^D}{\partial \rho^D} = \frac{\Psi}{[1 + (1 + R)\rho^D]^2}$$

$$\frac{\partial \rho^D}{\partial \beta} = \frac{(1 + R)s_x + s_g[(1+2\beta)s_x + (1+\beta)s_g(1+\beta)s_x - 2\beta^2 s_x]}{1 - (1 + 2\beta)s_x}$$

(13)

(14)

These two partial derivatives make it possible to formulate Proposition 2:

**Proposition 2** Corruption boosts public debt accumulation when the policymaker assigns more importance to the output objective than to price stability.

**Proof.** $\frac{\partial d^D}{\partial \rho^D} < 0$ in any case and $\frac{\partial \rho^D}{\partial \beta} > 0$ when $s_x \to 1$ and $s_x \to 0$. In consequence, $\frac{\partial d^D}{\partial \beta} = \frac{\partial d^D}{\partial \rho^D} \times \frac{\partial \rho^D}{\partial \beta} < 0$ for $s_x \to 1$ and $s_x \to 0$. ■

Corruption raises the shadow price of collecting regular taxes relative to collecting seigniorage revenues, which leads the government to review the way

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$^2$This is for instance the case with $s_x = 0.8$ and $s_x = s_g = 0.1$.

$^3$As an example, corruption increases the intratemporal loss when the three objectives are weighted equally in (3) (i.e. $s_x = s_x = s_g = \frac{1}{3}$).
of financing public expenditure (that is, the split between seigniorage, taxation and borrowing). A rise in corruption involves reducing inflation and taxes if priority is given to output stabilization, as seen in Section 3, so the policymaker borrows more to compensate for the lost revenue. Interestingly, the model implies that the public debt amount is likely to be larger in nations where the problem of bad governance is more acute, which fits empirical observations.

It follows from (8) that:

\[
\frac{\partial L^D_{\text{inter}}}{\partial \rho^D} = \frac{2 (1 + R) \rho (\Xi - \Omega)}{\Omega [1 + (1 + R) \rho^D]^3}
\]

By making use of (14) and (15), a third proposition can be established:

**Proposition 3** More corruption leads to lower intertemporal welfare losses by boosting public debt accumulation when the government is “weight-liberal” and cares more about output deviations than it does about price stability.

**Proof.** \( \Xi > \Omega \) as long as \( \beta > 0 \); therefore, according to (15), \( \frac{\partial L^D_{\text{inter}}}{\partial \rho^D} > 0 \) \( \forall \beta > 0 \). Moreover, from (14), \( \frac{\partial \rho^D}{\partial \beta} > 0 \) if \( s_x \to 1 \) and \( s_n \to 0 \). Consequently, constuct any change in \( \rho^D \).

Proposition 3 draws its theoretical rationale from the inflation bias under discretion. The positive sign of \( \frac{\partial L^D_{\text{inter}}}{\partial \rho^D} \) as long as \( \beta > 0 \) means that the debt stock carried over from the first period into the second is inefficiently low in equilibrium, since a smaller value of \( \rho^D \), and thus a larger amount of debt (see (13)), would entail a decrease in the intertemporal loss component. As shown by Beetsma and Bovenberg (1997), inflation expectations in the second period are endogenous from the government’s standpoint when setting debt policy. The government then is induced to employ debt policy strategically in order to influence future inflation expectations and economic performance. Under discretion, long-term inflation expectations are too high from an *ex ante* perspective. Therefore, the policymaker can alleviate the long-run inflation bias by issuing less debt: as the distortionary tax rate needed to meet future debt payment obligations will be lower, the temptation to engage in a surprise monetary expansion will be lessened as well, hence lower equilibrium inflation in the second period of the game. Such a strategic behavior is formally captured in the model by the ratio \( \frac{\Xi}{\Omega} \) in the expression for \( \rho^D \). The presence of this ratio raises the second-period cost of additional debt and thereby constitutes a credibility effect: given that \( \Xi > \Omega \) \( \forall \beta > 0 \), the effective discount factor is higher and, correspondingly, public debt is lower under discretion than under commitment (see Footnote 1).

Nonetheless, this trade-off between the cost of additional distortions in \( t = 1 \) and the gain in anti-inflation credibility in \( t = 2 \) turns out to be suboptimal,
for it generates a too low equilibrium debt stock compared with the benchmark solution corresponding to commitment. As already pointed out by Beetsma and Bovenberg (1997), the government is indeed induced to rely more heavily on the first-period sources of financing, especially through unanticipated inflation, in order to build up public assets and to contain future inflation expectations. As such an incentive is correctly anticipated by the private sector, the discretionary regime is characterized by an asset bias. It follows that the effective discount factor $\rho^D$ is too high from a social point of view. Accordingly, a rise in corruption makes it possible to reduce the asset bias by pushing debt accumulation in the direction of its second best, and so improves the intertemporal allocation of distortions.

It should nevertheless be stressed that this beneficial effect requires that the policymaker’s and society’s time preferences have to be similar, which is implicitly assumed to be the case in this analysis. In a slightly different model with myopic authorities focusing more on short-term performance than society, it is possible to have $\frac{\partial L}{\partial L^*} < 0$. The equilibrium debt stock would then be too large instead of too small, and more corruption would cause larger intertemporal losses by exacerbating the initial debt bias with a populist policymaker showing little concern about pursuing price stability.

## 5 Conclusion

The main finding of this note is that corruption can, in theory, make a country better off if its government is unable to make binding commitments and assigns a larger weight to output than to inflation stabilization. Admittedly, the case for a beneficial impact of deficient governance on the intertemporal allocation of distortions seems questionable. However, according to the model, the overall impact of corruption can theoretically still be positive in a country with excessive debt and running populist, short-termist policies, the condition being that the gain from lower intratemporal distortions must outweigh the intertemporal loss caused by public debt accumulation.

More broadly, this note supports the view that the degree of anti-inflation credibility could be an important factor in the fight against corruption since the motivation to really tackle such a challenge appears to be questionable under discretion. The uncertainty regarding the effect of reforms intended to strengthen institutions could partly explain why the issue of bad governance and political instability remains more prevalent in countries lacking credibility. This reinforces the possibility of a poor-institution trap highlighted by Huang and Wei (2006).

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5 In this model commitment leads to a second-best solution only because of the absence of lump-sum taxation.
References


Appendix

Derivation of Public Debt Accumulation and Welfare Losses in the Discretionary Regime

In each period $t$ ($t = 1, 2$), the government, which cannot commit to future policies, sets the inflation rate, $\pi_t$, and the tax rate, $\tau_t$, taking as given inflation expectations. The solution is found by working back in time. For a given stock of public debt $d_1$, the policy choices and welfare loss of the second period are derived first, under the rational expectations hypothesis (i.e. $\pi_2 = \pi_2^e$ ex post). The first-period monetary and fiscal policies and the public debt amount are computed subsequently, assuming that future policies will be selected optimally.

The second-period objective function can be written as:

$$L_2 = \frac{1}{2} \left\{ s_\pi \pi_2^2 + s_x (\pi_2 - \pi_2^e - \tau_2)^2 + s_g [\pi_2 + \beta \tau_2 - (1 + R) d_1 - g_2^e]^2 \right\} \quad (A1)$$

The government minimizes the above loss function with respect to $\pi_2$ and $\tau_2$. The first-order conditions are:

$$\pi_2 = \frac{s_g [(1 + R) d_1 + g_2^e] + (s_x - \beta s_\pi) \tau_2}{s_\pi + s_g} \quad (A2)$$

$$\tau_2 = \frac{\beta s_\pi [(1 + R) d_1 + g_2^e] - \beta s_\pi \pi_2}{s_x + \beta^2 s_g} \quad (A3)$$

Resolving the system (A2)-(A3) for $\pi_2$ and $\tau_2$ yields:

$$\pi_2 = \frac{(1 + \beta) s_x s_\pi [(1 + R) d_1 + g_2^*]}{\Omega} \quad (A4)$$

$$\tau_2 = \frac{\beta s_\pi s_g [(1 + R) d_1 + g_2^e]}{\Omega} \quad (A5)$$
where \( \Omega \equiv s_x (s_x + \beta^2 s_g) + (1 + \beta) s_x s_g \), as defined in the main text.

Make use of Equations (1) and (2) in the text to obtain the second-period output and government spending levels for a given value of \( d_1 \):

\[
x_2 = -\frac{\beta s_x s_g [(1 + R) d_1 + g_2^*]}{\Omega} \tag{A6}
g_2 - g_2^* = -\frac{s_x s_g [(1 + R) d_1 + g_2^*]}{\Omega} \tag{A7}
\]

Substituting (A4), (A6) and (A7) into (A1) yields the second-period welfare loss for any value of \( d_1 \):

\[
L_2 = \frac{s_x s_g \Xi [(1 + R) d_1 + g_2^*]^2}{2\Omega^2} \tag{A8}
\]

where \( \Xi \equiv s_x (s_x + \beta^2 s_g) + (1 + \beta)^2 s_x s_g \), as defined in Section 2.

Consequently, in \( t = 1 \), the policymaker faces the following optimization problem:

\[
\min L_1 + \rho L_2 = \frac{1}{2} \left\{ s_x \pi_1^2 + s_x (\pi_1 - \pi_1^*)^2 + s_x (\pi_1 + \beta \tau_1 + d_1 - g_1^*)^2 \right\} + \frac{\rho s_x s_g \Xi [(1 + R) d_1 + g_2^*]^2}{2\Omega^2} \tag{A9}
\]

Since \( \pi_1 = \pi_1^* \) \textit{ex post}, the first-order conditions for \( \pi_1 \) and \( \tau_1 \) are:

\[
\pi_1 = \frac{s_x (g_1^* - d_1) + (s_x - \beta s_g) \tau_1}{s_x + s_g} \tag{A10}
\]

\[
\tau_1 = \frac{s_x (g_1^* - d_1) - \beta s_g \pi_1}{s_x + \beta^2 s_g} \tag{A11}
\]

Resolving the system (A10)-(A11) for \( \pi_1 \) and \( \tau_1 \) yields:

\[
\pi_1 = \frac{(1 + \beta) s_x s_g (g_1^* - d_1)}{\Omega} \tag{A12}
\]

\[
\tau_1 = \frac{\beta s_x s_g (g_1^* - d_1)}{\Omega} \tag{A13}
\]

Combine the above two equations with (1) and (2) in the main text to obtain the levels of output and public expenditure in period one for any value of \( d_1 \):

\[
x_1 = -\frac{\beta s_x s_g (g_1^* - d_1)}{\Omega} \tag{A14}
\]

\[
g_1 - g_1^* = -\frac{s_x s_g (g_1^* - d_1)}{\Omega} \tag{A15}
\]

The government must decide how much debt to issue at \( t = 1 \). The first-order condition from (A9) is \( \frac{\partial L_1}{\partial \pi_1} + \rho \frac{\partial L_2}{\partial \pi_1} = 0 \). The partial derivative of \( L_1 \) with respect to \( d_1 \) is:
\[
\frac{\partial L_1}{\partial d_1} = s_g (\pi_1 + \beta \tau_1 + d_1 - g_1^*) 
\]  
\[\text{(A16)}\]

Substituting (A12) and (A13) into (A16), one gets:

\[
\frac{\partial L_1}{\partial d_1} = \frac{(1 + \beta) s_x s_g^2 (g_1^* - d_1)}{\Omega} + \frac{\beta^2 s_x s_g^2 (g_1^* - d_1)}{\Omega} - s_g (g_1^* - d_1) 
\]  
\[\text{(A17)}\]

The above expression can be simplified by putting all terms over a common denominator:

\[
\frac{\partial L_1}{\partial d_1} = - \frac{s_x s_g (g_1^* - d_1)}{\Omega} 
\]  
\[\text{(A18)}\]

Let us now consider the second-period loss:

\[
\rho \frac{\partial L_2}{\partial d_1} = \frac{(1 + R) \rho s_x s_g \Xi [(1 + R) d_1 + g_2^*]}{\Omega^2} 
\]  
\[\text{(A19)}\]

By making use of (A18) and (A19), the first-order condition \(\frac{\partial L_1}{\partial d_1} + \rho \frac{\partial L_2}{\partial d_1} = 0\) can be written as:

\[
\frac{s_x s_g (g_1^* - d_1)}{\Omega} = \frac{(1 + R) \rho s_x s_g \Xi [(1 + R) d_1 + g_2^*]}{\Omega^2} 
\]  
\[\text{(A20)}\]

Eliminating common terms in the above equation results in:

\[
g_1^* - d_1 = \frac{(1 + R) \rho \Xi [(1 + R) d_1 + g_2^*]}{\Omega} 
\]  
\[\text{(A21)}\]

which is (4) in the main text.

Let us note \(\rho^D \equiv \frac{(1 + R) \rho \Xi}{\Omega}\) (the superscript \(^D\) denoting discretion). It is straightforward to solve for the optimal amount of public debt under discretion from (A21):

\[
d_1^D = \frac{g_1^* - \rho^D g_2^*}{1 + (1 + R) \rho^D} 
\]  
\[\text{(A22)}\]

which is (5) in the main text.

Substituting (A22) for \(d_1\) into (A4)-(A7) gives the second-period policies and economic outcomes:

\[
\pi_2^D = \frac{(1 + \beta) s_x s_g \Psi}{\Omega [1 + (1 + R) \rho^D]} 
\]  
\[\text{(A23)}\]

\[
x_2^D = - \frac{\beta s_x s_g \Psi}{\Omega [1 + (1 + R) \rho^D]} 
\]  
\[\text{(A24)}\]

\[
g_2^D - g_2^* = - \frac{s_x \psi}{\Omega [1 + (1 + R) \rho^D]} 
\]  
\[\text{(A25)}\]

where \(\Psi \equiv (1 + R) g_1^* + g_2^*,\) as defined in Section 2.
Combining the previous expression for public debt with (A12)-(A15) then yields the first-period equilibrium values:

\[ D_1 = (1 + \beta) s_s g D_s \]  
\[ x_1 = -\frac{\beta s_s g D_s}{\Omega [1 + (1 + R) \rho D]} \]  
\[ g_1^D - g_1 = -\frac{s_s g D_s}{\Omega [1 + (1 + R) \rho D]} \]

(A26)  
(A27)  
(A28)

The equilibrium welfare loss under discretion can finally be derived by making use of (3) in the text:

\[ V^D = \frac{s_s g D_s \Xi}{2\Omega^2} \times \frac{\rho + \rho^D}{[1 + (1 + R) \rho D]^2} \times \Psi^2 \]  

(A29)

The first ratio in the right-hand side of (A29) is the intratemporal loss factor and the second ratio corresponds to the intertemporal loss factor.