Abstract
This paper extends the model of Chortareas and Miller (2003) to the case of a continuum of central banker types. We derive two main results. First, whether the central banker candidate is too selfish or not enough, he has the same incentive to accept the contract and to breach. Second, a too selfish central banker is more costly for society than a benevolent central banker.
1. INTRODUCTION

Chortareas and Miller (2003) [hereafter CM] consider a standard inflation bias model in which monetary policy is delegated through contracts but the central banker’s trade-off between social welfare and the incentive scheme remains a private information. They show that it is convenient for a benevolent central banker (CB) to pretend to be selfish, accept the contract, and breach it. Consequently, a surprising positive inflation occurs. An important feature of their paper is the assumption that there are only two types of CB candidates: the benevolent cares only about social welfare, whereas the selfish one implements the contract. This note extends the model of CM to the case of a continuum of CB types, that is, to the case where the CB’s degree of selfishness can be too low (and even equal to zero as in CM) but also too high\(^1\). Our results are as follows. First, it is shown that a selfish agent has an incentive to accept a contract designed for a less selfish one. Second, uncertainty about the CB’s selfishness may generate a negative inflation surprise and an output lower than the natural rate. Third, the social welfare loss resulting from the uncertainty is asymmetric: a degree of selfishness that is too high is more costly than one that is too low. This note is organized as follows. CM’s results are described in Section 2 as the benchmark case. Section 3 establishes the results for a continuum of CB types. The conclusion summarizes the changes produced by the introduction of this assumption.

2. CHORTAREAS AND MILLER’S (2003) MODEL

As in CM and following their notation, monetary policy delegation is represented by four equations.

\[
y = y^n + \alpha(\pi - \pi^c) + \varepsilon \tag{2.1}
\]

\[
\pi = m + v - \gamma \varepsilon \tag{2.2}
\]

\[
L_S = (y - y^*)^{2} + \beta(\pi - \pi)^{2} \tag{2.3}
\]

\[
L_{BC} = [(y - y^*)^2 + \beta(\pi - \pi)^2] - \xi(t_o - t\pi) \tag{2.4}
\]

The first two equations employ the formulation adopted by Walsh (1995). (2.1) is the Lucas supply function where \(y, y^n, \pi,\) and \(\pi^c\) denote output, natural output, inflation and inflation expectations, respectively. \(\varepsilon\) is a shock with mean zero and variance \(\sigma^2\). The policy instrument is the rate of growth of the money supply \((m)\). The link between this instrument and inflation is given by (2.2), where \(v\) is a control error (with \(E(v) = 0\), \(E(v^2) = \sigma^2\) and \(E(v\varepsilon) = 0\)) and \(\varepsilon\) represents the direct impact of the shock on inflation. (2.3) is the society’s loss function where \(y^*\) is the targeted output \((y^* = y^n + z, \text{ with } z > 0\) the expansionary bias) and \(\pi\) is the preferred value for inflation (zero by normalization). \(\alpha, \beta,\) and \(\gamma\) are positive parameters. The timing of events is as follows. The government chooses the monetary regime, the private sector forms rational expectations, random shocks occur and the authority sets monetary policy.

If the discretionary solution applies, the equilibrium is found by minimizing (2.3) subject to (2.1) and (2.2). This solution is characterized by the following expressions for, respectively, the reaction function, expected inflation, inflation and output (with \(\theta = 1/((\alpha^2 + \beta))\)):

\[
m_d = \theta \alpha z + \theta \alpha^2 E[m_d] - v + (\gamma - \theta \alpha) \varepsilon \tag{2.5}
\]

\[
E[\pi^c_d] = E[m_d] = \frac{\alpha z}{\beta} \tag{2.6}
\]

\(^1\)Related papers are Ciccarone and Marchetti (2010) and Campoy, Candel-Sánchez and Negrete (2009). Ciccarone and Marchetti (2010) extend the model of CM to the common agency context but retain the assumption that there exist only two types of CB. Campoy, Candel-Sánchez and Negrete (2009) show that with selfishness uncertainty the CB’s choice in a menu of contracts is a credible signal that reveals its true type to the private sector.
\[ \pi_d = \frac{\alpha z}{\beta} - \theta \alpha \varepsilon \quad \text{and} \quad y_d = y^n + \theta \beta \varepsilon \]  

(2.7)

If monetary policy delegation is implemented using a linear inflation contract, the CB’s loss function (2.4) incorporates the social loss and a personal incentive scheme \((t_o\) is the fixed reward and \(t\) the optimal penalization rate). \(\xi > 0\) represents the degree of selfishness of the CB, that is, the extent to which he responds to the incentive scheme rather than takes care of social loss. The monetary policy is found by minimizing (2.4) subject to (2.1) and (2.2). The government chooses \(t\) so as to minimize (2.3) subject to the policy outcomes. This yields:

\[ m_c = \theta \alpha z - \frac{1}{2} \theta t \xi + \theta \alpha^2 E[m_c] - v + (\gamma - \theta \alpha) \varepsilon \]  

(2.8)

\[ t^* = \frac{2z \alpha}{\xi} \]  

(2.9)

\[ E[m_c] = E[m_c] = 0 \]  

(2.10)

\[ \pi_c = -\theta \alpha \varepsilon \quad \text{and} \quad y_c = y^n + \theta \beta \varepsilon \]  

(2.11)

It is straightforward that the effectiveness of the contract depends on the degree of selfishness of the CB. The problem is that it is not perfectly known \textit{a priori}. CM assume that the government offers a contract for a selfish CB \((\xi = \xi > 0)\) and that a benevolent candidate \((\xi = 0)\) masquerades as a selfish one to become the CB. Then, \(\xi\) is the supposed degree of selfishness, whereas \(\xi\) is the actual one. \(\phi\) is the fraction of the private sector believing that \(\xi = \xi\), that is, that the CB will respect the contract. \(1 - \phi\) (with \(0 < \phi < 1\)) is the fraction knowing that \(\xi = 0\), that is, that the CB is benevolent and will breach. The fraction \(\phi\) sets its expectations according to the contract equilibrium, whereas the fraction \(1 - \phi\) sets them at the discretionary one. Using (2.6) and (2.10), the private sector aggregated expectations are then given by:

\[ E[m_b] = E[m_b] = (1 - \phi) \frac{z \alpha}{\beta} \]  

(2.12)

Combining these expectations (2.12) and the reaction function of the benevolent CB (that reduces to 2.3) gives the actual rate of money growth. The corresponding inflation and output are:

\[ \pi_b = \theta \left( \frac{\alpha z}{\beta} \right) (\alpha^2 (1 - \phi) + \beta) - \theta \alpha \varepsilon \quad \text{and} \quad y_b = y^n + \theta (\alpha^2 \phi z + \beta \varepsilon) \]  

(2.13)

CM’s main propositions are then as follows. First, a benevolent candidate has an incentive to accept a contract designed for a selfish CB and to breach it. Plugging (2.13) into (2.3), we obtain his expected loss in this case \(E[L_{CB}]_{\xi=0}\). Plugging (2.11) into (2.3), we obtain his expected loss if a truly selfish CB was appointed \(E[L_{CB}]_{\xi>0}\). The benevolent candidate’s expected loss is always lower under breaching since the following condition is always fulfilled.

\[ E[L_{CB}]_{\xi>0} - E[L_{CB}]_{\xi=0} = \frac{1}{\beta^2} \theta \left[ \alpha^2 (1 - \phi) + \beta \right]^2 \alpha^2 \phi z^2 > 0 \]  

(2.14)

Second, the benevolent CB who accepts and then breaches the contract produces an inflation surprise. Indeed, comparing (2.12) with inflation in (2.13) gives:

\[ \pi_b - E[\pi]_b = \theta z \alpha \phi > 0. \]  

(2.15)

\footnote{CM also show that if the CB’s selfishness is private information but the CB is selfish indeed (and not benevolent), then a negative inflation surprise occurs.}
3. A CONTINUUM OF CENTRAL BANKER TYPES

CM’s model highlights the costs of appointing a CB who assigns not enough importance to personal reward compared to social welfare. However, what are the costs of appointing a CB who assigns too much importance to his private welfare? We generalize the model by assuming that the actual degree of selfishness of the CB candidate (\( \xi \)) can be higher or lower than that selected to define the contract (\( \tilde{\xi} \)). As in CM, the government offers a contract for a CB with the degree of selfishness \( \xi = \tilde{\xi} > 0 \) (see 2.9). A candidate with the degree \( \xi = \tilde{\xi} + \Delta \) masquerades as a \( \tilde{\xi} \) one to become the CB. \( \Delta \) can be seen as a positive or negative "selfishness-gap" (\( \Delta \in [-\tilde{\xi}, +\infty[ \)). The CB candidate’s loss function is then:

\[
L_{CB,\Delta} = [(y - y^*)^2 + \beta(\pi - \bar{\pi})^2] - (\tilde{\xi} + \Delta)(t_o - t\pi)
\]  (3.1)

This framework includes CM’s model as a particular case where \( \Delta = -\tilde{\xi} \). The equilibrium is found by minimizing (3.1) subject to (2.1) and (2.2). Plugging the penalization rate (2.9) into the first-order condition gives the CB reaction function:

\[
m_{\Delta} = -\frac{\Delta}{\tilde{\xi}} \theta \alpha z + \theta \alpha^2 E[m]_{\Delta} - v + (\gamma - \theta \alpha) \varepsilon
\]  (3.2)

\( \phi \) remains the fraction of the private sector believing that the CB has a degree of selfishness \( \xi = \tilde{\xi} > 0 \) and will respect the contract. This fraction \( \phi \) sets its expectations according to the contract equilibrium (see 2.10). \( 1 - \phi \) (with \( 0 < \phi < 1 \)) is the fraction knowing that the actual degree of selfishness is \( \xi = \tilde{\xi} + \Delta \) and that the CB will breach. This fraction \( 1 - \phi \) sets its expectations according to the reaction function (3.2). Aggregated expectations follow as:

\[
E[\pi]_{\Delta} = E[m]_{\Delta} = - (1 - \phi) \frac{\alpha z \Delta}{\beta \tilde{\xi}}
\]  (3.3)

Inserting (3.3) back into (3.2) and using the resulting expression obtains us the realized inflation and output:

\[
\pi_{\Delta} = \left(\frac{-\Delta}{\tilde{\xi}}\right) \left(\frac{\alpha z}{\beta}\right) \theta \left(\alpha^2(1 - \phi) + \beta\right) + \theta \alpha \varepsilon \quad \text{and} \quad y_{\Delta} = y^n + \left(\frac{-\Delta}{\tilde{\xi}}\right) \theta \alpha^2 \phi z + \theta \beta \varepsilon
\]  (3.4)

The following propositions posit the findings of this note.

**Proposition 1.** Whether the CB candidate is too selfish or not enough, he has the same incentive to accept the contract and to breach.

**Proof.** We first calculate the CB’s expected loss when he breaches. Substituting (3.4) into (3.1) and taking expectations, we get:

\[
E[L_{CB,\Delta}]_{\xi + \Delta} = z^2 + \frac{\Delta \alpha^2 \tilde{\xi}^2}{\beta \tilde{\xi}} \left(2(\phi - 1) + \frac{\Delta}{\tilde{\xi}} \theta \left[\alpha^2(\phi^2 - 1) - \beta\right]\right) - (\tilde{\xi} + \Delta) t_o + \theta \beta \sigma_z^2
\]  (3.5)

If the CB does not breach, he brings about the optimal contract inflation rate (see 2.11). Plugging the inflation rate into (2.11) and the private sector expectations (3.3) into (2.1), we get the output. Substituting this outcome and the inflation rate into (3.1) and taking expectations, the CB’s excepted loss when he abides by the contract is given by:

\[
E[L_{CB,\Delta}]_{\xi} = z^2 + \frac{\Delta \alpha^2 \tilde{\xi}^2}{\beta^2 \tilde{\xi}^2} (\phi - 1) [\Delta \alpha^2 (\phi - 1) + 2 \beta \tilde{\xi}] - (\tilde{\xi} + \Delta) t_o + \theta \beta \sigma_z^2
\]  (3.6)
The CB breaches when $E[L_{CB,\Delta}|_\xi] > E[L_{CB,\Delta}|_{\xi+\Delta}]$. Computing (3.6) – (3.5) yields:

$$\frac{\Delta^2 \alpha^2 z^2}{\beta^2 z^2} \theta \left[ \alpha^2 (1 - \phi) + \beta \right]^2$$

(3.7)

Whatever the sign of $\Delta$, this expression is positive. As a consequence, a CB who is too selfish has the same incentive to cheat as one who is not enough selfish. This result replicates and extends CM’s finding (Proposition 2). For example, remember that the benevolent CB in CM is the particular case where $\Delta = -\xi$ in our model. A CB candidate with the symmetric excessive selfishness ($\Delta = \xi$) has the same incentive to cheat as the benevolent one.

**Proposition 2.** Unexpected inflation can be positive or negative and output can be higher or lower than the natural rate depending on whether the CB assigns not enough or too much importance to personal reward.

**Proof.** Ignoring any supply shock and comparing (3.3) with the inflation in (3.4) yields:

$$\pi_\Delta - E[\pi]_\Delta = -\frac{\Delta}{\xi} \theta \alpha \phi z$$

(3.8)

When the CB places not enough weight on his private welfare ($\Delta < 0$), $\pi_\Delta - E[\pi]_\Delta > 0$: the actual inflation rate exceeds the expected one. In this case, $y_\Delta > y^n$ (see 3.4). When the CB is too selfish ($\Delta > 0$), $\pi_\Delta - E[\pi]_\Delta < 0$: the actual inflation rate is lower than the expected one and then $y_\Delta < y^n$. This result replicates and extends CM’s finding (Proposition 1). A CB who is too selfish reduces inflation expectations (see 3.3). However, the expected inflation is higher than their actual value because incomplete information exists about the CB type ($\phi > 0$).

**Proposition 3.** A CB candidate who is too selfish relative to the contract offered by the government is more costly for the society than one who is not enough selfish.

**Proof.** Using (3.4) to substitute for inflation and output in (2.3), we get the expected social loss when the CB breaches the contract:

$$E[L_{S,\Delta}|_{\xi+\Delta}] = z^2 + \frac{\Delta^2 \alpha^2 z^2}{\beta^2 z^2} \theta \left[ \alpha^2 (\phi - 1)^2 + \beta \right] + \theta \beta \sigma^2 z^2 + \frac{2\Delta}{\xi} \theta \alpha^2 z^2$$

(3.9)

Whatever the sign of $\Delta \neq 0$, the three first terms of (3.9) are the same ones. If $\Delta < 0$, the last term is negative. If $\Delta > 0$, the last term is positive. Hence, the cost of appointing a wrong candidate is asymmetric: for a given absolute value of the selfishness-gap, a CB who is too selfish deteriorates social welfare more sharply than one who is not enough. The intuition behind this result is as follows. As long as a fraction of the private sector believes that the CB will respect the contract ($\phi > 0$), a negative (positive) selfishness-gap produces a positive (negative) inflation surprise and an output higher (lower) than the natural rate. Because of the expansionary bias ($z$), the net cost of less inflation but less output when the CB is too much selfish exceeds the net cost of more output but more inflation when the CB is not enough selfish. The higher the degree of selfishness selected to define the contract ($\xi$) or the higher the degree of conservatism ($\beta$), the smaller the asymmetry.

---

3Tillmann (2008) is a related paper. He shows that a CB who is too conservative is more costly than a one who is too liberal.
4. Conclusion

The aim of this paper has been to extend the model of Chortareas and Miller (2003) and to explore the effects of appointing a CB who assigns too much importance to his private welfare. Our results replicate and extend CM’s findings that only take into account the case of a benevolent CB. We have shown that a CB candidate who is too selfish has the same incentive to breach the contract by which monetary policy is delegated than a not enough selfish one. Thus, when the CB’s selfishness is private information, inflation surprise can be negative or positive and output can be lower or higher than the natural rate. Finally, we find that a CB who is not enough "self-interested" is cheaper for society than a CB who is too "self-interested".

References