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### Inflation inertia and optimal delegation of monetary policy

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#### Abstract

This paper analyzes the relationship between the optimal weight on output gap in the central bank's loss function and the degree of inertia in a hybrid version of New Keynesian model with a pure discretionary inflation targeting. I show that under endogenous persistence of inflation dynamics, even in discretionary monetary policy regime, a Rogoff's (1985) conservative central banker does not necessarily improve social welfare.

# 1 Introduction

In this paper, I investigate the relationship between the optimal weight on output gap in the central bank's loss function and the parameters contained in a hybrid New Keynesian model when the central bank conducts a discretionary monetary policy. This issue has not been fully discussed in the existing literatures. Among the parameters, I focus on the degree of inertia in a hybrid New Keynesian Phillips curve. This is because the recent empirical studies such as Galí et al. (2005) support the presence of inertial inflation dynamics.

In the literature, the seminal work by Rogoff (1985) suggests that to reduce social loss generated by the inflation bias under discretionary optimal monetary policy, it is optimal to appoint a central banker who places a higher weight on inflation than society. While Rogoff (1985) uses a traditional Lucas-type Phillips curve, Clarida et al. (1999) use a basic New Keynesian model and show that the Rogoff's (1985) result holds under exogenous inflation persistence due to serial correlation of cost shocks.<sup>1</sup>

The results of this paper show that under endogenous persistence of inflation dynamics, even in discretionary monetary policy regimes, Rogoff's (1985) conservative central bankers do not necessarily improve social welfare.<sup>2</sup>

## 2 Model and Calculation

### 2.1 Hybrid New Keynesian Model

The private economy is characterized by the hybrid version of New Keynesian Phillips curve,

$$\pi_t = (1 - \psi)\beta E_t \pi_{t+1} + \psi \pi_{t-1} + \kappa x_t + v_t. \quad (1)$$

Here,  $x_t$ ,  $\pi_t$  and  $v_t$  denote output gap, inflation rate and cost shock in period  $t$ , respectively. Parameters  $\beta$  and  $\kappa$  are positive constants, where  $\beta$  is the discount factor and  $\kappa$  is the impact of one unit of output gap on inflation. A constant parameter  $\psi \in [0, 1]$  represents the degree of inertia in supply relation. If  $\psi = 0$ , then the model is identical to a basic New Keynesian model, which is a purely forward-looking macroeconomic model. If  $\psi = 1$ , then it becomes the traditional backward-looking macroeconomic model used in Svensson (1997). I assume that  $\{v_t\}_{t=0}^{\infty}$  follows an AR(1) process. That is, dynamics of  $v_t$  are given by

$$v_{t+1} = \rho v_t + \varepsilon_{t+1},$$

where  $\rho \in [0, 1)$  and  $\varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$ .

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<sup>1</sup>For details, see section 2.4.

<sup>2</sup>Along the line of Rogoff (1985), a central banker is called conservative if she places a larger relative weight on inflation than society.

To measure social welfare, as in Clarida et al. (1999) and Walsh (2003), I adopt the standard social loss function such that

$$\begin{aligned} L^s &= E_0 \sum_{t=0}^{\infty} \beta^t L_t^s, \\ L_t^s &= \frac{1}{2}(\pi_t^2 + \lambda^s x_t^2), \end{aligned} \tag{2}$$

where  $L_t^s$  is social loss in period  $t$ , parameter  $\lambda^s$  is the relative weight that the representative household places on output gap relative to inflation.

## 2.2 Inflation Targeting under Discretion

Suppose that the central bank pursues an inflation targeting under discretion. In this regime, in each period, the central bank minimizes a discounted sum of the current and future loss with the current state given. I set the period-loss function of the central bank in such a way that

$$L_t^c = \frac{1}{2}(\pi_t^2 + \lambda^c x_t^2), \quad \forall t \geq 0,$$

where  $\lambda^c$  is the relative weight selected by the central bank that may be different from  $\lambda^s$ . Since the (relevant) state variables in period  $t$  are  $\pi_{t-1}$  and  $v_t$ , the Bellman equation for the central bank's optimization problem is <sup>3</sup>

$$\begin{aligned} V(\pi_{t-1}, v_t) &= \min_{\pi_t, x_t} \left\{ \frac{1}{2}(\pi_t^2 + \lambda^c x_t^2) + \beta E_t V(\pi_t, v_{t+1}) \right\}, \\ \text{s.t.} \quad \pi_t &= (1 - \psi)\beta E_t \pi_{t+1} + \psi \pi_{t-1} + \kappa x_t + v_t. \end{aligned} \tag{3}$$

Thus, the necessary condition for an optimum can be obtained as

$$\pi_t = \frac{\lambda^c}{\kappa}(\beta\psi E_t x_{t+1} - x_t), \quad \forall t \geq 0. \tag{4}$$

The equations (1) and (4), the hybrid New Keynesian Phillips curve and the optimal monetary policy rule, determine the equilibrium dynamics of the model economy as the sequences of inflation and output gap.

## 2.3 Numerical Analysis

Since the optimal sequences of inflation and output gap,  $\{(\pi_t, x_t)\}_{t=0}^{\infty}$ , depend on  $\lambda^c$ , in equilibrium, the social loss (2) can be expressed as a function of parameter  $\lambda^c$ . Therefore, the optimal policy weight (denoted by  $\lambda^*$ ) can be selected by minimizing the social loss in equilibrium with respect to  $\lambda^c$ . Moreover, note that from (4) the social loss in equilibrium depends on  $\psi$ , so that the selected  $\lambda^*$  depends on  $\psi$  as well. Hence, I can obtain the relationship between the degree of inertia,  $\psi$ , and the optimal policy weight,  $\lambda^*$ .

Table 1: Baseline Parameter Value

$\lambda^s$	$\beta$	$\rho$	$\kappa$	$\sigma_\varepsilon$
0.25	0.99	0	0.05	0.015

I analyze it numerically. In doing so, I set the baseline parameter value in Table 1.<sup>4 5</sup> The numerical analysis reveals the following fact:

**Result 1 (Nonmonotonicity of Optimal Policy Weight)** *For a set of parameters with plausible magnitudes, there is  $\psi^*$  such that  $\lambda^*$  decreases with  $\psi$  for  $\psi \in [0, \psi^*]$  and increases with  $\psi$  for  $\psi \in [\psi^*, 1]$ .*

Figure 1 illustrates Result 1 based on our numerical example. Result 1 is intuitively plausible. A conservative central banker tries to stabilize inflation actively. Endogenous persistence in inflation dynamics helps such a stabilizing action directly, because a part of future inflation is controlled by current inflation through the economic agents' partially backward-looking behaviors. Thus, appointing a more conservative central banker improves a trade-off between inflation and output gap as long as the degree of inflation inertia is not too high. This is because the cost of stabilizing inflation generated by an expansion of output gap is relatively small. However, when the degree of inertia is sufficiently high, stabilizing inflation yields a large output gap: see Figure 2. Hence, in this case stronger conservatism leads to a worse trade-off between inflation and output gap. As a consequence, there is a turning point  $\psi^*$  in the relation between the degree of inflation inertia  $\psi$  and the optimal weight  $\lambda^*$ .

**Remark 1 (Nonoptimality of Conservative Central Bankers)** *When the degree of inflation inertia is sufficiently large, then it is optimal to appoint a more flexible inflation targeter than society.*

## 2.4 Discussion

The results of Clarida et al. (1999) claim that in a basic New Keynesian model without inertia, if the cost shock  $\{v_t\}_{t=0}^\infty$  is not serially correlated as related literatures often assumes, then the optimal weight on output gap in inflation targeting under discretion is identical to the social preference  $\lambda^s$ . In other words, appointing a central banker sharing the social preference is optimal. In fact, since that optimal weight  $\lambda^*$  is given by

$$\lambda^* = (1 - \beta\rho)\lambda^s, \tag{5}$$

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<sup>3</sup>As explained below,  $x_{t-1}$  is an irrelevant state variable eventually in the analysis and so it is often omitted in literature.

<sup>4</sup>The values of parameters are the same as in Jensen (2002) and Walsh (2003).

<sup>5</sup>The qualitative result is robust with respect to the parameter values.

$\rho = 0$  implies that  $\lambda^* = \lambda^s$ .<sup>6</sup> The intuition for this result is as follows. Since

$$v_{t+j} = \rho^j v_t + \sum_{i=0}^j \rho^i \varepsilon_{t+j-i}, \quad (6)$$

if  $\rho > 0$ , the future values of the cost shocks can be partially forecast by public. Noting that in the basic New Keynesian model, equilibrium inflation in inflation targeting under discretion is given by<sup>7</sup>

$$\pi_{t+j} = \frac{\lambda^c}{\lambda^c(1 - \beta\rho) + \kappa^2} v_{t+j}, \quad (7)$$

by (6) and (7), the expected inflation rate in period  $t + j$  at period  $t$  is

$$E_t \pi_{t+j} = \frac{\lambda^c \rho^j}{\lambda^c(1 - \beta\rho) + \kappa^2} v_t. \quad (8)$$

If  $\rho > 0$ , the coefficient of  $v_t$  in (8) is increasing in  $\lambda^c$ . Hence, intuitively, the rational agents, who know that a conservative central banker (i.e., one with lower  $\lambda^c$ ) reacts to the cost shocks harder, expect stable future inflation. This behavior contributes to stabilizing current inflation.<sup>8</sup> Thus, it may be called the expectations effect. Clearly, it disappears if  $\rho = 0$ .

However, Resut 1 demonstrates that this policy implication does not necessarily hold when inflation dynamics has endogenous persistence. In this case, it raises social welfare to appoint a more conservative central banker. This is because, in addition to the expectations effect, there is an inertia effect mentioned in the previous section. Note that the inertia effect is generated through a mechanism which is different from the expectations effect.

Equation (5) also means that in the absence of endogenous persistence of inflation dynamics, if  $\{v_t\}_{t=0}^{\infty}$  is serially correlated, the optimal weight on output gap is lower than  $\lambda^s$  and it monotonically decreases with the degree of exogenous persistence  $\rho$ . However, when the inflation persistence is endogenous, by the mechanism mentioned above, there is a critical value  $\psi^*$  after which  $\lambda^*$  increases with  $\psi$ : see Figure 1. That is, under inertial inflation dynamics, the behavior of the optimal policy weight is not monotone, so that stronger inflation persistence does not necessarily require a more conservative central banker: the central bank should place a higher weight on the loss from income fluctuation when inflation inertia is intense enough. Besides, the former result,  $\lambda^* < \lambda^s$ , can be reversed if inflation dynamics exhibits very strong inertia. In this case, since inflation behaves stably by itself, the gain of stabilizing income fluctuation more actively

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<sup>6</sup>See Vestin (2006).

<sup>7</sup>For the derivation, see Chapter 8 of Walsh (2010).

<sup>8</sup>In precise, since the New Keynesian Phillips curve of the basic New Keynesian model is

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + v_t,$$

the current inflation is stable if so is the expected future inflation. This is due to the forward-looking pricing behavior of the monopolistic competitive firms.

is relatively large. Hence, the central bank should place a higher weight on output gap than society, that is,  $\lambda^* > \lambda^s$ .

Next, let us clarify the difference between the expectations effect and inertia effect. Figure 3 illustrates the behaviors of the optimal policy weight for alternative values of  $\rho$ . Largely, the optimal policy weights are lower when degree of serial correlation of cost shock is large. This is because in such a case, the expectations effect is relatively strong and thus the benefit of conservative central bankers is large. The turning point  $\psi^*$  is smaller when  $\rho$  is large. Since the inertia effect generated from positive value of  $\psi$  is augmented in this case, which is illustrated in Figure 4, a relatively small value of  $\psi$  becomes the critical value at which the benefit of less conservative central bankers overtakes that of more ones.

The results suggest that under inertial inflation dynamics, the policy implication on the optimal weight on output gap in the central bank's loss function is not simple as the literature claims. The parameter value should be considered more carefully in the face of monetary policy delegation problem. As shown in empirical studies such as Ball et al.(1988) and Nishizaki and Watanabe (2000), the slopes of the Phillips curves have becoming gentler. Joining it to the result displayed in Figure 1 expands the possibility of optimality of flexible monetary policy, which is claimed in Remark 1.

One of the remaining problems is to find the degree of inertia in inflation dynamics on which monetary economists reach a consensus. According to Fact 1, the concrete policy implication about the optimal weight for the real economy depends mainly on the true degree of inertia. Rudebusch (2002) estimates  $\psi = 0.71$  for the U.S. data. Galí et al. (2005) estimate  $\psi$  by three methods and the values of the estimators are 0.349, 0.374 and 0.260.<sup>9</sup> Fuhrer (1997) demonstrates that the case  $\psi = 1$  can not be rejected. Thus, there has not been a general agreement with the value of  $\psi$ .

### 3 Conclusion

The optimal delegation problem of monetary policy does not have a monotone answer under inertial inflation dynamics supported by the recent empirical works. The result in this paper clarifies the two effects of monetary policy delegation on macroeconomic fluctuations: the expectations and inertia effects. In particular, it finds that Rogoff's (1985) conservative central bankers can be harmful in terms of social welfare when inflation inertia is strong. This is an undiscovered policy implication in the literature on theory of monetary policy.

### References

- Ball, G., Mankiw, G. and D. Romer (1988), "The new Keynesian economics and the output-inflation tradeoff", *Brookings Papers on Economic Activity* **1**, 1-65.
- Clarida, R., Galí, J. and M. Gertler (1999), "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature* **37**, 1661-1707.

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<sup>9</sup>In Galí et al. (2005), they use real marginal costs in place of output gaps of the hybrid New Keynesian Phillips curve.

- Fuhrer, J.,C. (1997), “The (Un)Importance of Forward-Looking Behavior in Price Specifications”, *Journal of Money, Credit and Banking* **29**, 338-350.
- Galí, J. and M. Gertler (1999), “Inflation dynamics: A structural econometric analysis”, *Journal of Monetary Economics* **44**, 195-222.
- Galí, J., Gertler, M. and J.D. López-Salido (2005), “Robustness of the estimates of the hybrid New Keynesian Phillips curve”, *Journal of Monetary Economics* **52**, 1107-1118.
- Nishizaki, K. and T. Watanabe (2000), “Output-Inflation Trade-off at Near-Zero Inflation Rates”, *Journal of the Japanese and International Economies* **14**, 304-326.
- Rogoff, K. (1985), “The Optimal Degree of Commitment to an Intermediate Monetary Target”, *Quarterly Journal of Economics* **100**, 1169-1189.
- Rudebusch, G., D. (2002), “Assessing Nominal Income Rules for Monetary Policy with Model and Data Uncertainty”, *Economic Journal* **112**, 402-432.
- Svensson, L. (1997), “Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets”, *European Economic Review* **41**, 1111-1147.
- Vestin, D. (2006), “Price-level versus inflation targeting”, *Journal of Monetary Economics* **53**, 1361-1376.
- Walsh, C. (2003), “Speed Limit Policies: The Output Gap and Optimal Monetary Policy”, *American Economic Review* **93**, 265-278.
- Walsh, C. (2010), “*Monetary Theory and Policy*”, 3rd ed., MIT Press.

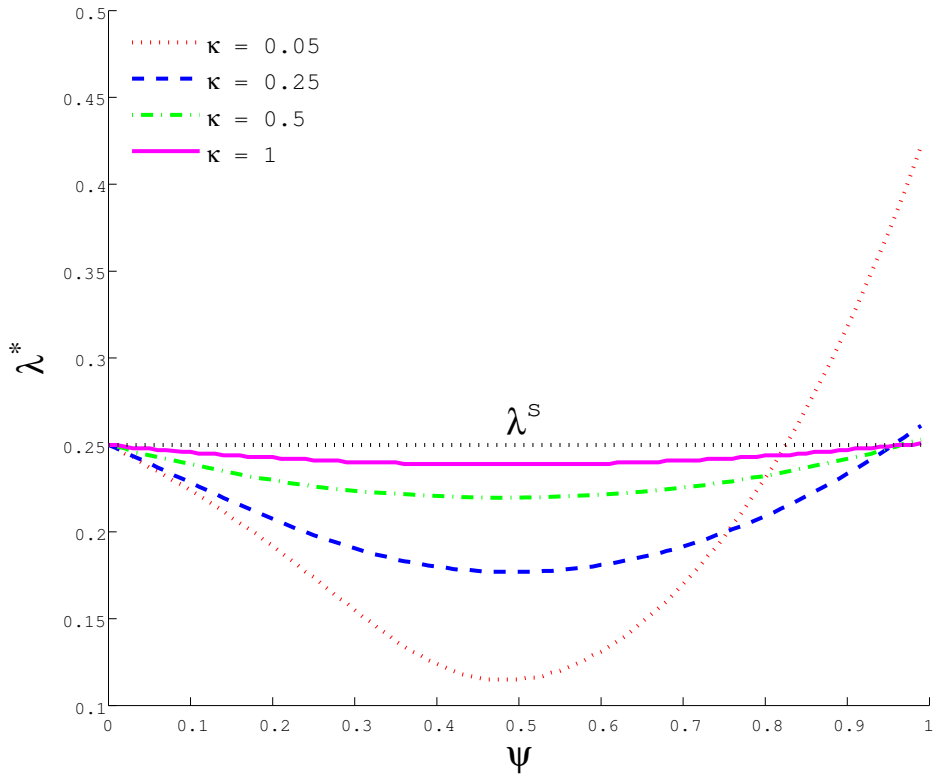


Figure 1: relationship between  $\psi$  and  $\lambda^*$  in various values of  $\kappa$



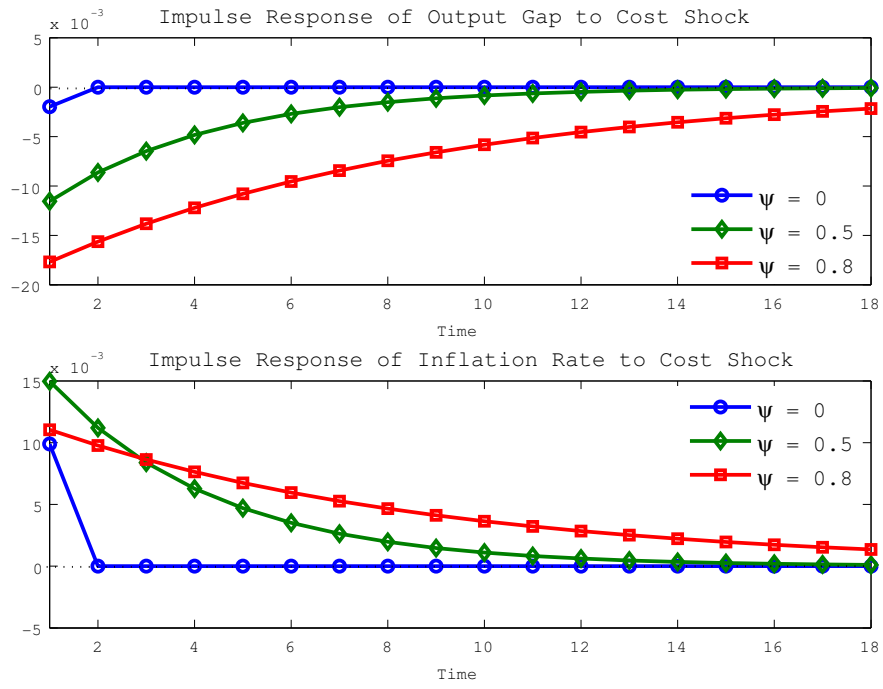


Figure 2: impulse responses of output gap and inflation rate to cost shock with various values of  $\psi$  ( $\rho = 0$ )

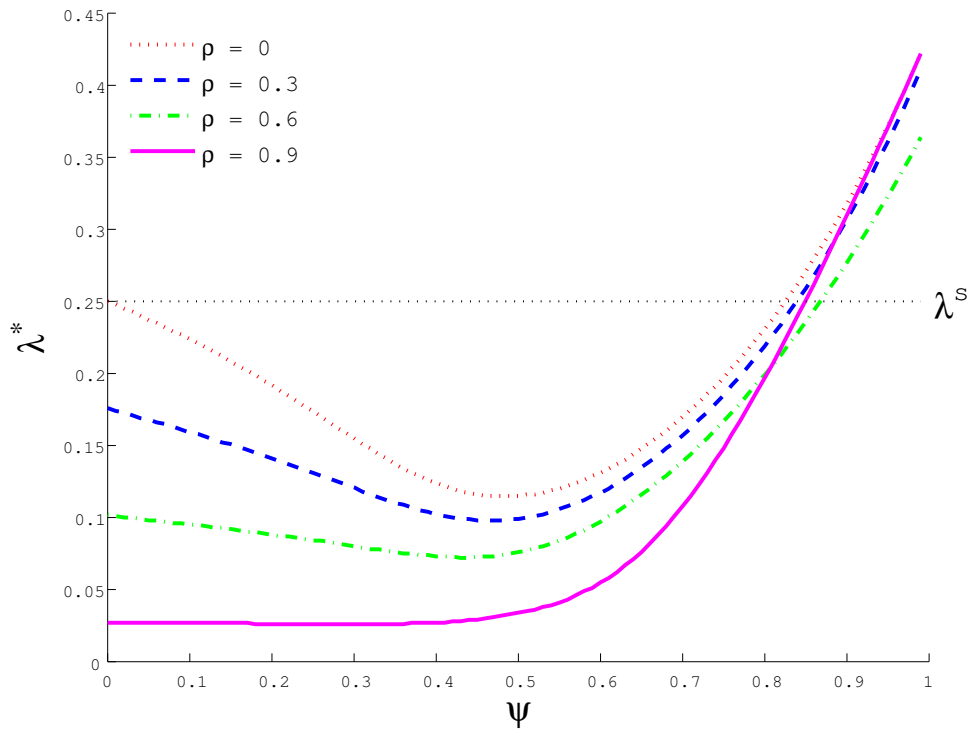


Figure 3: relationship between  $\psi$  and  $\lambda^*$  in various values of  $\rho$

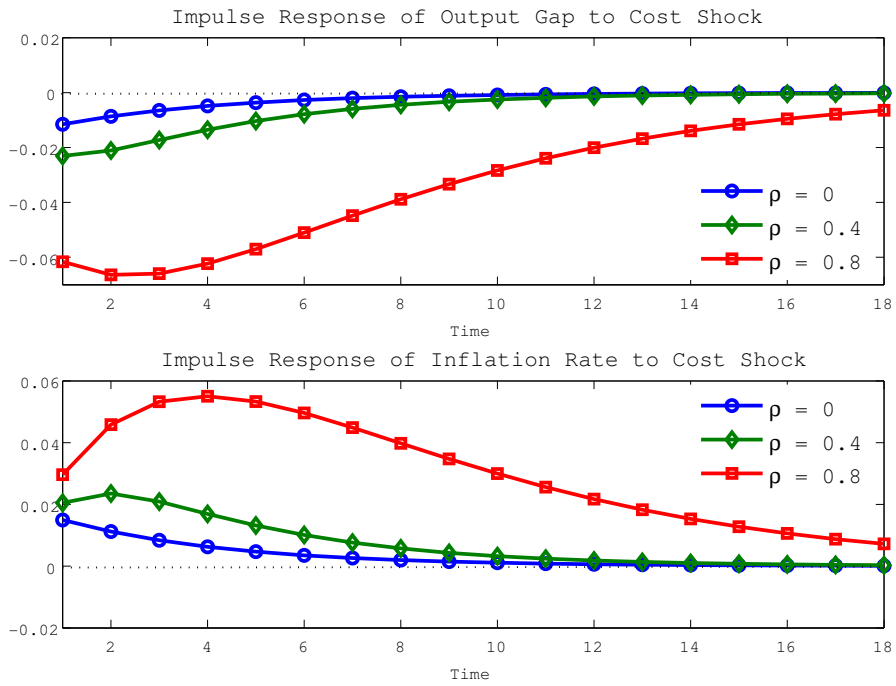


Figure 4: impulse responses of output gap and inflation rate to cost shock with various values of  $\rho$  ( $\psi = 0.5$ )