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Product differentiation in a spatial Cournot model with asymmetric demand

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Abstract

This paper considers a spatial discrimination Cournot model with asymmetric demand. We use the geographical interpretation of the linear market and introduce differentiated products. We analyze a location-quantity game and show that agglomeration or dispersed locations may arise, depending on parameter combinations. The degree of differentiation plays an important role in location choice if the demand is asymmetric. The higher the degree of differentiation between the products the more likely is agglomeration. Only cases with a low degree of differentiation and a relatively low difference in market size leads to the absence of agglomeration in the larger market.

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1. Introduction

Since the seminal contribution of Hotelling (1929) spatial models have received consistent attention by economists to answer the question whether firms would agglomerate or disperse. If firms compete in prices most theoretical models of spatial competition have found that firms will never agglomerate.[1] The reason is that price competition drives profits down to zero as the locations of firms offering identical products become nearer.

In the case of quantity competition, with full coverage of the market, Anderson and Neven (1991) and Hamilton et al. (1989) obtain that the equilibrium location of firms is characterized by agglomeration in the middle of the linear market. Both approaches use a linear market with a uniform distribution of consumers. In this context, it is obvious to interpret this model as a geographical location model, instead of a model with varying consumer tastes. Shimizu (2002) analyzes the influence of product differentiation in a location-quantity model with a uniform distribution of consumers and obtains the result that agglomeration in the center of the market remains unaffected by the degree of differentiation.

Gupta et al. (1997) use a location-quantity model with nonuniform consumer distribution and show that agglomeration occurs under a wide variety of consumer distributions. However, these authors show as well that dispersed equilibria are also consistent with Cournot competition. Liang et al. (2006) use a barbell a la Hwang and Mai (1990) and derive the influence of asymmetric demand on location choice. These authors show that agglomeration occurs if transportation costs are relatively low and dispersion occurs if transportation is relatively costly.

It is the purpose of this paper to answer the question if location choice, and thereby agglomeration, is affected by product differentiation in a spatial Cournot model with asymmetric demand or if Shimizu´s result remains stable that agglomeration is unaffected by the degree of differentiation between the products. Our analysis indicates that the degree of product differentiation has an important influence on agglomeration in the spatial Cournot model with asymmetric demand.

This paper is organized as follows. In Section 2 we develop the spatial Cournot model with asymmetric demand and product differentiation. Section 3 concludes.

2. Model

Following Hwang and Mai, the spatial structure is a line ranging from 0 to 1. Markets are concentrated in two locations connected by a transportation route, e.g. a highway. The markets are located at the endpoints of the line and the population density between these spatially separated markets is zero. To obtain an asymmetric demand structure, we have to assume that market 1 and market 2 represent different market sizes. Such models have been used to explain intra-industry trade in international trade theory[2], but could be applied to regional markets inside a country as well, for example two cities connected with a highway.

There are two firms A and B that produce at constant marginal production cost (zero without loss of generality). Each firm faces linear transportation costs of t to move one unit of output one unit of distance. Further, we assume that $t \leq \frac{1}{2}$ in order to ensure that both firms serve the whole market. Firms are able to discriminate between consumers in both markets since they control transportation. We assume that consumer arbitrage is prohibitively costly. The locations of the firms are denoted by x_j *(j=A, B)*. Without loss of generality, we assume that

[1] See, e.g., d'Aspremont et al. (1979) and Hamilton et al. (1989).

[2] See, e.g., Neven and Phlips (1985).

firm A locates to the left of firm B, i.e. $x_A \leq x_B$. Further, we assume that firms compete in quantities.

We consider the following two stage game: in the first stage, the two firms simultaneously choose their locations. In the second stage, both firms select simultaneously their quantities, given the location decision. The game is solved by backward induction.

The firms face a demand curve expressing the price of their product in terms of the production levels in market 1 and 2:

$$p_1^j = 1 - \frac{1}{a\gamma}(q_1^j + \theta q_1^{-j}), \tag{1}$$

$$p_2^j = 1 - \frac{1}{a}(q_2^j + \theta q_2^{-j}), \tag{2}$$

where p_i^j is the delivered price in market i (*i=1,2*) by firm j *(j=A,B)* and the demanded quantities in market i are denoted as q_i^j and q_i^{-j} (*-j=A,B* and $-j \neq j$), a and γ are positive constants and the relative market size is denoted by γ. If $\gamma > 1$, market 1 is relatively larger than market 2, to simplify the analysis, we assume that this condition is always satisfied.

The degree of differentiation between the products is denoted as θ. We set $\theta \in (0,1]$, to analyze the case of substitutes, where $\theta = 1$ corresponds to completely homogeneous products.[3]

The analysis starts with the second stage. The profit of firm j can be written as

$$\Pi^j = (1 - \frac{1}{a\gamma}(q_1^j + \theta q_1^{-j}))q_1^j - tx_j q_1^j + (1 - \frac{1}{a}(q_2^j + \theta q_2^{-j}))q_2^j - t(1-x_j)q_2^j. \tag{3}$$

Standard calculation of the Cournot-Nash-equilibrium[4] yields

$$q_1^{*j} = \frac{a\gamma}{4-\theta^2}(2 - \theta - 2tx_j + \theta t x_{-j}), \tag{4}$$

$$q_2^{*j} = \frac{a}{4-\theta^2}(2 - \theta - 2t + \theta t + 2tx_j - \theta t x_{-j}). \tag{5}$$

In the first stage each firms selects a profit-maximizing location given the rival´s location. Substitution of (4) and (5) into (3) and differentiation with respect to location gives

$$\frac{\partial \Pi^{*j}}{\partial x_j} = \frac{4t}{4-\theta^2}(-q_1^{*j} + q_2^{*j}), \tag{6}$$

$$\frac{\partial^2 \Pi^{*j}}{\partial x_j^2} = \frac{8at^2(1+\gamma)}{(\theta^2-4)^2} > 0. \tag{7}$$

The profit function for firm j is strictly convex with respect to location x_j, see (7), which suggests a corner solution. The Nash equilibrium location can be obtained by comparing the profits at the two corners. Because of the assumption $x_A \leq x_B$, we consider three possible solutions $(x_A, x_B) = (0,0)$, (0,1) and (1,1).

[3]We exclude the case $\theta = 0$, since this implies that products are completely differentiated, each producer is then a monopolist of its own brand.

[4]Equilibrium values are marked with "*" in the superscript.

Comparison of firm A's profits yields:

$$\Pi^{*A}(0,1) - \Pi^{*A}(1,1) = \frac{4at}{(\theta^2-4)^2}(2\gamma - 2 + t - t\gamma + \theta - \gamma\theta + t\theta\gamma) > 0. \tag{8}$$

It follows that, profits are necessarily higher to firm A, if it locates in market 1. Note that this result holds for all values of the degree of differentiation.

Given firm A locates in market 1, comparison of firm B´s profits can be written as:

$$\Pi^{*B}(0,0) - \Pi^{*B}(0,1) = -\frac{4at}{(\theta^2-4)^2}(-2\gamma + 2 - t - \theta + \theta\gamma + \theta t + \gamma t). \tag{9}$$

From (9) we can see that, in the first stage, the solution can be either (0,0) or (0,1), depending on parameter combinations. Two effects affect the location decision of firm B: the *demand effect* moves firm B to market 1, while the *competition effect* pushes firm B away from firm A. Setting (9) equal to zero and solving for t gives the following critical value of agglomeration in market 1:

$$t^* = \frac{(\gamma-1)(2-\theta)}{\gamma-1+\theta}. \tag{10}$$

We sum up the equilibrium locations of the firms in the following proposition:

Proposition 1: In equilibrium, firm A locates at $x_A^* = 0$ and firm B locates at

$$x_B^* = \begin{cases} 0, & t \le \frac{(\gamma-1)(2-\theta)}{\gamma-1+\theta} \\ 1, & t > \frac{(\gamma-1)(2-\theta)}{\gamma-1+\theta}. \end{cases}$$

Since it is the purpose of this paper to explore the impact of product differentiation on agglomeration in the barbell-model, we take a closer look at condition (10). The effect of differentiation between the products on agglomeration is given by

$$\frac{\partial t^*}{\partial \theta} = -\frac{\gamma^2-1}{(\theta-1+\gamma)^2} < 0. \tag{11}$$

The derivative (11) shows that a lower degree of differentiation, i.e. a higher value of θ, leads to a lower critical value of agglomeration in market 1 and hence agglomeration occurs for a smaller range of parameter combinations compared to a higher degree of differentiation, i.e. a lower value of θ. The reason for this is that the *competition effect* declines with rising degree of differentiation and therefore the incentive to choose a location in the larger market increases and agglomeration becomes more likely. This result is different from the one derived by Shimizu (2002), who could prove that with a uniform distribution of consumers on a linear market and differentiated products, agglomeration occurs at the center independently of the degree of differentiation.

To illustrate the effect of product differentiation, we calculate numerical values for (10). The results are reported in Table 1.

The interpretation of these critical values of agglomeration for the transportation cost is the following, all transportation cost rates below the critical value imply agglomeration, while transportation cost rates above these values result in dispersed firm locations. Note that the highest possible value of t equals .5. All values in Table 1 that are higher than .5 are highlighted in italics since they imply agglomeration for all combinations of t, θ and γ in that respective cell. The case of homogeneous products (as in Liang et al. (2006)) corresponds to

$\curvearrowleft \theta \backslash \gamma \rightarrow$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
.1	*.95*	*1.27*	*1.43*	*1.52*	*1.58*	*1.63*	*1.66*	*1.69*	*1.71*	*1.73*
.2	*.60*	*.90*	*1.08*	*1.20*	*1.29*	*1.35*	*1.40*	*1.44*	*1.47*	*1.50*
.3	.43	*.68*	*.85*	*.97*	*1.06*	*1.13*	*1.19*	*1.24*	*1.28*	*1.31*
.4	.32	*.53*	*.69*	*.80*	*.89*	*.96*	*1.02*	*1.07*	*1.11*	*1.14*
.5	.25	.43	*.56*	*.67*	*.75*	*.82*	*.88*	*.92*	*.96*	*1.00*
.6	.20	.35	.47	*.56*	*.64*	*.70*	*.75*	*.80*	*.84*	*.88*
.7	.16	.29	.39	.47	*.54*	*.60*	*.65*	*.69*	*.73*	*.76*
.8	.13	.24	.33	.40	.46	*.51*	*.56*	*.60*	*.64*	*.67*
.9	.11	.20	.28	.34	.39	.44	.48	*.52*	*.55*	*.58*
1	.09	.17	.23	.29	.33	.38	.41	.44	.47	*.50*

Table 1: Simulated critical values of agglomeration.

the case of the last row $\theta = 1$. Two effects are obvious; first; increasing market size in market 1 leads to a higher critical value,[5] and therefore agglomeration occurs for a larger set of values, second; as shown in (11), increasing product differentiation leads to a higher critical value and agglomeration is more likely. The simulated critical values show that only dispersed firm locations exist if the relative difference in market size and the degree of product differentiation is low.

3. Conclusion

We show that in contrast to the result derived by Shimizu (2002) for a linear market with uniform distributed consumers, the degree of differentiation plays an important role in location choice if the demand is asymmetric since our results differ significantly from the results of Liang et al. (2006), who analysed a similar model with homogeneous goods. The higher the degree of differentiation between the products the more likely is agglomeration. Only cases with a low degree of differentiation and a relatively low difference in market size leads to the absence of agglomeration in the larger market.

[5]This effect can be shown by $\frac{\partial t^*}{\partial \gamma} = \frac{2\theta - \theta^2}{(\theta - 1 + \gamma)^2} > 0$.

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