Abstract
Results from unit root tests applied to the bilateral China - US real exchange rate do not support purchasing power parity between the two countries. However, tests of the real equivalent exchange rate for the Chinese yuan versus a traded-weighted basket of currencies support purchasing power parity. Due to severe non-normality, critical values for tests of the real equivalent exchange rate are obtained from the wild bootstrap.
Purchasing Power Parity and the Chinese Yuan

1. Introduction

The theory of purchasing power parity (PPP) states that international arbitrage should eliminate any difference in the common currency price of identical goods in two countries. In macroeconomic terms, this implies equality between common currency price levels, thus a real exchange rate that is equal to unity. However, a variety of trade frictions (such as transportation costs, non-traded goods, and tariffs) may result in a persistent difference between common currency price levels. In the empirical literature, a stationary real exchange rate is interpreted as evidence consistent with purchasing power parity (PPP). This interpretation allows the mean of the real exchange rate to differ from unity due to the presence of trade frictions; however, deviations from the mean are quickly arbitraged away. In contrast, if the real exchange rate contains a unit root, then the common currency price levels wander apart for extended periods of time. Deviations from PPP are not quickly arbitraged away.

A vast literature has developed applying various unit root tests to real exchange rates to test the PPP hypothesis. Sarno and Taylor (2002) and Sarno (2005) provide overviews of the empirical literature in testing purchasing power parity. The general consensus is that for the currencies of major industrialized nations, PPP is a valid long-run international parity condition and that mean reversion displays significant non-linearities, with small deviations having a longer half-life than larger deviations. Econometric studies tend to find that deviations from PPP are highly volatile and that the volatility of relative prices is considerably lower than the volatility of nominal exchange rates, but recent studies using a non-linear framework to model such departures provide evidence that PPP may hold.

The conventional procedures for testing the PPP use a null hypothesis that the process generating the real exchange rate has a unit root, with an alternative hypothesis that all of the roots lie on the unit circle. However, as has been noted by Benninga and Protopapadakis (1988), Dumas (1992) and Sercu, Uppall and Van Hulle (1995), the presence of transaction costs may imply a non-linear process that may have different rates of adjustment for different sized deviations. Michael, Nobay and Peel (1997) and Taylor, Peel and Sarno (2001), using models that allow for a non-linear adjustment process in the error terms, find that the exchange rates of major currencies are well characterized by nonlinearly mean reverting processes since 1973. Using a model that allows for nonlinear reversion and for shocks from both exchange rates and relative prices, Sarno and Valente (2005) find that for most countries, long-run PPP holds, that the relative importance of the sources of shocks varies over time and that the speed of reversion is consistent with nominal rigidities suggested by conventional open economy models.

Evidence on the validity of PPP in “Developing” countries is relatively rare. The most inclusive study to date has been done by Alba and Park (2003), who use data for 65 developing countries from 1976-1999. Overall, they find weak support for PPP using linear panel unit root tests, though the evidence is much stronger for the post 1980’s period. Bahmani-Oskooee, Kutan and Zhou (2008) test for PPP in 88 developing countries and find support for stationary real exchange rates twice as often when allowing for nonlinear adjustment to the mean compared to tests with only linear adjustment. They also find that PPP holds more often for countries experiencing high inflation and for countries with more flexible exchange rates.

All of the previous tests for Chinese PPP have focused on the relative valuation of the yuan versus another single currency, usually the US dollar. Beginning with Yu (2000), followed by Yang and Xiangsheng (2004), Fink and Rahn (2005) and Coudert and Couharde (2007), linear models are used that conclude there are significant departures from PPP and
that it doesn’t hold for the yuan/renminbi. None of these previous studies allow for non-linear adjustment to PPP. The study by Ahmad and Rashid (2008) tests for non-linear adjustments to PPP for China and four other South Asia countries and find that nonlinear tests are more successful in validating PPP.

This paper extends the empirical literature on Chinese PPP in three ways. First, we test for PPP for the Chinese yuan versus both the U.S. dollar and a trade-weighted index of currencies. Secondly, we conduct a wide variety of tests, including those that allow for nonlinear reversion to mean or a discrete change in mean. Third, the wild bootstrap is used to demonstrate that results supporting PPP are robust to correction for non-normality and heteroscedasticity of the test residuals.


The real exchange rate between the United States and China in a given month \( RER_t \) is constructed as:

\[
RER_t = \left( \frac{e_t \times P_{c,t}}{P_{us,t}} \right) \tag{1}
\]

The nominal exchange rate measured in dollars per yuan is denoted \( e_t \). The price levels in China and the US are denoted respectively as \( P_{c,t} \) and \( P_{us,t} \). The monthly data span the period from January 1986 through May 2010. The data series were obtained from International Financial Statistics (IFS) and World Development Indicator (WDI) from the International Monetary Fund, with World Economic Outlook (WEO) data used to fill in some missing values. A time plot of the US-China real exchange rate is provided in Figure 1.

The RER series is subjected to a battery of unit root tests.\(^1\) The first of these is the augmented Dickey-Fuller (ADF) test. The test equation is:

\[
\Delta RER_t = \alpha + (\beta \times RER_{t-1}) + \sum_{k=1}^{K} (y_k \times \Delta RER_{t-j}) + \varepsilon_t \tag{2}
\]

where \( \Delta RER_t \) is the change in the real exchange rate, \( \alpha, \beta, \) and the \( y_k \)'s are coefficients, and \( \varepsilon_t \) is a white noise error term. Under the null hypothesis of a unit root, the \( \beta \) coefficient is equal to zero. The number of augmenting lags (K) differs depending on whether the Akaike (AIC) or Schwarz (SIC) information criteria is used; however, in all cases the test fails to reject the null hypothesis of a unit root.\(^2\) Results are presented in the second and third rows of Table 1.

The ADF-GLS test of Elliott, Rothenberg, and Stock (1996) has been used in studies such as Taylor (2002) due to potential gains in power over the standard ADF test. This test replaces the real exchange rate in (2) with the GLS detrended real exchange rate. Both AIC and SIC criteria indicate an optimal augmenting lag length of 12. Results of this test are displayed in the fourth line of Table 1. This test also fails to reject the presence of a unit root in RER.

A unit root test developed by Perron and Vogelsang (1992) next is applied to RER. This test allows for a one-time change in the level of the series both under the null and alternative hypothesis.\(^3\) The PV test equation is:

\[
\Delta RER_t = \alpha + (\beta \times RER_{t-1}) + (\delta \times DU_t) + (\theta \times DTB_t) + \sum_{k=1}^{K} y_k \times \Delta RER_{t-j} + \varepsilon_t \tag{3}
\]

\(^1\) Unless otherwise noted, the null hypothesis of each test is that the series contains a unit root, and the alternative hypothesis is a stationary series.

\(^2\) In this case, the modified AIC lag selection criterion results in the same lag length as AIC. Similarly, the modified SIC criteria results in the same lag length as SIC.

\(^3\) The innovative outlier (IO) model is used for the test.
where the shift indicators $\text{DTB}_t$ is equal to one for the period in which the level shift occurs and equal to zero for all other periods; $\text{DU}_t$ is equal to zero for period up until the level shift and equal to one for all periods afterward. The variable $\text{DTB}_t$ captures the immediate effect of the level shift, while $\text{DU}_t$ captures the after effects. The time of the level shift and the augmentation lag $K$ are chosen to maximize the evidence against the unit root null hypothesis. Asymptotic critical values are non-standard but are provided by PV. Results are provided in the fifth row of Table 1. Again, the null hypothesis of a unit root cannot be rejected.

A test developed by Kwiatkowski, Phillips, Schmidt, and Shin (1992) also is applied to the US-Chinese real exchange rate. The KPSS test differs because it has a null hypothesis of stationarity versus an alternative of a unit root. Newey-West standard errors are used to correct for serial correlation in the test equation. The test result is shown in the sixth row of Table 1. The KPSS test rejects stationarity in favor of the alternative of a unit root even using a 1% critical value.

The US-China RER is subjected to one additional type of unit root test. The nonlinear unit root test of Kapetanios, Shin, and Snell (2003) allows for ESTAR behavior. This means that the real exchange rate may display unit root behavior within boundaries but if it passes outside those bounds then it quickly returns. The RER thus may wander within bounds determined by the degree of trade frictions, but if it moves outside the bounds then arbitrage quickly moves it back inside the limits. This test has been widely employed in the PPP literature. Although fully parameterized ESTAR models are quite complicated and may suffer from identification problems, a first-order Taylor approximation yields the simple test equation:

$$\Delta \text{RER}_t = \delta^* \text{RER}^3_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta \text{RER}_{t-j} + \epsilon_t$$

(4)

The mean is removed from the real exchange rate prior to testing. The null hypothesis $H_0: \delta = 0$ then is tested versus an alternative $H_1: \delta < 0$ using the t-statistic. The null hypothesis of a unit root is rejected in favor of nonlinear reversion to trend if the test statistic lies beyond a lower critical value. Test results with the augmenting lag length chosen using AIC and SIC criteria are displayed in rows 7-8 of Table 1. This test also fails to reject the null hypothesis of a unit root in RER.

In summary, the tests used in this study fail to provide any support for PPP between the US dollar and Chinese yuan. This result holds even when the tests allow for a shift in the mean of the series or for nonlinear adjustments toward the mean. The lack of support for PPP is consistent with exchange rate interventions by the Central Bank of China.

3. PPP for the Chinese effective real exchange rate.

The real equivalent exchange rate (REER) provides a comparison of the real exchange rate for the Chinese yuan versus a weighted basket of other currencies. Monthly observations for the REER are used for the period from January 1986 through September 2006. The data was obtained from International Financial Statistics of the International Monetary Fund. Their construction of the REER follows the methodology outlined in Bayoumi et al (2006). A plot of the REER is provided in Figure 2.

An ADF test is used to investigate whether the REER is consistent with purchasing power parity. The test equation is the same as shown in equation (1) above. Both the AIC and SIC criteria indicate an optimal augmenting lag length of zero lags. The resulting ADF test statistic does not follow a t-distribution. Asymptotic critical values are provided by KSS.

---

4 The test statistic does not follow a t-distribution. Asymptotic critical values are provided by KSS.
test statistic is -4.5510. This test statistic lies beyond the 1% critical value of -3.4526, thus providing seemingly strong support for PPP. However, Arghyrou and Gregoriou (2007) show in their study of PPP that the ADF test may suffer from size distortions when the residuals are non-normal. Specifically, the null hypothesis may be rejected too often. A Jarque-Bera test indicates that the residuals of the test equation indeed are highly non-normal. Therefore, test conclusions should be based on critical values that allow for non-normality. Critical values corrected for non-normality are obtained by using the wild bootstrap procedure. This involves estimating the ADF test equation (2) by OLS and retaining the estimated residuals $\epsilon_t$ as well as the t-statistic for testing the null hypothesis. In implementing the wild bootstrap 100,000 sets of new residuals $\epsilon_t^*$ are generated according to:

$$\epsilon_t^* = \epsilon_t u_t$$

The $u_t$ variable is drawn from the two-point distribution suggested by Mammen (1993):

$$u_t = \begin{cases} \frac{1 - \sqrt{5}}{2} & \text{with probability } p = \frac{1 + \sqrt{5}}{20}, \\ \frac{1 + \sqrt{5}}{2} & \text{with probability } (1 - p). \end{cases}$$

The ADF test equation with the null hypothesis is used to create 100,000 artificial data sets. Each artificial data set is constructed by combining the original estimates of the $\gamma_j$ coefficients with one of the generated $\epsilon_t^*$ series.

The null hypothesis is true by construction for each artificial data set. In addition, the $u_t$ terms are mutually independent drawings from a distribution that is independent of the original data and has the properties $E(u_t) = 0$, $E(u_t^2) = 1$, and $E(u_t^3) = 1$. These properties imply that any non-normality present in the original $\epsilon_t$ residuals from equation (1) remains in the generated $\epsilon_t^*$ for each artificial data set.

In the wild bootstrap procedure, the artificial data sets are subjected to the ADF test to generate a vector of ordered test statistics. This vector then is used to construct the empirical distribution of the test statistics under the null hypothesis. The lower (wild bootstrapped) critical values of the test are based upon this empirical distribution. The test statistic remains equal to -4.5510. However, the empirical critical values are considerably wider than those associated with the standard ADF test. The 5% empirical critical value is equal to -4.0935, while the 1% empirical critical value is equal to -4.9758. The null hypothesis is rejected using the 5% empirical critical value. Thus, results of the ADF test with bootstrapped critical values still provide evidence in favor of PPP. However, the evidence is somewhat weakened by the use of the wild-bootstrapped critical values because the null cannot be rejected at a 1% level.

### 4. Conclusions

The results reported above for this research stand in stark contrast to previous results for unit root test of PPP. While tests of the China-US real exchange rate do not support purchasing power parity, tests of the real equivalent exchange rate for the Chinese yuan versus a trade-weighted basket of currencies supports purchasing power parity. A possible explanation is that the switching of the currency regime between different types of pegs

---

5The Jarque-Bera test statistic of 4604.12 has a marginal significance level of 0.0000.
versus the dollar has led to departures from equilibrium that are mitigated by using a trade-weighted basket.

References


Table 1
Tests of RER

<table>
<thead>
<tr>
<th>Test</th>
<th>Augmenting Lag Length</th>
<th>Test Statistic</th>
<th>5% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>12</td>
<td>-1.9297</td>
<td>-2.8717</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-2.0252</td>
<td>-2.8712</td>
</tr>
<tr>
<td>ADF-GLS</td>
<td>12</td>
<td>-0.3339</td>
<td>-1.9419</td>
</tr>
<tr>
<td>PV</td>
<td>6</td>
<td>-3.7533</td>
<td>-4.4400</td>
</tr>
<tr>
<td>KPSS</td>
<td>----</td>
<td>1.2174**</td>
<td>0.4630</td>
</tr>
<tr>
<td>KSS</td>
<td>12</td>
<td>-1.1649</td>
<td>-2.9300</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-1.5294</td>
<td>-2.9300</td>
</tr>
</tbody>
</table>

**Denotes significance using a 1% test size.