Competing impure public goods and the sustainability of the theater arts

Tyler Pugliese  
Economics Dept., Rochester Institute of Technology

Jeffrey Wagner  
Economics Dept., Rochester Institute of Technology

Abstract
The general purpose of this paper is to extend the literature regarding public good provision when consumers may contribute via consumption of an impure public good and/or by donating directly to the public good. Standard models pose consumer utility as a function of one impure public good and one or more private goods. Our model features two competing impure public goods and two private goods: one that is a conventional substitute good and one that is a numeraire. We build most directly upon Kotchen’s (2005) model of “green” consumption of impure public goods. We propose national and local live theater arts as an example of competing impure public goods. Our model shows that if local and national live theater are substitutes, and the national live theater (such as the Met) is strengthened via technological change (for instance, via simulcasts into local venues), the overall sustainability of the live theater arts may be diminished.

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1. Introduction

There is a rich literature regarding the economics of public good provision when consumers may contribute by consuming a product that jointly yields private and public benefits and/or by making a donation that directly supports the public good. The economics of contributing via consumption of a joint/mixed/impure product was set forth by Cornes and Sandler (1984, 1994). The impact of a donation mechanism alongside joint production was analyzed by Andreoni (1989, 1990) and Vicary (1997, 2000). Most recently, Kotchen (2005) extended this literature with a model in which consumers choose among three products: a numeraire good, an impure public good that generates both a private and public characteristic, and a private conventional good substitute for the impure public good that generates only the private characteristic. He then adds the donation possibility to this generalization and conducts comparative statics analysis within the extended framework. Although Kotchen’s model generates a number of insights, all of the aforementioned models feature a single impure public good competing against one or more private goods. The general purpose of this paper is to extend this line of inquiry to a framework in which consumers allocate consumption and donation resources among private goods and two competing impure public goods. We find that the seemingly straightforward step of adding a second impure public good to the consumer choice framework greatly complicates the comparative statics results. While this is in some sense discouraging, the generalized model nevertheless enables us to point to specific sources of the complications that can arise in real-world policy-making.

While the extension we pursue is of theoretical interest, we were motivated to explore it by a specific empirical question: Could the New York Metropolitan Opera’s relatively recent deployment of live simulcasts in hundreds of theaters across the U.S. be expected to strengthen or weaken local theater arts quantity/quality, and therefore strengthen or weaken the long-run sustainability of the theater arts in general? As Baumol and Bowen (1966) forewarned, technological progress presents something of a double-edged sword to the sustainability of the theater arts, not unlike the challenge technological progress poses to environmental quality and other public goods. On one hand, the popularity of such technologically sophisticated Met simulcasts can be taken as evidence that the sustainability of the theater arts has been enhanced. On the other hand, if such simulcasts from a national institution drive live local theater from the market, the effect of simulcast technology on the overall sustainability of theater arts is ambiguous. Our instinct is that the Met and local theater productions can be considered competing impure public goods, and that therefore a conceptual structure like Kotchen’s, extended to two impure public goods, could help us understand how a number of exogenous factors combine to affect the sustainability of the theater arts. Our resulting model confirms that if local and national live theater are substitutes, and the national live theater (such as the Metropolitan Opera of New York and its simulcasts into local venues) is strengthened via technological change, the overall sustainability of the live theater arts may be diminished.

2. Basic Concepts and Notation

We proceed by setting forth Kotchen’s basic framework and then we introduce the notation for the second impure public good. Kotchen follows Cornes and Sandler (1984, 1994) in

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1 Recent data suggests that 1.8 million tickets to simulcasts were sold in 2009 and sales were on pace to surpass 2.25 million in 2010. [http://online.wsj.com/article/SB10001424052702303695604575182474127585754.html?mod=WSJ_newsreel_lifeStyle]
utilizing the characteristics approach to consumer demand that was developed by Gorman (1980) and Lancaster (1971). Consider a representative consumer who has well-defined preferences $U$ over four characteristics: $X$, $Y$, $W$, and $Z$. Characteristic $X$ could be mass media entertainment or caffeine that is generated by consumption of conventional private goods such as movie theater tickets in the former case and by coffee in the latter (Kotchen) case. Characteristic $Y$ is theatrical arts quality, cultural quality, or environmental quality on a broad, national scale that is provided by impure public goods such as the New York Metropolitan Opera in the former case or by shade-grown coffee in the latter case. Characteristic $Y$ is valued by consumers to the extent that it captures what national arts endeavors such as the Met generate in utility to consumers in the current generation, as well as what they preserve for consumption by future generations on a national (if not global) level. We contrast this source of utility from national public goods with characteristic $W$, which represents what is valued by consumers in local theater arts in the former case and local environmental quality in the latter case. Characteristic $W$ captures what consumers feel is distinctive about live theater focused upon local issues and produced by local artists. Such productions therefore yield local benefits as well as public benefits that are passed to future generations and become part of the national arts legacy. It is of course true that citizens of New York City may view the Met as a “local” theater; however, our belief is that local support for the Met is not so much with regard to preserving local theater as it is for preserving the Met as a national institution that produces and preserves theater that resonates in every state and village. Finally, following Kotchen, we define characteristic $Z$ as capturing in a numeraire manner all other characteristics of value to the consumer beyond characteristics $X$, $Y$, and $W$.

Turning now to the goods that generate these characteristics, we follow Kotchen in allowing for the impure public goods to generate multiple characteristics in (constant) linear fashion. Good $l$ (local live theater) generates characteristics $X$, $Y$ and $W$ according to:

\begin{align*}
X & = \alpha_l l \\
Y & = \beta_l l \\
W & = \gamma l 
\end{align*}

Good $n$ (national live theater) generates characteristics $X$ and $Y$ (and not $W$) according to:

\begin{align*}
X & = \alpha_n n \\
Y & = \beta_n n 
\end{align*}

Good $c$ (conventional good such as movie tickets or DVDs) generates only characteristic $X$:

\begin{align*}
X & = c 
\end{align*}

Good $z$ (numeraire good) generates only characteristic $Z$:

\begin{align*}
Z & = z 
\end{align*}

The last element in the set-up to note, again following Kotchen, is that since goods $l$ and $n$ are impure public goods, the representative consumer gains utility from others’ provision of these goods. Thus, the amounts of $Y$ and $W$ that are produced are given by the sums of what the representative agent chooses and what all other agents choose. The notation for this aspect will be specified below, where we set forth the representative agent’s maximization problem.

3. Optimal Choices in the Presence of a Conventional Substitute

We follow Kotchen in this section by introducing a conventional substitute to our model in which a representative consumer has exogenous income $m$ to allocate across that conventional good, a numeraire good, and two impure public goods (one local and one national). Given our assumptions, the consumer’s problem is:
\[
\text{Max}\left[ U(Z, X, Y, W) \right] = Z + P_c X + (P_n - P_c) Y + \frac{(P_l - P_n)}{\gamma} \tilde{W} + \frac{(P_l - P_n)}{\gamma} \tilde{W} = m + \tilde{W} \left[ \frac{\alpha_n P_n}{\beta_n} \gamma + \frac{P_l}{\gamma} \beta_n \gamma \right] + \frac{P_l}{\gamma} \beta_n \gamma \left[ \frac{P_n}{\beta_n} - \frac{\alpha_n P_c}{\beta_n} \right]
\]

Note that in Kotchen’s eq. (2), \( W = 0; \alpha_n = \beta_n = 1; \alpha_l = \beta_l = 0; \) and his \( P_n \) is equivalent to our \( P_n \). In our work below, we maintain Kotchen’s assumption that \( \alpha_n = \beta_n = 1 \); and in that spirit, to simplify notation, we set \( \alpha_l = \beta_l = 1 \) as well to obtain (13):

\[
\text{Max}\left[ U(Z, X, Y, W) \right] = \left( P_n - P_c \right) Y + \left( P_l - P_n \right) \tilde{W} = m + \left( P_n - P_c \right) \tilde{Y} + \left( P_l - P_n \right) \tilde{W}
\]

Assuming that \( P_l > P_n > P_c \) (to ensure positive implicit characteristic prices) and that an interior solution exists, we now wish to conduct comparative statics on the demand for national and local theater quality, \( \hat{Y} \) and \( \hat{W} \) (analogous to Kotchen’s environmental quality, \( \hat{Y} \)) with respect to our exogenous parameters. (Superscript \( c \) denotes the case in which there is a conventional substitute good available.) As Kotchen sets forth, the demand for national theater quality in our model is an implicit function of the price of characteristics \( Y \), \( X \) and \( W \), as well as effective income (the right-hand side of eq. (13)). In the case of \( Y \), we have \( \hat{Y} \) of \( (\pi_y, \pi_x, \pi_w, \Gamma) \), where \( \pi_y = P_n - P_c \), \( \pi_x = P_c \), \( \pi_w = \frac{1}{\gamma} (P_l - P_n) \), and

\[
\Gamma = m + \tilde{W} \left( \frac{1}{\gamma} (P_l - P_n) \right) + \tilde{Y} (P_n - P_c). \quad \hat{W} \text{ is defined analogously.}
\]

As Kotchen describes, we

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2 While we model the representative agent’s utility as increasing in the linear sum of public good contributions from others, we are grateful to the anonymous referee for calling our attention to the possibility that a best-shot technology may be more appropriate in the national/international theater arts context. We should like to explore this possibility more fully in future research.
would then want to differentiate this function with respect to each parameter and substitute
the appropriate Slutsky decompositions into each of those results.

The first comparative static we wish to consider is the effect of a change in \( P_n \) on \( \hat{Y}^e \).
Letting subscripts denote partial derivatives, we have the following:

\[
\hat{Y}_{P_n}^e = (\bar{Y}_x^e - \hat{Y}_x^e \hat{Y}_y^e)(1) + 0 + (\bar{Y}_x^e - \hat{W}^e \hat{Y}_y^e) \left( -\frac{1}{\gamma} \right) + \hat{Y}_t^e \left( -\frac{\hat{W}^e}{\gamma} + \hat{Y}^e \right) \tag{14}
\]

A bit of algebra simplifies (14) to:

\[
\hat{Y}_{P_n}^e = (\bar{Y}_x^e - \hat{Y}_x^e) + \hat{Y}_t^e \left( \hat{I}^e - \hat{n}^e \right) \tag{15}
\]

Note first that Kotchen’s result immediately follows from our eq. (14) by setting \( \bar{Y}_x^e \) and \( \hat{I}^e \)
equal to zero. (Another way to see the difference in our models is to write eq. (15) as
\[
\hat{Y}_{P_n}^e = (\bar{Y}_x^e - \hat{Y}_x^e) - \bar{F}_x^e + \hat{Y}_t^e \hat{I}^e, \text{ where the two new terms follow Kotchen’s result in the}
parentheses.) The interpretation of these terms and the resulting sign is as follows. The first
term on the right hand side is the own price substitution effect (negative). The next term is
the cross-price effect, which is positive if \( W \) and \( Y \) are substitutes and negative if they are
complements. Thus, the terms in the first bracket yield a negative sign if \( W \) and \( Y \) are
substitutes (the case we feel is most likely). The \( \hat{Y}_t^e \) term is positive for normal goods (which
we assume) and negative for inferior goods. Lastly, the terms in the right hand bracket
represent the relative quantities of \( l \) and \( n \), respectively. This of course can be positive
or negative. If it is negative (which is to say, if \( \hat{I}^e \leq \hat{n}^e \)), then \( \hat{Y}_{P_n}^e < 0 \) as Kotchen finds in his
analysis when only one impure public good and a conventional substitute are available.
Indeed, his result immediately obtains from our eq. (15) above by setting \( \bar{Y}_x^e \) and \( \hat{I}^e \)
equal to zero. However, if \( \hat{I}^e > \hat{n}^e \), and/or if \( Y \) and \( W \) are complements rather than substitutes, \( \hat{Y}_{P_n}^e \) can
be positive. When \( P_n \) rises, \( n \) falls and therefore \( \hat{Y}^e \) falls. But if \( l \) is a substitute for \( n \), then
\( \hat{Y}^e \) rises with \( l \), (since both \( l \) and \( n \) produce \( Y \)). The greater \( l \) that is chosen relative to \( n \), the
more likely \( \hat{Y}_{P_n}^e \) is to be positive.

Using the same methodology, we have the following additional comparative statics
and analogous interpretations:

\[
\hat{W}_{P_n}^e = (\bar{W}_x^e - \hat{W}_x^e) + \hat{W}_t^e (\hat{I}^e - \hat{n}^e) \tag{16}
\]

\[
\hat{Y}_{P_n}^e = (\bar{Y}_x^e - \hat{Y}_x^e) + \hat{Y}_t^e (\hat{n}^e - \hat{\tilde{c}}^e) \tag{17}
\]

\[
\hat{W}_{P_n}^e = (\bar{W}_x^e - \hat{W}_x^e) + \hat{W}_t^e (\hat{n}^e - \hat{\tilde{c}}^e) \tag{18}
\]

Turning now to the impact that the price of the local impure public good may have upon the
quality of the national public good, we have:

\[
\hat{Y}_{P_l}^e = \bar{Y}_x^e + \hat{Y}_t^e (-\hat{I}^e) \tag{19}
\]

\[
\hat{W}_{P_l}^e = \bar{W}_x^e + \hat{W}_t^e (-\hat{I}^e) \tag{20}
\]

Here the results are clearer in that we have only one ambiguous term rather than two. The
first term of eq. (19) is the cross-price effect, which is positive if characteristics \( W \) and \( Y \) are
substitutes and negative if they are complements. Thus, if the public good characteristics are
complements, the sign of eq. (19) is clearly negative and an increase in the price of local live
theater can reduce the quality of nationally provided theater. However, if the public good
characteristics are substitutes, an increase in the price of \( l \) raises the implicit price of \( W \),
leading consumers to favor more Y as a substitute. This substitution can be strong enough to cause an overall increase in Y when the price of local theater tickets increases.

The last set of comparative statics to examine in this context regards technology parameters. Unfortunately, the results (available from the authors upon request) are quite complicated and do not yield clear implications without a number of assumptions. The result for $\hat{W}_{\beta_n}$ deserves special consideration, however, as it speaks to the empirical question with which our paper began: could a technological improvement in the provision of a national public good—e.g., the introduction of live simulcasts of the New York Metropolitan Opera—reduce the provision of live local theater? That comparative static is given by:

$$W_{\beta_n}^* = \left( P_n - P_c \alpha_n \right) \left[ \frac{\beta_l}{\gamma} \left( \frac{W_n^*}{\gamma_s} - \hat{W}_n^* \gamma + \hat{W}_n^* \gamma \right) - \left( \frac{\gamma}{\gamma_s} - \hat{Y} \gamma + \hat{Y} \gamma \right) \right]$$  \hspace{1cm} (21)

The answer to our question is in the affirmative if either the first or second bracketed terms in eq. (21) are negative and the other is positive. In terms of the first bracketed term, even though $P_n > P_c$ by assumption, this does not imply that $P_n > \alpha_n P_c$. Thus, the first bracketed term could be positive or negative. Likewise, inspection of the terms within the second set of brackets reveals that the entire term can be positive or negative. Thus, while our comparative static analysis is not able to determine this sign, the analysis nevertheless shows why $\hat{W}_{\beta_n}$ can be negative, and therefore how a technological improvement in the provision of one public good can undermine provision of related public goods.

4. Donation Parameters

As with Kotchen’s impure public good model, our model can include a donation option. Considering how dependent theaters typically are upon these donations, it is important to extend our model likewise in this direction.\(^3\)

The representative consumer’s problem is thus:

$$\max U(Z, X, Y, W) \left[ z + P_c c + P_l l + P_n n = m \right] \text{ when } c > 0 \text{ and } d = 0$$

(22)

(which is the same as eq. (8)) and $X$, $Y$, and $W$ are defined as in eqs. (9)-(11) and

$$\max U(Z, X, Y, W) \left[ z + P_d d + P_l l + P_n n = m \right] \text{ when } c = 0 \text{ and } d > 0$$

(23)

where $X$, $Y$, and $W$ are defined as follows:

$X = \alpha_n l + \alpha_n n$  \hspace{1cm} (24)

$Y = \beta_n l + \beta_n n + \hat{Y} + d$  \hspace{1cm} (25)

$W = \gamma l + \hat{W} + d$  \hspace{1cm} (26)

While donations in Kotchen’s model generate characteristic $Y$, donations in the present model generate both public characteristics $Y$ and $W$. As in the previous section, to simplify notation in the work that follows, we set $\alpha_i = \beta_i = 1$ for all $i$ but preserve parameter $\gamma$ for the time being. As Kotchen (2005, 295-296) notes, the dual constraints emerge in our eqs. (22) and (23) from reasonable assumptions regarding relative prices. In our model, those assumptions

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are that $P_i > P_d > P_n > P_c$, $P_c + P_d > P_i$, and $P_d > \frac{P_n - P_i}{\gamma}$. For instance, we might have $P_i = 20$; $P_d = 16$; $P_n = 10$; $P_c = 5$; and $\gamma = 1.4$. This implies that a consumer would not choose to make a donation and consume the conventional good. For analogous to Kotchen’s observation in the one-impure-public good framework, any amounts of characteristics $X$, $Y$, and $W$ the consumer seeks by a combination of $c$ and $d$ can be acquired at a lower cost by increasing consumption of good $l$ and reducing consumption of $c$ and donations $d$. Thus, $c + d$ cannot be part of an optimal bundle.

Now, as in the previous sections, a good bit of algebra can be carried out with regard to eqs. (23)-(26) to transform the maximization problem from one based upon goods to one based upon characteristics. Doing so yields:

$$
\begin{align*}
\text{Max} \left[U(Z, X, Y, W)\right] & = Z + \left[\frac{P_i + (\gamma - 1)P_n}{\gamma} - P_d\right] X + \left[\frac{P_d + P_n - P_i}{\gamma}\right] Y + \left[\frac{P_i - P_n}{\gamma}\right] W = \\
n + \left[\frac{P_d + P_n - P_i}{\gamma}\right] \tilde{Y} + \left[\frac{P_i - P_n}{\gamma}\right] \tilde{W}
\end{align*}
$$

when $c = 0$ and $d > 0$. (Note that since eq. (22) is the same as eq. (13) (the conventional good substitute is purchased in lieu of making donations), we analyze only the problem expressed in eq. (23)). The demand for characteristic $Y$ can be expressed as $\hat{y}^d = \hat{y}^d(\pi_y, \pi_x, \pi_w, \Gamma)$, where the superscript $d$ denotes the scenario in which donations are made, and implicit prices of the characteristics $X$, $Y$, and $W$ are of course given by $\pi_y = \frac{P_d + (\gamma - 1)P_n}{\gamma}$, $\pi_x = \frac{P_i + (\gamma - 1)P_n}{\gamma} - P_d$, $\pi_w = \frac{P_d + P_n - P_i}{\gamma}$, and $\Gamma = n + \tilde{W}\left[\pi_y \Gamma\right] + \tilde{Y}\left[\pi_x \Gamma\right]$.

We are able to derive a number of comparative statics (available from the authors upon request). As was the case in Section 3 (the case in which there is a conventional substitute good available), we are not able to definitively sign any of the comparative statics when a donation mechanism is also in place. Indeed, consistent with our intuition, the presence of a donation mechanism serves to further complicate the relationship between the two impure public goods in our model. To round out our analysis, we are also able to generalize eq. (27) by retrieving the technology parameters $\alpha$ and $\beta$ that were previously set equal to one. Our purpose in doing so is to return to the primary empirical question of our interest: could a technological improvement in the provision of a national public good—e.g., the introduction of live simulcasts of the New York Metropolitan Opera—reduce the provision of live local theater, and therefore on balance undermine the sustainability of the theater arts? The generalization of eq. (27) yields:

$$
\begin{align*}
\text{Max} \left[U(Z, X, Y, W)\right] & = Z + X \left[\frac{P_i + P_n(\gamma - \beta_i) - P_d}{\beta_n}\right] + Y \left[\frac{\alpha_n}{\beta_n}(\gamma - \beta_i) + c_i\right] + \left[\frac{\alpha_n}{\beta_n}(\gamma - \beta_i) + \beta_i\right] + \frac{P_n}{\beta_n}
\end{align*}
$$

Note, however, that these relative prices, as well as those in eq. (13), would be more complicated if all $\alpha$'s and $\beta$'s had not been set equal to one.
We are then theoretically in the position of being able to derive $\hat{W}^d_{\beta_n}$ and have confirmed that its sign, analogous to the sign of $\hat{W}^c_{\beta_n}$, is indeterminate. Thus in both contexts in which a conventional substitute good is available and when a donation mechanism is also available, we are able to say that technological improvement in the provision of one impure public good has a potentially deleterious effect upon the provision of related impure public goods. If this effect is strong enough, such technological improvement that may be expected to raise the overall sustainability of the theater arts may actually reduce sustainability.

5. Directions for Future Research

One area for future research is to consider possible policy intervention tools that can raise the sustainability of the local good when there is evidence that the market is not naturally providing the socially efficient rate of support. One analogue to this situation may be the use some local communities have made of tax increment financing of historic downtown properties following the construction of the interstate highway system on the perimeters of towns. In such cases, the tax revenue increment is collected from new businesses established along the non-local public good (highway) and transferred to local, historic areas adversely affected by the shift in economic activity. Secondly, in the process of utilizing Kotchen’s economic model of environmental quality provision to analyze the sustainability of the theater arts, we realized that while these topics may be considered separately, consumer choices in one realm are likely related to consumer decisions in the other realm. That is to say, consumers who are inclined to support environmental quality with their purchases and/or donations are quite likely to be consumers who are inclined to support the arts. Thus, where Kotchen finds that the introduction of a donation mechanism can reduce environmental quality, our model sheds light on a further possibility: consumers who raise their consumption of—or donation to—one public good (such as the theater arts) may significantly reduce their consumption of—or donation to—another public good (such as environmental quality). We wish to explore this dynamic further in future research.

References


5 The derivation is available from the authors upon request.


