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Estimating value at risk and optimal hedge ratio in Latin markets: a copula-based GARCH approach

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Abstract

In this paper we use a copula-based GARCH model to estimate conditional variances and covariances of the bivariate relationships between U.S. market with Brazilian, Argentinean and Mexican markets. To that we used daily prices of S&P500, Ibovespa, Merval and IPC from January 2009 to December 2010, totaling 483 observations. The results allows to conclude that both the volatility of Latin markets, such as its dependence with the U.S. decreased in the period, resulting in lower estimates for the VaR and Hedge, compared with those based on the unconditional variance and covariance, emphasizing that after theeffects of the 2007/2008 U.S. crisis, these Latin markets can again be considered as options for international diversification for investors with assets of the U.S. market in their portfolio.

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1. Introduction

Managing and monitoring major financial assets are routine for many individuals and organizations. Therefore careful analysis, specification, estimation and forecasting the dynamics of returns of financial assets, construction and evaluation of portfolios are essential skills in the toolkit of any financial planner and analyst (Caporini and McAleer, 2010).

Within this context, the knowledge of the stochastic behavior of correlations and covariances between asset returns is an essential part in asset pricing, portfolio selection and risk management (Baur, 2006). The study of volatility is therefore of great importance in finance, particularly in derivative pricing and risk management of investments. Traditionally the calculation of estimates of the volatility of financial returns as well as its application in determining the value at risk (VaR) or hedge a portfolio rely on the daily changes in asset prices (Goodhart and O'hara, 1997).

Since the proposal of Generalized Auto-Regressive Conditional Heteroscedastic (GARCH) family models by Engle (1982) and Bollerslev (1986) to account for variance heterogeneity in financial time series, a huge number of multivariate extensions of GARCH models have been introduced. The most consolidated models in literature are the Constant Conditional Correlation (CCC-GARCH) model of Bollerslev (1990), the BEKK model of Engle and Kroner (1995) and later the Dynamic Conditional Correlation (DCC-GARCH), developed by Engle and Sheppard (2001) and Tse and Tsui (2002). These models are based on multivariate Gaussian distributions, where care has to be taken to result in positive definite covariance matrices.

However, this assumption is unrealistic, as evidenced by numerous empirical studies, in which it has been shown that many financial asset returns are skewed, leptokurtic, and asymmetrically dependent (Longin and Solnik, 2001; Ang and Chen, 2002; Patton, 2006). Hence, these characteristics should be considered in the specifications of any effective hedging model or estimative of a portfolio's VaR.

These difficulties (Gaussian assumption and joint distribution modelling) can be treated as a problem of Copulas. A copula is a function that links univariate marginals to their multivariate distribution. Since it is always possible to map any vector of random variables into a vector with uniform margins, we are able to split the margins of that vector and a digest of the dependence, which is the copula. The concept of copula was introduced by Sklar (1959) and studied by many authors such as Deheuvels (1979), Genest and MacKay (1986). The use of copulas for modeling the residual dependence between assets was recently appeared in empirical studies (Jondeau and Rockinger, 2006; Ausin and Lopes, 2010; Min and Czado, 2010).

In this sense, the present study attempts to improve the effectiveness of dynamic hedging and VaR estimation by specifying the joint distribution of bivariate shocks of U.S. and Latin American financial markets (Brazil, Argentina and Mexico) considering the period after the recent financial crisis of 2007/2008, in order to avoid possible vestiges of the it. The sample is formed by daily prices of S&P500, Ibovespa, Merval and IPC from January, 3, 2009 to December, 31, 2010, totaling 483 observations. These Latin American emerging markets rank among the most mature markets within the universe of emerging countries and they actually attract a particular attention from global investors thanks to their great market openness (Arouri *et al.*, 2008).

Recent studies employ various bivariate conditional volatility models to estimate a time-varying hedge ratio and VaR, demonstrating that generally the dynamic strategy can result in greater risk reduction than the static one (Brooks and Chong, 2001; Choudhry, 2003). The superiority of the time-varying hedge ratio essentially comes from taking account of the changing joint distribution of returns (Hsu, Tseng and Wang, 2008).

We fitted a copula-based GARCH model for the estimation of the conditional volatilities of the bivariate relationship of the Latin Markets with the U.S. market. Thus, we made a forecast of conditional variances and covariances of these markets in order to estimate the optimal hedge ratio and the VaR of each index, taking in consideration the American market, due to its influence. Without the assumption of multivariate normality, the joint distribution can be decomposed into its marginal distributions and a copula, which can then be considered both separately and simultaneously.

2. Multivariate Volatility Modeling

Multivariate models of volatility have attracted considerable interest during the last decade. This may be associated with increased availability of financial data, the increasing of the processing capacity of computers, and the fact that the financial sector began to realize the potential advantages of these models.

But when it comes to the specification of a multivariate GARCH model, there is a dilemma. On one hand, the model should be flexible enough to be able to represent the dynamics of variance and covariance. On the other, as the number of parameters in a multivariate GARCH model often increases rapidly with the size of assets, the specification must be parsimonious enough to allow the model to be easily estimated with relative ease, as well as allowing a simple interpretation of its parameters.

A feature that must be taken into account in the specification is the restriction of positivity (covariance matrices must necessarily take its determinants defined as positive). Based on this idea, consider the model with multivariate GARCH parameterization VECH, proposed by Bollerslev, Engle and Wooldridge (1988), represented by (1).

$$vech(H_t) = A_0 + \sum_{j=1}^{q} \beta_j vech(H_{t-1}) + \sum_{j=1}^{p} A_j vech\left(\varepsilon_{t-j}, \varepsilon'_{t-j}\right). \quad (1)$$

In [1], *vech* is the operator that contains the lower triangle of a symmetric matrix into a vector; H_t describes the conditional variance; the error term is$\varepsilon_t = H_t^{1/2}\eta_t, \eta_t \sim iidN(0,1)$. The disadvantage of this model is that it has a large number of parameters and in order to ensure the positivity of H_t, restrictions must be imposed.

Thus, emerges the BEKK parametrization as an alternative, as suggested by Engle and Kroner (1995). The BEKK parameterization, which essentially takes care of the problems mentioned above about the VECH model, is defined as shown in (2).

$$H_{t+1} = C'C + B'H_tB + A'\epsilon_t\epsilon'_t. \quad (2)$$

The matrices A, B and C, which contain the coefficients for the case with two assets, are defined as:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},\ B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix},\ C = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}. \quad (3)$$

In (2), H_{t+1} is a conditional covariance matrix. In the bivariate case, the parameter *B* explains the relationship between the past conditional variances with the current ones (GARCH). The parameter *A* measures the extent to which conditional variances are correlated with past squared errors, i.e. it captures the effects of shocks or volatility (ARCH). The total number of estimated parameters in bivariate occasion is eleven. In this case, the BEKK parameterization, the volatilities of the equation (2) have the forms (4) and (5).

$$h_{11,t+1} = c_{11}^2 + b_{11}^2 h_{11,t} + 2b_{11}b_{12}h_{12,t} + b_{21}^2 h_{22,t} + a_{11}^2\varepsilon_{1,t}^2 + 2a_{11}a_{12}\varepsilon_{1,t}\varepsilon_{2,t} + a_{21}^2\varepsilon_{2,t}^2. \quad (4)$$

$$h_{22,t+1} = c_{12}^2 + c_{22}^2 + b_{12}^2 h_{11,t} + 2b_{12}b_{22}h_{12,t} + b_{22}^2 h_{22,t} + a_{12}^2\varepsilon_{1,t}^2 + 2a_{12}a_{22}\varepsilon_{1,t}\varepsilon_{2,t} + a_{22}^2\varepsilon_{2,t}^2. \quad (5)$$

However, the BEKK model parameterization has the disadvantage of being difficult to interpret its estimated parameters. The formulations (4) and (5) show that even for the case of bivariate modeling, the interpretation of the coefficients can be confusing because there are no parameters that are governed exclusively by an equation (Baur, 2006).

Thus, an approach to circumvent the problem of interpretation of the parameters is the model of conditional covariance matrix, observed indirectly through the matrix of conditional correlations. The first such model was the constant conditional correlation (CCC) proposed by Bollerslev (1990) and Bollerslev and Wooldridge (1992). The conditional correlation was assumed to be constant and only the conditional branches are variable in time. The CCC model can be defined as the formulation (6).

$$H_t = D_t R D_t = \left(\rho_{ij}\sqrt{h_{ii,t}h_{jj,t}}\right). \tag{6}$$

In the formulation (6) $D_t = diag\left(h_{11,t}^{1/2} \dots h_{NN,t}^{1/2}\right)$, where $h_{ii,t}$ is defined similarly to any univariate GARCH model; $R = \left(\rho_{ij}\right)$ is a symmetric positive definite matrix, with $\rho_{ii} = 1, \forall i$, i.e., R is the matrix containing the constant conditional correlationsρ_{ij}.

However, the assumption that the conditional correlation is constant over time is not convincing, since, in practice, the correlation between assets undergoes many changes over time. Thus, Engle and Sheppard (2001) and Tse ans Tsui (2002) introduced the model of dynamic conditional correlation (DCC). The DCC model is a two-step algorithm to estimate the parameters which makes it relatively simple to use in practice. In the first stage, the conditional variance is estimated by means of univariate GARCH model, respectively, for each asset. In the second step, the parameters for the conditional correlation, given the parameters of the first stage, are estimated. Finally, the DCC model includes conditions that make the covariance matrix positive definite at all points in time and the covariance between assets' volatility a stationary process. The DCC model is represented by the formulation (7).

$$H_t = D_t R_t D_t. \tag{7}$$

Where,

$$R_t = diag\left(q_{11,t}^{-1/2} \dots q_{NN,t}^{-1/2}\right) Q_t diag\left(q_{11,t}^{-1/2} \dots q_{NN,t}^{-1/2}\right). \tag{8}$$

Since the square matrix of order N symmetric positive defined $Q_t = \left(q_{ij,t}\right)$ has the form proposed in (9).

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha u_{t-1}u_{t-1}^{'} + \beta Q_{t-1}. \tag{9}$$

In (9), $u_{i,t} = \varepsilon_{i,t}/\sqrt{h_{i,t}}$; $\bar{Q}$ is the $N \times N$ matrix composed by unconditional variance of u_t; α and β are non-negative scalar parameters satisfying $\alpha + \beta < 1$.

All of the models mentioned in the previous section are estimated under the assumption of multivariate normality. The use of a copula function, on the other hand, allows us to consider the marginal distributions and the dependence structure both separately and simultaneously (Hsu, Tseng and Wang, 2008). Therefore, the joint distribution of the asset returns can be specified with full flexibility, which is more realistic.

In that sense, Hansen (1994) proposes a GARCH model in which the first four moments are conditional and time varying. For the conditional mean and volatility, he built on the usual GARCH model. To control higher moments, he constructed a new density, which is a generalization of the Student-t distribution while maintaining the assumption of a zero mean and unit variance, in order to model the GARCH residuals. The conditioning is obtained by defining parameters as functions of past realizations (Jondeau and Rockinger, 2006). The conditional volatility model proposed by Hensen (1994), and later discussed in Theodossiou (1998) and Jondeau and Rockinger (2003) is represented by formulation (10).

$$h_{i,t}^2 = c_i + b_i h_{i,t-1}^2 + a_i \varepsilon_{i,t-1}^2. \tag{10}$$

Where $\varepsilon_{i,t} = h_{i,t}z_{i,t}, z_{i,t} \sim skewed - t(z_i|\eta_i, \phi_i)$. The density of skewed-t distribution is represented by formulation (11).

$$d(z|\eta, \phi) = \begin{cases} bc\left[1 + \frac{1}{\eta-2}\left(\frac{bz+a}{1-\phi}\right)^2\right]^{-\eta+1/2}, z < -\frac{a}{b} \\ bc\left[1 + \frac{1}{\eta-2}\left(\frac{bz+a}{1-\phi}\right)^2\right]^{-\eta+1/2}, z > -\frac{a}{b} \end{cases}. \tag{11}$$

In (11), $a \equiv 4\phi c \frac{\eta-2}{\eta-1}$; $b \equiv 1 + 3\phi^2 - a^2$; $c \equiv \frac{\Gamma(\eta+1/2)}{\sqrt{\pi(\eta-2)}\Gamma(\eta-2)}$; η and ϕ are the kurtosis and asymmetry parameters, respectively. These are restricted to $4 < \eta < 30$ and $-1 < \phi < 8$.

3. Copula distribution functions

Dependence between random variables can be modeled by copulas. A copula returns the joint probability of events as a function of the marginal probabilities of each event. This makes copulas attractive, as the univariate marginal behavior of random variables can be modelled separately from their dependence (Kovadinovic and Yan, 2010).

The concept of copula was introduced by Sklar (1959). However, only recently its applications have become clear. A detailed treatment of copulas as well as of their relationship to concepts of dependence is given by Joe (1997) and Nelsen (2006). A review of applications of copulas to finance can be found in Embrechts *et al.* (2003) and in Cherubini *et al.* (2004).

For ease of notation we restrict our attention to the bivariate case. The extensions to the *n*-dimentional case are straightforward. A function $C : [0,1]^2 \to [0,1]$ is a *copula* if, for $0 \leq x \leq 1$ and $x_1 \leq x_2$, $y_1 \leq y_2$, (x_1, y_1), $(x_2, y_2) \in [0,1]^2$, it fulfills the following properties:

$$C(x,1) = C(1,x) = x, \;\; C(x,0) = C(0,x) = 0. \tag{12}$$

$$C(x_2, y_2) - C(x_2, y_1) - C(x_1, y_2) + C(x_1, y_1) \geq 0. \tag{13}$$

Property (12) means uniformity of the margins, while (13), the *n-increasing property* means that $P(x_1 \leq X \leq x_2, y_1 \leq Y \leq y_2) \geq 0$ for (X,Y) with distribution function *C*.

In the seminal paper of Sklar (1959), it was demonstrated that a Copula is linked with a distribution function and its marginal distributions. This important theorem states that:

(i) Let *C* be a copula and F_1 and F_2 univariate distribution functions. Then (14) defines a distribution function *F* with marginals F_1 and F_2.

$$F(x,y) = C\big(F_1(x), F_2(y)\big), \;\; (x,y) \in R^2. \tag{14}$$

(ii) For a two-dimensional distribution function *F* with marginals F_1 and F_2, there exists a copula *C* satisfying (14). This is unique if F_1 and F_2 are continuous and then, for every $(u,v) \in [0,1]^2$:

$$C(u,v) = F\big(F_1^{-1}(u), F_2^{-1}(v)\big). \tag{15}$$

In (15), $F_1^{-1}$and F_2^{-1} denote the generalized left continuous inverses of F_1 and F_2.

4. Method

In order to estimate the optimal hedge ratio and the VaR of Brazilian, Argentinean and Mexican markets, considering the American market, we collected data of the daily prices of S&P500, Ibovespa, Merval and IPC, from January, 3, 2009 to December, 31, 2010, totaling 483 observations. These indices were chosen because they are commonly used in academic papers as proxies for the financial markets in these countries. Both are compounds by the stocks that are more representative in terms of liquidity and value. We considered the period after the recent financial crisis of 2007/2008, in order to avoid possible vestiges of it that could cause some bias in the results.

The ADF test (Dickey Fuller Aumented) was initially employed in prices and their logarithmic differences (returns), to eliminate problems of non-stationarity. The ADF test, proposed by Dickey and Fuller (1981) is represented by (16).

$$\Delta P_t = \gamma P_{t-1} + \sum_{i=1}^{n} \delta_i \Delta P_{t-i} + \varepsilon_t. \tag{16}$$

In the formulation (1), ΔP_t is the price change at time t, γ and $\delta_i$are constant, and ε_t is a white noise series. If the null hypothesis cannot be rejected, the price series $\{P\}$ contains a unit root, with non-stationarity. The equations are estimated by Ordinary Least Squares and the parameter values are compared to critical values from tables generated by Dickey and Fuller (1981), based on Monte Carlo simulations.

We used a vector autoregressive (VAR) to obtain the average estimate of the return and the residuals series of each index. The mathematical form of the bivariate VAR model used is represented by (17).

$$VAR(L,A) = \begin{cases} \Delta L_{j,t} = \beta_0 + \sum_{i=1}^{n} \beta_i \Delta L_{j,t-i} + \sum_{j=1}^{m} \beta_j \Delta A_{t-j} + \varepsilon_{1,t} \\ \Delta A_t = \alpha_0 + \sum_{i=1}^{n} \alpha_i \Delta A_{t-i} + \sum_{j=1}^{m} \alpha_j \Delta L_{j,t-j} + \varepsilon_{2,t} \end{cases}. \tag{17}$$

In (17), ΔL_t and ΔA_t are, respectively the daily returns of Latin American j and U.S. markets; β_k and α_k are regression parameters; $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are, correspondingly, the estimated residuals of returns.

Subsequently, using the residuals that were obtained through the VAR applied to the series, we used the copula-based GARCH model, represented by (10). Through this, the estimates of conditional variances and covariances of these markets were obtained, taking in consideration the American market, due to its influence as a benchmark. We compared the unconditional variance and the copula-based GARCH model centered on the posterior predictive distribution of the hedge ratio and VaR for a portfolio of the margins. The optimal hedge ratio is defined as the ratio of U.S. index holdings to a Latin index position that minimizes the risk of the hedged portfolio. The VaR is a lower quantile of the distribution of a portfolio. The absolute value of the $(1-\alpha)*100\%$ VaR from the predictive distribution of a portfolio gives the loss that is not exceeded with probability α.The hedge ratio and the VaR are represented, respectively, by formulations (18) and (19).

$$\delta_t = \frac{cov(\Delta L_{j,t}, \Delta A_t)}{var(\Delta A_t)}. \tag{18}$$

$$VaR_{j,t} = \mu_j - F^{-1}(1-\alpha) * \sqrt{var(\Delta L_{j,t})}. \tag{19}$$

In formulations (18) and (19), δ_t is the optimal hedge ratio in *t*; $VaR_{j,t}$ is the value at risk estimate for the market *j* at the instant *t*; ΔL_t and ΔA_t are, respectively the daily returns of Latin American *j* and U.S. markets; F is the probability distribution function of the returns, in this case the skewed-t; μ_j is the mean of the returns of the Latin market *j*; cov and *var* are the covariance and variance functions;

5. Results

Initially, we performed the ADF test of unit root in all series in level and first difference of logarithm (daily returns). Results are shown in Table 1.

Table 1. Unit Root test of USA, Brazil, Mexico and Argentina.

Variable	ADF test	p-value
S&P500	-0.6020	0.8679
Ibovespa	-1.6507	0.4565
Merval	0.6752	0.9917
IPC	-0.3236	0.9191
Δln(S&P500)	**-15.0330**	0.0000
Δln(Ibovespa)	**-15.7937**	0.0000
Δln(Merval)	**-15.8186**	0.0000
Δln(IPC)	**-14.8784**	0.0000

*Bold values are significant at the 1% level

As the presence of a unit root in all series was confirmed, we calculated the daily returns by the difference of logarithms of prices. Table 2 displays the descriptive statistics of these returns, whereas Figure 1 shows the temporal evolution of these series.

The results in Table 2 confirm the fact that Brazil, Argentina and Mexico being emerging countries, should have a higher standard deviation, representing greater risk and therefore requiring higher returns, as it is verified by higher values for mean and median. The U.S. market, by contrast had lower mean and median and lower standard deviation of returns, representing a more stabilized economy. It is also noticed that all sets of returns are leptokurtic, a fact quite common, being widely recognized by financial professionals.

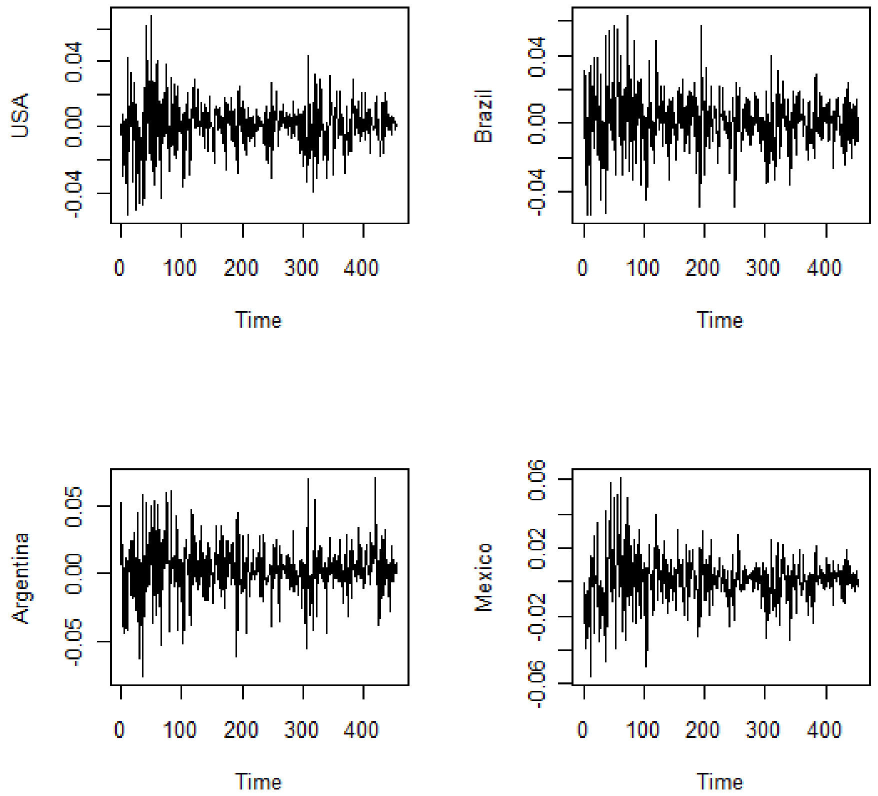


Figure 1. Time series of daily returns of S&P500 (USA), Ibovespa (Brazil), Merval (Argentina) and IPC (Mexico).

Table 2. Descriptive statistics of daily returns of Ibovespa and S&P500.

Statistic	**Δln(USA)**	**Δln(Brazil)**	**Δln(Argentina)**	**Δln(Mexico)**
Mean	0.0007	0.0011	0.0025	0.0011
Median	0.0012	0.0014	0.0026	0.0020
Minimum	-0.0543	-0.0540	-0.0770	-0.0563
Maximum	0.0684	0.0638	0.0712	0.0618
Standard deviation	0.0151	0.0171	0.0201	0.0142
Skewness	-0.0734	0.0432	-0.1404	0.0428
Kurtosis	5.3436	4.6304	4.7111	6.0122

Figure 2 endorses these results. It confirms visually the greater dispersion of the daily returns of the Latin American markets compared to the U.S. It is noteworthy that there is a volatility cluster at the begin of the observations, extending for about 100 trading days. It was the vestiges of the American financial crisis.

Subsequently, it was estimated a copula-based GARCH model to obtain the estimated variances and covariances of the bivariate relationship of the U.S. and Latin markets. Table 3 presents the results of these models.

Table 3. Results of the estimated copula-based GARCH models for the bivariate relationships of daily log-returns of S&P500 with Ibovespa, Merval and IPC.

	Brazil			**Argentina**			**Mexico**		
Variable	Coef.	Error	Prob.	Coef.	Error	Prob.	Coef.	Error	Prob.
USA a_i	0.1130	0.0437	0.0097	0.0968	0.0356	0.0066	0.1089	0.0486	0.0251
USA b_i	0.8799	0.0536	0.0000	0.8657	0.0253	0.0000	0.8825	0.0408	0.0000
USA d.f.	6.3504	2.4676	0.0101	7.9201	2.9564	0.0074	6.3718	3.5838	0.0754
Latin a_i	0.0836	0.0215	0.0001	0.0766	0.0277	0.0057	0.0555	0.0133	0.0000
Latin b_i	0.8938	0.0519	0.0000	0.8984	0.1201	0.0000	0.9435	0.0128	0.0000
Latin d.f.	7.9471	2.9435	0.0069	3.7965	1.7292	0.0281	4.6953	1.4610	0.0013
AIC			-11.8120			-11.5980			-12.6720

*All the values, except USA d.f. in the model with Mexico, are significant at 5% level.

The results in Table 3 indicate that the conditional volatility of all markets studied was significantly affected at the level of 5% by the past squared shocks, and lagged volatility. Moreover, these impacts had similar magnitudes in the models estimated for the three bivariate relationships. Nevertheless, the shape of the probability distribution of conditional volatilities estimated had difference between the analyzed markets regarding to the number of degrees of freedom of the skewed-t function. The degrees of freedom were 7.94 for Brazil, 3.79 for Argentina and 4.69 for Mexico. These results emphasize that Latin market’s returns are skewed and leptokurtic.

Complementing, the estimated volatilities and dynamic correlations are shown, respectively, in Figure 2 and 3, for the bivariate relationships proposed in this study. Beyond, the Q statistics are presented in Table 4, in order to verify the serial dependence of the residues of GARCH estimates.

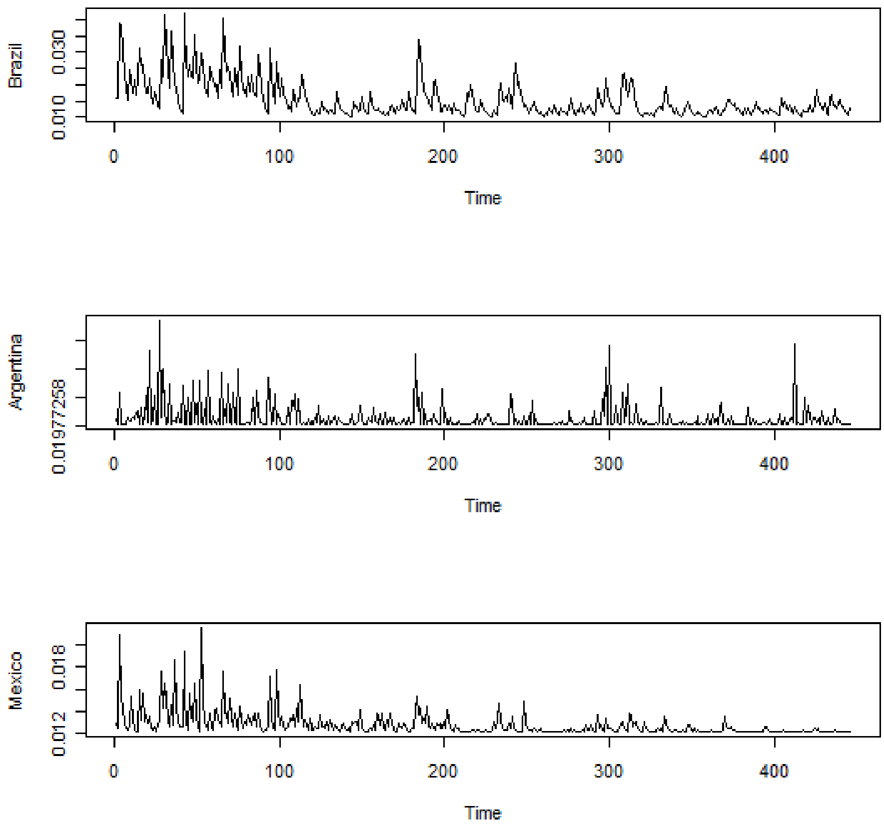
Table 4. Ljung-Box Q statistic for residuals of daily returns of S&P500 (USA), Ibovespa (Brazil), Merval (Argentina) and IPC (Mexico) estimated by copula-based GARCH model.

	USA/Brazil		**Brazil**		**USA/Argentina**		**Argentina**		**USA/Mexico**		**Mexico**	
Lag	Q	prob.	Q	prob.	Q	prob.	Q	prob.	Q	prob.	Q	prob.
1	0.011	0.915	1.568	0.210	1.517	0.218	0.041	0.838	0.000	0.998	0.227	0.634
2	0.904	0.636	3.102	0.212	1.685	0.430	1.423	0.490	0.116	0.943	0.291	0.865
3	0.904	0.824	4.965	0.174	4.932	0.176	1.439	0.696	0.117	0.989	0.462	0.927
4	2.287	0.683	5.762	0.217	5.167	0.270	1.442	0.836	1.964	0.742	0.462	0.977
5	5.325	0.377	5.762	0.330	5.795	0.326	1.685	0.890	4.625	0.463	1.085	0.955
6	7.012	0.319	7.805	0.252	6.267	0.393	1.720	0.943	6.496	0.369	1.439	0.963
7	7.060	0.422	7.870	0.344	6.293	0.506	3.013	0.883	6.514	0.481	3.558	0.829
8	7.753	0.457	9.780	0.280	6.300	0.613	3.372	0.908	7.284	0.506	4.632	0.796
9	14.909	0.093	15.629	0.075	6.551	0.683	4.023	0.909	14.153	0.117	16.144	0.064
10	15.042	0.130	16.827	0.078	6.833	0.741	4.792	0.904	14.372	0.156	16.170	0.095

* None of the values are significant at 5% level.

The results in Table 4 suggest that the estimated residuals from the copula-based GARCH model do not exhibit significant serial correlation. Therefore, the estimated models were able to fit the sample of bivariate relationships between the daily returns of the U.S. market with the Latin countries, filtering the serial dependence and the heteroscedastic dynamic behavior of data. Thus, the estimates of variance and covariance of the studied markets are valid for the computation of the optimal hedge ratio and the value of assets at risk in question.

The plots of Figure 2 emphasizes that after the observation 100, the vestigies of the American financial crisis of 2007/2008 began to disappear, returning to the stability period. It is notable that with the resumption of normal variability of returns in these markets, Mexico was more stable, followed by Brazil and Argentina.



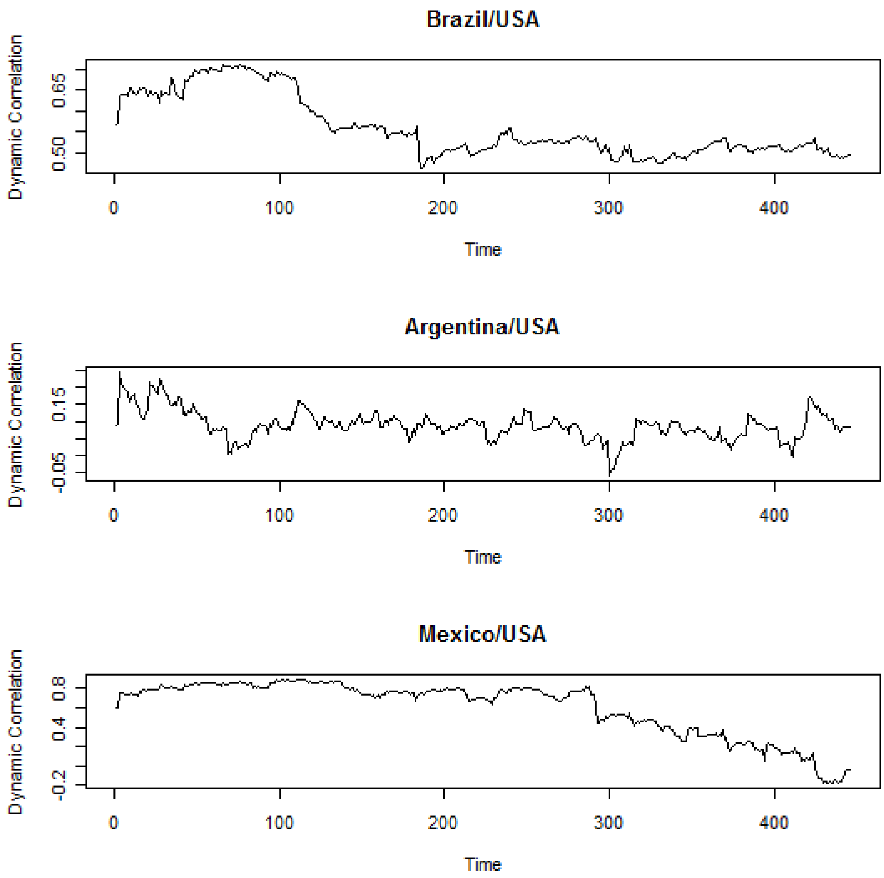


Figure 3. Estimated conditional correlation of the bivariate relationships of daily log-returns of S&P500 with Ibovespa, Merval and IPC.

Figure 3, which exposes the dynamic correlations estimated for the series of returns of the markets studied, suggests the existence of a pattern behavior. At begin of the analyzed period, the dependence among the markets were higher, decreasing with the path of the sample. The Argentinean market was the first to show this reduction, followed by the Brazilian and the Mexican. It is worth noting also that the correlation coefficient between the Mexican and American markets was the highest among Latin American countries, followed by Brazil and Argentina.

Based on the conditional variance and covariance estimated one step ahead of the sample used to estimate the GARCH models, we calculated, for the bivariate relationships of daily returns of the Latin markets with the U.S., the optimal hedge ratio and the value at risk. Table 5 presents these estimates and the calculated values for these measures based on the unconditional variance and covariance of the financial assets in question.

Table 5. Optimal hedge ratio and value at risk of the bivariate relationships of daily log-returns of S&P500 with Ibovespa, Merval and IPC estimated by conditional and unconditional variances and covariances.

	Brazil		**Argentina**		**Mexico**	
Measures	Conditional	Uncond.	Conditional	Uncond.	Conditional	Uncond.
Hedge	0.1206	0.5522	0.0244	0.0777	0.0603	0.6173
VaR(5%)	-0.0193	-0.0325	-0.0293	-0.0370	-0.0077	-0.0268
VaR(1%)	-0.0258	-0.0431	-0.0394	-0.0495	-0.0105	-0.0356
VaR(0,1%)	-0.0333	-0.0555	-0.0511	-0.0640	-0.0138	-0.0459

Analyzing the results shown in Table 5, we found that the initial estimates for the optimal hedge ratio, calculated by the model of conditional volatility in question, are substantially lower than those based on unconditional variances and covariances of the sample. This demonstrates that the dependence among Latin and U.S. markets has reduced in a great extent in the period after the 2007/2008 crisis. This is because a lower coefficient of hedging is related to a decrease in the covariance between assets. This finding reinforces the analysis of the plots of estimated dynamic correlations over the studied period, as shown in Figure 3.

Some authors have evidenced that dependence among stock markets of other countries increases drastically during the crisis (Hon, Strauss and Yong, 2006; Khalid and Rajaguru, 2007; Huyghebaert and Wang, 2010). Thus, as the effects of the crisis were disappearing, the dependence between Latin and U.S. markets, here represented by covariance, was returning to normal levels.

Nevertheless, not only the covariance between markets in different countries reduces after the turbulence of a crisis, but also the volatility of each of these markets. In this study, this fact was evidenced by the value at risk estimates obtained using the conditional volatility model. For all Latin markets, the VaR estimated was substantially lower than that obtained based on the unconditional standard deviation of the returns examined. This result is reinforced by the plots of Figure 2.

This stabilization of the analyzed markets is very relevant to the international portfolio diversification because over the last ten years, the volatility of Latin American financial markets has become a key determinant for explaining the risk-taking behaviors of investors, especially the substitution in their portfolios between different categories of securities (Dufrenot, Mignon and Péguin-Feissolle, 2010).

6. Concluding Remarks

In this paper we analyzed the difference in the estimation of optimal hedge ratio and value at risk when it is employed a model able to capture the inherent characteristics of financial asset returns, and the relation of dependence between markets. To that, we used daily data from Brazilian, Argentinean, Mexican and U.S. markets.

Initially, we estimated copula-based multivariate GARCH models, in order to estimate the conditional variance and covariance of the bivariate relationships of the U.S. market with these three Latin markets. We found that both the volatility of Latin markets as its covariance with the U.S. market, tended to decrease due to the disappearance of the 2007/2008 crisis and subsequent stabilization of the economies.

These results were reflected in the estimation of optimal hedge ratio and value at risk to returns of the Latin markets. As the estimation was done one step beyond the sample, capturing a moment practically free of the crisis' vestiges, we obtained hedge ratio and value at risk estimates well below those calculated based on the unconditional variance and covariance.

Thus, the use of models unable to correctly estimate the conditional volatility of an asset produces incorrect results, prompting investors to achieve diversification of its portfolio ineptly. As an empirical result, it was found that, over the crisis, Latin markets may again be considered relevant options for international diversification of investors with positions in U.S. assets.

As suggestions for future studies, we highlight the application of a similar model to estimate the optimal hedge ratio and value at risk of Asian and European markets, taking into account the influence of a greater market.

7. References

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