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Unconcerned groups and the majority rule

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Abstract

When preferences are defined over two alternatives, the (relative) majority rule is characterized in terms of the four axioms U, P, I, and G. U is unanimity. P is the condition that the union of two unconcerned (that is, indifferent) groups of individuals creates an unconcerned group. I asserts that the preferences of the individuals of an unconcerned group can be cancelled out without altering the result. G states that, for any sufficiently small group G that is not unconcerned and for any group H with the same size as G but without members in common with G, it is possible to make the union of G and H unconcerned.

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1. Introduction

There are several axiomatic characterizations of the (relative) majority rule when preferences are defined over two alternatives. Those by May (1952, p. 682), Fishburn (1973, p. 58; 1983, p. 33), and Llamazares (2006, p. 319) assume that the set of individuals is fixed but their preferences are variable. Xu and Zhong's (2010, p. 120) characterization is formulated for the symmetric case: the set of individuals is variable but their preferences are held fixed. Aşan and Sanver (2002, p. 411), Woeginger (2003, p. 91; 2005, p. 9), and Miroiu (2004, p. 362) provide characterizations when both the set of individuals and their preferences can vary. This paper proposes another two characterizations of the majority rule in the latter framework. Though the universal set *I* of individuals is assumed to be finite, the fact that the characterizations hold for universal sets of any finite size makes them also hold for any countably infinite set *I*.

Except the unanimity axiom **U**, the other four axioms postulated (**P**, **I**, **G**, and **SET**) hinge on the concept of "unconcerned group", namely, a group whose preference is indifference. Axiom **P** captures the idea that unconcerned groups are persistent: the union of two unconcerned groups creates an unconcerned group. Axiom **I** makes an unconcerned group irrelevant when the group joins another group whose members have all the same preference. Axiom **G** establishes conditions that generate unconcerned groups: if *G* is not an unconcerned group but has sufficiently small size, then it is possible for any group *H* without members in common with *G*, but with the same size as *G*, to make the group $G \cup H$ unconcerned. Finally, the simple equal treatment condition **SET** by Xu and Zhong (2010, p. 120) asserts that two individuals with strict, opposite preferences constitute an unconcerned group.

It is shown that for universal sets of any size, the majority rule is characterized by the set of axioms {**U**, **P**, **I**, **SET**}. When the universal set *I* consists of just one individual, **P**, **I**, and **SET** are obviously redundant. When *I* consists of two or three members, **P** is redundant. When *I* has at least four members, none of the four axioms is dispensable. Finally, since **SET** and **G** are equivalent in the presence of **U** and **I**, the majority rule is also characterized by the set of axioms {**U**, **P**, **I**, **G**}.

2. Definitions and axioms

Let $\mathbb{N}$ designate the set of positive integers and *I* be any non-empty finite set whose *n* members designate individuals. A group is a non-empty subset of *I*. There are two alternatives: x and y. A preference over $\{x, y\}$ is represented by a number from the set $\{-1, 0, 1\}$. If the number is 1, x is preferred to y; if -1, y is preferred to x; if 0, x is indifferent to y. A preference profile for a group G is a function $p_G : G \to \{-1, 0, 1\}$ assigning a preference over $\{x, y\}$ to each member of G. For $r \in \{1, \ldots , n\}$, P_r is the set of all preference profiles for groups with exactly r members and $P_{\leq r} = P_1 \cup \ldots \cup P_r$ is the set of all preference profiles for groups with at most r members.

For preference profile p_G and group $H \subset G$, p_H is the restriction of p_G to H; that is, the preference profile p_H such that, for all $i \in H$, $p_H(i) = p_G(i)$. For preference profile p_G and $i \in G$, p_i abbreviates $p_G(i)$. If p_G and p_H are preference profiles of disjoint groups G and H, then (p_G, p_H) is the preference profile corresponding to the group $G \cup H$. Preference profile p_G is unanimous if there is $a \in \{-1, 0, 1\}$ such that, for all $i \in G$, $p_i = a$. The unanimous preference profile for G with common preference a is denoted by (a^G). If $G = \{i\}$, (a^i) stands for $(a^{\{i\}})$. For any finite set S, $|S|$ is the number of members of S.

Definition 2.1. A social welfare function on $P_{\leq n}$ is a mapping $f : P_{\leq n} \to \{-1, 0, 1\}$.

A social welfare function on $P_{\leq n}$ transforms preference profiles for any group G having n or fewer members into a preference of group G over $\{x, y\}$. Specifically, $f(p_G) = 1$ means that, given the preference profile p_G, the group G prefers x to y; $f(p_G) = -1$, that G prefers y to x; and $f(p_G) = 0$, that G is indifferent between x and y.

Definition 2.2. The majority rule on $P_{\leq n}$ is the social welfare function $\mu_{\leq n} : P_{\leq n} \to \{-1, 0, 1\}$ on $P_{\leq n}$ such that, for all $p_G \in P_{\leq n}$: (i) if $\sum_{i \in G} p_i > 0$, then $\mu_{\leq n}(p_G) = 1$; (ii) if $\sum_{i \in G} p_i < 0$, then $\mu_{\leq n}(p_G) = -1$; and (iii) if $\sum_{i \in G} p_i = 0$, then $\mu_{\leq n}(p_G) = 0$.

U. *Unanimity*. For each group $G \subseteq I$ and each $a \in \{-1, 0, 1\}$, $f(a^G) = a$.

Axiom **U** states that, if all the members in a group G have the same preference, then that common preference defines the preference of the group G.

A group G is an unconcerned group, given a social welfare function f and a preference profile p_G for G, if $f(p_G) = 0$.

IUG. *Independence of an unconcerned group*. For each $p_G \in P_{\leq n}$, each group $H \subseteq I \backslash G$, and each $p_H \in P_{\leq n}$, if $f(p_H) = 0$, then $f(p_G, p_H) = f(p_G)$.

Borrowed from Xu and Zhong (2010, p. 120), **IUG** states that the preference of a group G does not depend on the preferences of the members of an unconcerned subgroup of G.

I. *Independence of an unconcerned group given unanimous preferences*. For each $p_G \in P_{\leq n}$, each group $H \subseteq I \backslash G$, and each $p_H \in P_{\leq n}$, if $f(p_H) = 0$ and p_G is unanimous, then $f(p_G, p_H) = f(p_G)$.

Axiom **I** is obtained from **IUG** by requiring p_G to be unanimous. According to **I**, when an unconcerned group joins a group with a unanimous preference profile, then the preference of the new group is determined by the group with the unanimous preference profile. In other words, the preferences of the members of an unconcerned group are cancellable if the cancellation of those preferences creates a unanimous preference profile.

P. *Persistence of unconcerned groups*. For each $p_G \in P_{\leq n}$, each group $H \subseteq I \backslash G$, and each $p_H \in P_{\leq n}$, if $f(p_G) = f(p_H) = 0$, then $f(p_G, p_H) = 0$.

Axiom **P** holds that the union of two unconcerned, disjoint groups generates an unconcerned group. Equivalently, if a group is not unconcerned, then, for every partition of the group into two groups, one of them should not be unconcerned.

Remark 2.3. IUG implies both **I** and **P**. The conjunction of **I** and **P** does not imply **IUG**.

Whereas **I** is **IUG** plus the constraint that p_G is unanimous, **P** is **IUG** plus the constraint that $f(p_G) = 0$. For $n \geq 2$, it will be next defined a social welfare function g on $P_{\leq n}$ that satisfies both **I** and **P**, but fails to satisfy **IUG**. For $p_G \in P_{\leq n}$, define $z(p_G) = |\{i \in G: p_i = 0\}|$. With $P = \{p_G \in P_{\leq n}$: for some $i \in G$ and $j \in G$, $p_i = 1$ and $p_j = -1\}$, let g be the social welfare function on $P_{\leq n}$ such that: (i) for all $p_G \in P_{\leq n} \backslash P$, $g(p_G) = \mu_{\leq n}(p_G)$; and (ii) for all $p_G \in P$, $g(p_G) = 1$ if $z(p_G)$ is even or zero, and $g(p_G) = -1$ if $z(p_G)$ is odd. It can be easily verified that $g(p_G) = 0$ if and only if, for all $i \in G$, $p_i = 0$. Therefore, if $g(p_G) = g(p_H) = 0$, with $G \cap H = \varnothing$, then $(p_G, p_H) = (0^{G \cup H})$, so $g(p_G, p_H) = 0$. This proves that g satisfies **P**. With respect to **I**, suppose p_G is unanimous, $g(p_H) = 0$, and $G \cap H = \varnothing$. This means that $p_H = (0^H)$ and that, for some $a \in \{-1, 0, 1\}$, $p_G = (a^G)$. In view of this, $(p_G, p_H) \in P_{\leq n} \backslash P$. Consequently, since $p_G \in P_{\leq n} \backslash P$, $g(p_G, p_H) = \mu_{\leq n}(p_G, p_H) = \mu_{\leq n}(p_G) = g(p_G)$. **IUG** does not hold because $g(0^k) = 0$ would have to imply $g(1^i, -1^j, 0^k) = g(1^i, -1^j)$, which is not the case: $g(1^i, -1^j, 0^k) = -1$ and $g(1^i, -1^j) = 1$.

SET. *Simple equal treatment*. For all $i \in I$ and $j \in I \backslash \{i\}$, $f(1^i, -1^j) = 0$.

Taken from Xu and Zhong (2010, p. 120), **SET** asserts that, with preferences given by the preference profile $(1^i, -1^j)$, $\{i, j\}$ constitutes an unconcerned group.

G. *Genesis of unconcerned groups*. For each $p_G \in P_{\leq n}$ such that $|G| \leq n/2$ and each group $H \subseteq I \backslash G$ such that $|H| = |G|$, if $f(p_G) \neq 0$, then, for some $p_H \in P_{\leq n}$, $f(p_G, p_H) = 0$.

According to **G**, unconcerned groups can be generated by duplicating the size of a group that is not unconcerned. More specifically, suppose G is not unconcerned: $f(p_G) \neq 0$. Then **G** requires that, for every group H with no member in common with G but with the same number of members as G, there is some preference profile p_H for H making the group $G \cup H$ unconcerned given the preference profile (p_G, p_H).

Remark 2.4. **U** and **I** make **G** and **SET** equivalent.

Assume that f is a social welfare function on $P_{\leq n}$ that f satisfies **U** and **I**. As **G** and **SET** are vacuously true for $n = 1$, let $n \geq 2$. To show that **G** implies **SET**, choose $i \in I$ and $j \in I \backslash \{i\}$. By **U**, $f(1^i) = 1$. By **G**, for some $a \in \{-1, 0, 1\}$, $f(1^i, a^j) = 0$. If $a = 1$, then, by **U**, $f(1^i, a^j) = 1$: contradiction. If $a = 0$, then, by **U**, $f(a^j) = 0$. Given this, by **I**, $f(1^i, a^j) = f(1^i)$. By **U**, $f(1^i) = 1$, so $f(1^i, a^j) = 1$: contradiction. Therefore, $a = -1$. To show that **SET** implies **G**, suppose $p_G \in P_{\leq n}$ and $H \subseteq I \backslash G$ are such that $f(p_G) = a \neq 0$, $|G| \leq n/2$, and $|H| = |G|$. Let $p_G = (1^J, -1^K, 0^L)$, where $J \cup K \cup L = G$. By **U**, $p_G \neq (0^G)$. Accordingly, $J \cup K \neq \varnothing$. Consider any bijection $\beta : H \to G$. Define p_H to be such that, for all $i \in H$, $p_i = -p_{\beta(i)}$. For $S \in \{J, K, L\}$, let S^* abbreviate $\{i \in H$: for some $j \in S$, $\beta(i) = j\}$. Hence, $p_H = (-1^{J^*}, 1^{K^*}, 0^{L^*})$. It must be shown that $f(p_G, p_H) = 0$. That is, $f(1^J, -1^{J^*}, -1^K, 1^{K^*}, 0^L, 0^{L^*}) = 0$. By **U**, $f(0^L, 0^{L^*}) = 0$. This and **I** imply $f(p_G, p_H) = f(1^J, -1^{J^*}, -1^K, 1^{K^*})$. By **SET**, for all $i \in J^*$, $f(-1^i, 1^{\beta(i)}) = 0$. Since β induces a bijection between J and J^*, by **I**, the preferences of each pair $(i, \beta(i)) \in J^* \times J$ cancel out. As a result, $f(1^J, -1^{J^*}, -1^K, 1^{K^*}) = f(-1^K, 1^{K^*})$. The same reasoning can be applied to K and K^*, so that, by **I**, one can remove all the members of $K^* \times K$ except some pair $(k, \beta(k))$ to reach the conclusion that $f(-1^K, 1^{K^*}) = f(-1^k, 1^{\beta(k)})$, which is equal to 0 by **SET**.

Remark 2.5. For all $n \in \mathbb{N}$, $\mu_{\leq n}$ satisfies **U**, **SET**, **G**, and **IUG** (and a fortiori **I** and **P**).

Let $n \in \mathbb{N}$. It should not be difficult to verify that $\mu_{\leq n}$ satisfies **U**, **SET**, and **IUG**. With respect to **G**, suppose $\mu_{\leq n}(p_G) \neq 0$, where $|G| \leq n/2$. Hence, there is $H \subseteq I \backslash G$ such that $|H| = |G|$. Consider any bijection $\beta : H \to G$. Let p_H satisfy, for all $i \in H$, $p_i = -p_{\beta(i)}$. Clearly, $\mu_{\leq n}(p_G, p_H) = 0$.

3. Results

Lemma 3.1. For $n \geq 3$, let f be a social welfare function on $P_{\leq n}$. If $f = \mu_{\leq n}$ on $P_{\leq 2}$, then $f = \mu_{\leq n}$ if and only if f satisfies **U**, **P**, and **I**.

Proof. "⇒" Remark 2.5. "⇐" Writing μ instead of $\mu_{\leq n}$ and taking the fact that $f = \mu$ on $P_{\leq 2}$ as the base case of an induction argument, choose $r \in \{3, \ldots, n\}$ and assume that $f = \mu$ on $P_{\leq r-1}$. It must be shown that $f = \mu$ on P_r. To this end, consider any $p_G \in P_r$. Case 1: $\mu(p_G) = 0$. Case 1a: for some $i \in G$, $p_i = 0$. By the induction hypothesis, $f(p_i) = \mu(p_i) = 0$. Since $\mu(p_G) = 0$, $\mu(p_i) = 0$ implies $\mu(p_{G\setminus\{i\}}) = 0$. By the induction hypothesis, $f(p_{G\setminus\{i\}}) = \mu(p_{G\setminus\{i\}})$. By **P**, it follows from $f(p_i) = f(p_{G\setminus\{i\}}) = 0$ that $f(p_G) = 0$. Case 1b: for all $i \in G$, $p_i \neq 0$. Since $\mu(p_G) = 0$ and $r \geq 3$, there must be $i \in I$ and $j \in I\setminus\{i\}$ such that $p_i = 1$ and $p_j = -1$. By the induction hypothesis, $f(p_{\{i,j\}}) = \mu(p_{\{i,j\}}) = 0$. In addition, $\mu(p_G) = 0$ implies $\mu(p_{G\setminus\{i,j\}}) = 0$. By the induction hypothesis, $f(p_{G\setminus\{i,j\}}) = \mu(p_{G\setminus\{i,j\}}) = 0$. By **P**, $f(p_{\{i,j\}}) = 0$ and $f(p_{G\setminus\{i,j\}}) = 0$ imply $f(p_G) = 0$. Case 2: $\mu(p_G) \neq 0$. Letting $\mu(p_G) = a$, define $M = \{i \in G: p_i = -a\}$ and $Z = \{i \in G: p_i = 0\}$. Case 2a: $M = \varnothing$. If $Z = \varnothing$, by **U**, $f(p_G) = a = \mu(p_G)$. If $Z \neq \varnothing$, by **U**, $f(p_Z) = 0$. Moreover, $\{i \in G: p_i = a\} = G\setminus Z$. Hence, by **I**, $f(p_Z) = 0$ implies $f(p_G) = f(p_{G\setminus Z})$. By **U**, $f(p_{G\setminus Z}) = a$, so $f(p_G) = a = \mu(p_G)$. Case 2b: $M \neq \varnothing$. Since $\mu(p_G) = a$, there is a partition $\{H, R\}$ of $\{i \in G: p_i = a\}$ such that $|R| = |M|$ and $H \neq \varnothing$. By the induction hypothesis, $f(p_{G\setminus H}) = \mu(p_{G\setminus H}) = 0$. Given that p_H unanimous, by **I**, $f(p_{G\setminus H}, p_H) = f(p_H)$. By **U**, $f(p_H) = a$. As a result, $f(p_G) = a = \mu(p_G)$.■

Lemma 3.2. For $n \geq 2$, let f be a social welfare function on $P_{\leq n}$. Then $f = \mu_{\leq n}$ on $P_{\leq 2}$ if and only if f satisfies **U**, **I**, and **SET**.

Proof. "⇒" Remark 2.5. "⇐" By **U**, $f = \mu_{\leq n}$ on P_1. With respect to P_2, let $(a^i, b^j) \in P_2$. Case 1: $a = b$. By **U**, $f(p_i, p_j) = a = \mu_{\leq n}(p_i, p_j)$. Case 2: $a \neq b$ and $\{a, b\} = \{1, -1\}$. By **SET**, $f(p_i, p_j) = 0 = \mu_{\leq n}(p_i, p_j)$. Case 3: $a \neq b$ and $\{a, b\} \neq \{1, -1\}$. This means that $0 \in \{a, b\}$. Without loss of generality, suppose that $a = 0$, so $b \in \{1, -1\}$. By **U**, $f(p_i) = a = 0$. Since p_j is unanimous, by **I**, $f(p_i, p_j) = f(p_j) = b = \mu_{\leq n}(p_i, p_j)$.■

Proposition 3.3. For all $n \in \mathbb{N}$, a social welfare function f on $P_{\leq n}$ satisfies **U**, **P**, **I**, and **SET** if and only if f is the majority rule on $P_{\leq n}$.

Proof. "⇐" Remark 2.5. "⇒" Lemmas 3.1 and 3.2.■

Xu and Zhong (2010, p. 120) characterize the majority rule in terms of **IUG**, **SET**, **SD**, and **WPP** when a preference profile is held fixed and different groups can be formed.

SD. *Self-determination*. For each $p_i \in P_1$, $f(p_i) = p_i$.

WPP. *Weak Pareto principle*. For each group $G \subseteq I$ and each $a \in \{-1, 1\}$, $f(a^G) = a$.

The conjunction of **SD**, **WPP**, and **P** implies **U**. Given that **IUG** implies **P**, the majority rule is characterized by **U**, **IUG**, and **SET**. Proposition 3.3 provides another characterization of the majority rule for the single preference profile case because, in Lemmas 3.1 and 3.2, the proof that $f(p_G) = \mu_{\leq n}(p_G)$ only depends on the preference profiles p_H such that $H \subseteq G$. By Remark 2.3, Proposition 3.3 refines the characterization in terms of **U**, **IUG**, and **SET** by weakening **IUG**. Remark 3.4 shows that **SET** need not be postulated for all pairs of individuals, as simple equal treatment for groups $\{i, j\}$, $\{j, k\}$, and $\{k, r\}$ implies simple equal treatment for $\{i, r\}$ given **IUG**.

Remark 3.4. If f satisfies **IUG** and $f(1^i, -1^j) = f(-1^j, 1^k) = f(1^k, -1^r) = 0$, then $f(1^i, -1^r) = 0$.

Suppose $f(1^i, -1^j) = f(-1^j, 1^k) = f(1^k, -1^r) = 0$. By **IUG**, $f(1^i, -1^j) = 0$ implies $f(1^i, -1^j, 1^k, -1^r) = f(1^k, -1^r) = 0$. By **IUG**, $f(-1^j, 1^k) = 0$ implies $f(1^i, -1^j, 1^k, -1^r) = f(1^i, -1^r)$. Since $f(1^i, -1^j, 1^k, -1^r) = 0$, $f(1^i, -1^r) = 0$.

Remark 3.5. If $n \in \{2, 3\}$, then **P** is redundant in Proposition 3.3.

Lemma 3.2 proves the case $n = 2$. To show that $f = \mu_{\leq n}$ on P_3 if **U**, **I**, and **SET** hold, let $p_G \in P_3$. If, for some $i \in G$, $p_i = 0$, then, by **U**, $f(p_i) = 0$. By **I**, $f(p_G) = f(p_{G\setminus\{i\}})$. By Lemma 3.2, $f(p_{G\setminus\{i\}}) = \mu_{\leq n}(p_{G\setminus\{i\}})$. And since $p_i = 0$, $\mu_{\leq n}(p_{G\setminus\{i\}}) = \mu_{\leq n}(p_G)$. If, for all $i \in G$, $p_i \neq 0$, then there are $i \in G$ and $j \in G\setminus\{i\}$ such that $\{p_i, p_j\} = \{1, -1\}$. Let k be the only member of $G\setminus\{i, j\}$. By **SET**, $f(p_i, p_j) = 0$. By **I**, $f(p_G) = f(p_{G\setminus\{i,j\}}) = f(p_k)$. By **U**, $f(p_k) = p_k$. Since $\{p_i, p_j\} = \{1, -1\}$, $\mu_{\leq n}(p_G) = \mu_{\leq n}(p_{G\setminus\{i,j\}}) = \mu_{\leq n}(p_k) = p_k$. Therefore, $f(p_G) = \mu_{\leq n}(p_G)$.

The following examples show that no axiom in Proposition 3.3 is redundant when $n \geq 4$.

Example 3.6. Let $f : P_{\leq n} \to \{-1, 0, 1\}$ satisfy, for all $p_G \in P_{\leq n}$, $f(p_G) = 0$. Then: (i) f satisfies **I**, **P**, and **SET**; (ii) does not satisfy **U**; and (iii) is not the majority rule on $P_{\leq n}$.

Example 3.7. Let $f : P_{\leq n} \to \{-1, 0, 1\}$ satisfy, for all $p_G \in P_{\leq n}$ and $a \in \{1, -1\}$: (i) if, for all $i \in G$, $p_i = a$, then $f(p_G) = a$; and (ii) otherwise, $f(p_G) = 0$. Then: (i) f satisfies **U**, **P**, and **SET**; (ii) does not satisfy **I**; and (iii) is not the majority rule on $P_{\leq n}$.

Example 3.8. Let $f : P_{\leq n} \rightarrow \{-1, 0, 1\}$ be such that, for all $p_G \in P_{\leq n}$: (i) if $|G| \geq 4$ and $|\{i \in G: p_i = 1\}| = |\{i \in G: p_i = -1\}| = |G|/2$, then $f(p_G) = 1$; and (ii) otherwise, $f(p_G) = \mu_{\leq n}(p_G)$. Then: (i) f satisfies **U**, **I**, and **SET**; (ii) does not satisfy **P**; and (iii) is not the majority rule on $P_{\leq n}$.

Example 3.9. With r being any linear ordering on I, let $f : P_{\leq n} \rightarrow \{-1, 0, 1\}$ be such that, for all $p_G \in P_{\leq n}$: (i) if, for some $i \in G$, $p_i \neq 0$, then $f(p_G)$ coincides with the preference of the member i of G such that $p_i \neq 0$ and no member of G appears before i in r; and (ii) otherwise, $f(p_G) = 0$. Then: (i) f satisfies **U**, **I**, and **P**; (ii) does not satisfy **SET**; and (iii) is not the majority rule on $P_{\leq n}$.

Proposition 3.10. For all $n \in \mathbb{N}$, a social welfare function f on $P_{\leq n}$ satisfies **U**, **P**, **I**, and **G** if and only if f is the majority rule on $P_{\leq n}$.

Proof. Remark 2.4, Remark 2.5, and Proposition 3.3.■

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