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A log-periodic fit for the flash crash of May 6, 2010

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Abstract

We show that a two-harmonic log-periodic formula fits the high-frequency data from the Dow Jones Industrial Average index, which encompass the recent episode known as the "flash crash" of May 6, 2010.

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1. Introduction

The log-periodicity hypothesis proposes that financial crashes, like material rupture, can be governed by log-periodic formulas (Sornette and Vanneste 1992, Sornette and Johansen 2001, Sornette and Zhou 2002, Sornette 2003). One-harmonic and two-harmonic log-periodic equations have been employed to fit daily data (Sornette and Zhou 2002), and a three-harmonic formula has been used to fit anti-bubbles (bear markets) in the Nikkei (from 1990 to 1998) and future gold prices (from 1980 to 1998) (Johansen and Sornette 1999). The three-harmonic formula has also been shown by us to fit intraday data for both bubble and anti-bubble episodes (Matsushita *et al.* 2006). There are other works using intraday data (Cajueiro *et al.* 2009), and Cajueiro *et al.* (2009) also offer a literature review.

One explanation of the physics underlying the log-periodic formulas posits that cooperative imitation between traders in a bull market is responsible for log-periodicity to emerge because crashes could possibly result from the buildup of correlations. Self-reinforcing imitation in a bubble episode builds up strains that push the market to a critical point, from which sooner or later many traders will place the same order (that is, “sell”) at the same time, thereby provoking a crash (Sornette 2003). It is imitation that makes the system periodic on the eve of a crash, and in that sense crashes are outliers with properties that are statistically distinct from the rest of the population.

Here, we show that a two-harmonic log-periodic formula fits well the data sampled at a one-minute frequency from the Dow Jones Industrial Average (DJIA) index ranging from September 1, 2009 to May 31, 2010. The data purposely encompass the recent episode known as the “flash crash” of May 6, 2010. On this day, the DJIA suffered its largest intraday decline, that is, 998.5 points. Most of the losses occurred between 2:40 pm and 3:00 pm, with a peak at 2:45 pm (Figure 1). The stocks of Accenture, for example, briefly traded for one cent. The crash, however, was followed by an almost immediate rebound.

What triggered the crash remains unknown, but some observers point to possible causes like computer-automated trades and error by human traders. However, if stock markets are viewed as complex systems, there is no need for a trigger to explain a crash (see Bak and Paczuski 1995 for the general case, and Mazzeu *et al.* 2011 for the flash crash). In particular, under the log-periodic hypothesis, after the critical time a crash may suddenly occur without any early warning signs. An initial rumor that the trigger was a trader who had typed a sell order for 16 billion shares of Procter & Gamble instead of 16 million was later dismissed by regulators. On October 1st, 2010 the Securities and Exchange Commission issued a report blaming a sloppily executed sell order of one mutual-fund group (Waddel & Reed), which started to sell \$4.1 billion of “E-Mini” futures contracts through robot trading, taking account only of volume, not time or price. Some analysts blame an intermarket sweep order, anxiety over Greece’s bailout package, the British election’s outcome, and simply two previous days’ declines in the index. Even if not the main cause, robot trading through electronic platforms (such as Direct Edge and BATS), which executes trades in milliseconds, certainly played a role in magnifying the crash. Also, thanks to increasing high-frequency trading, correlations previously only seen across hours or days in trading time-series are now possibly showing up in timescales of seconds or minutes (Smith 2010). The buildup of correlations brought by high-frequency trading may thus explain the log-periodic nature of the flash crash.

2. Results

As observed, we collected data from the DJIA index sampled at a one-minute frequency from September 1, 2009 to May 31, 2010, totaling 65,534 observations. In line with the previous literature (Sornette and Johansen 2001, Sornette and Zhou 2002, Johansen and Sornette 1999) we considered in the analysis the price index itself rather than the returns. Recent developments of the log-periodic power law model consider returns (Lin *et al.* 2009). Indeed, in Lin *et al.* (2009) stochastic conditional expectations of returns describe continuous updates of the investors' beliefs and sentiments.

Let $Z(t)$ be the time series of the DJIA index, where $t=1,\dots,65,534$ minutes. Log-periodic cycles with a smooth trend component are described by a sum of log-periodic harmonics, that is,

$$\ln Z(\tau) = A + B\tau^{\alpha} + \sum C_j \tau^{\alpha_j} \cos(j\omega_j \ln(\tau) + \phi_j)\,, \qquad (1)$$

where $\tau = t/30,000$ is a reparameterized starting time on the onset of the high-frequency bubble. Because the sample is large ($1 \le t \le 65,534$), we reparameterize it and divide t by 30,000 in order to stabilize the estimation numerical method. (Taking 60,000 instead rendered the estimation unstable.) Term $A + B\tau^{\alpha}$ is the trend across time, and A, B, and α give its shape. Parameters ω_j, C_j, and ϕ_j are, respectively, the angular log-frequency, amplitude, and phase of the j^{th} harmonic. We set $\tau = (t - t_c)/30,000 > 0$, where t_c is the critical time. Unlike in our previous work (Matsushita *et al.* 2006), here α_j, ω_j, and ϕ_j need not be equal, and $j=2$ allows more flexibility for the adjustment.

From the starting date used for the fitting procedure $t_{\text{start}} = 1$ (3:09 pm of September 18, 2009), the estimated critical time $t_c = 3,552$ corresponds to the lower price observed at 9:32 am on October 2, 2009. (It is worth remarking that, on this day, there was the announcement of the US employment indicators.) The gaps between the trading days were ignored. From $t = 24,335$ onwards, the series closely followed the log-periodic path given by the adjusted model. This data point corresponds to 11:17 am of December 18, 2009. Figure 2 shows that the two-harmonic fits well the natural logs of the DJIA index.

Table 1 shows the results for the parameters adjusted by nonlinear regression (performed using SAS 9.2). The mean square error of the fit in Figure 2 is 0.000124. For comparison, the case where $j=1$ rendered a mean square error of 0.000255, which is almost two times greater than the mean square error for $j=2$. That the two-harmonic log-periodic power law model (LPPL2) fits the data better than that of one harmonic (LPPL1) can be further justified using the information criterion of Akaike (AIC) and the Schwarz Bayesian information criterion (SBIC). Because we have a large sample ($n = 61,983$) the cost of increasing the number of parameters is outweighed by the benefit of a better fit. Indeed, in the AIC

$$\text{AIC(LPPL1)} = -512,849 > \text{AIC(LPPL2)} = -557,529$$

and in the SBIC

$$\mathrm{SBIC(LPPL1)} = -512,785 > \mathrm{SBIC(LPPL2)} = -557,430\ .$$

For robustness, we tested the stability of the fitting parameters by varying the size of the fit intervals through different time windows (Jiang *et al.* 2010). The windows were both squeezed and stretched. Figures 3 and 4 show overlaid fits with very similar patterns. In Figure 3 the size of the fit intervals were changed in steps of 592 minutes by squeezing the windows using a fixed end time at 10:09 am of May 25, 2010, and a starting time increasing from 3:09 pm of September 18, 2009 to 1:48 pm of October 26, 2009. In Figure 4 the same was done but now by stretching the windows considering a fixed starting time at 3:09 pm of September 18, 2009, and the end point increasing from 10:06 am of April 19, 2010 to 10:09 am of May 25, 2010. The values at the end of the range were extrapolated until 10:09 am of May 25, 2010 on the basis of the adjusted parameters.

3. Conclusion

The flash crash of May 6, 2010 in the DJIA index was a crash in high-frequency trading. Log-periodic crashes are the result of the buildup of correlations thanks to imitation between traders in bubble episodes. Although robot trading may not be the culprit in triggering the flash crash, high-frequency trading may have generated correlations in timescales of seconds. This may explain the fact that we cannot dismiss the hypothesis of a log-periodic nature for the flash crash. In particular, this study showed that a two-harmonic formula fits well the data.

Table 1. Log-periodic fit for the flash crash of May 6, 2010

coefficient	*estimate*	*approx. standard error*	*approx. 95% confidence limits lower*	*upper*
A	9.289	0.001	9.288	9.290
B	–0.037	0.001	–0.038	–0.036
C_1	0.0081	0.0001	0.0079	0.0083
C_2	0.0178	0.0001	0.0176	0.0178
α	–0.60	0.01	–0.61	–0.58
α_1	1.42	0.03	1.36	1.47
α_2	2.10	0.01	2.07	2.13
ω_1	17.20	0.03	17.15	17.26
ω_2	9.98	0.01	9.96	10.00
ϕ_1	12.81	0.01	12.78	12.83
ϕ_2	19.55	0.01	19.53	19.56

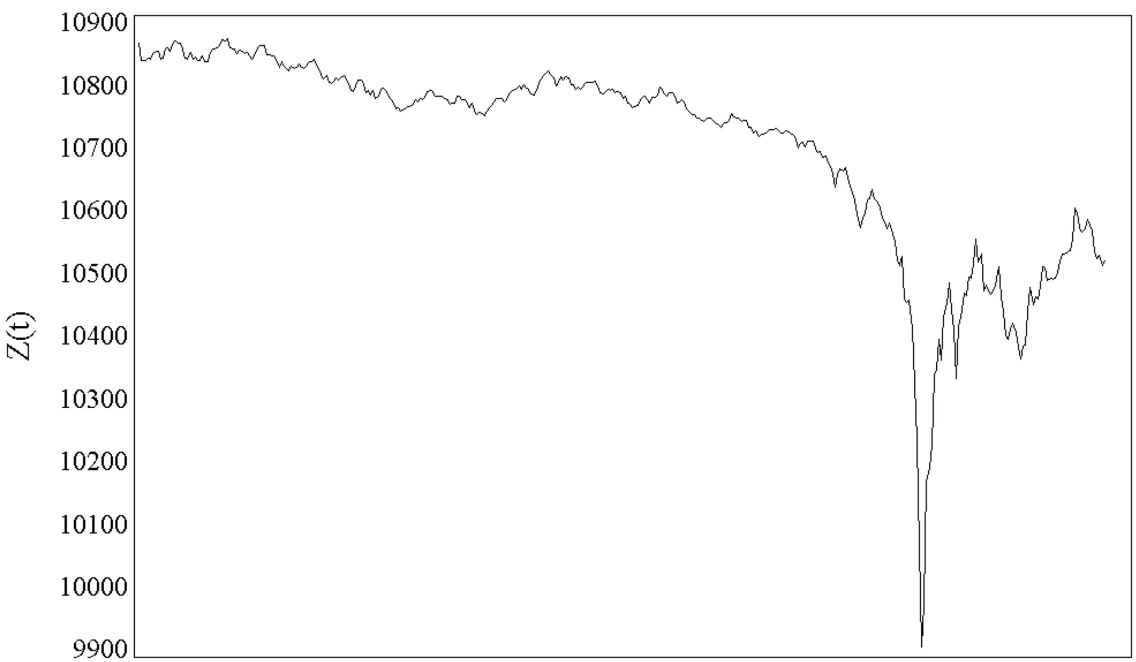

Figure 1. Daily chart of the Dow Jones Industrial Average index during the May 6, 2010 flash crash.

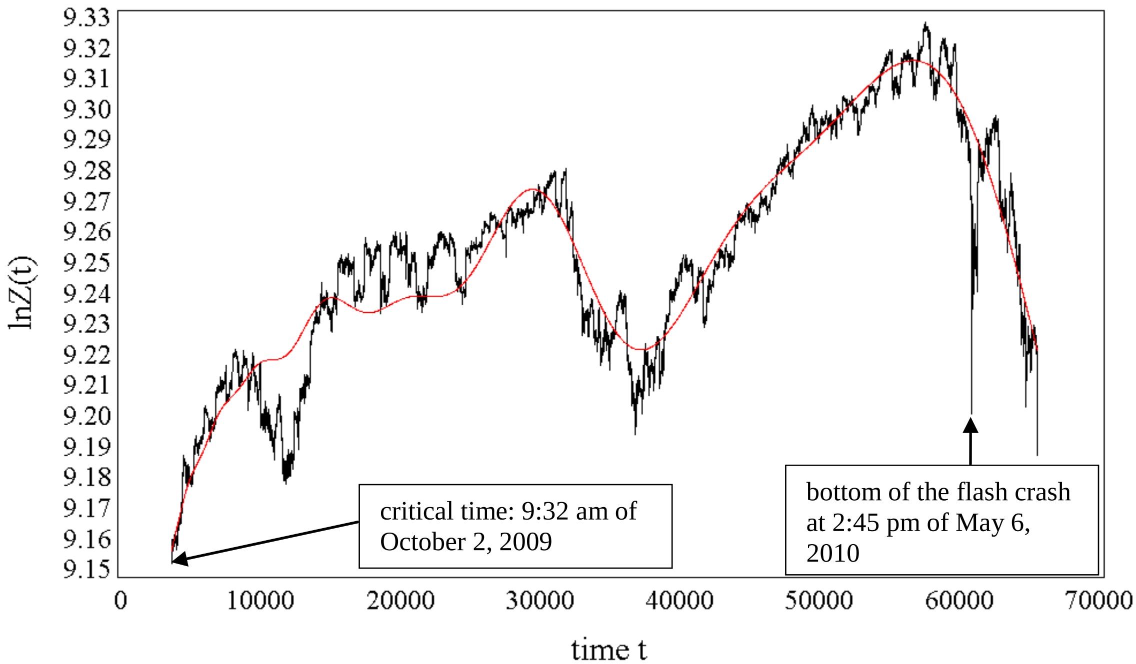

Figure 2. Critical time interval of the natural log of the Dow Jones Industrial Average index sampled at a one-minute frequency (October 2, 2009 to May 31, 2010) and a two-harmonic log-periodic fit. See Table 1.

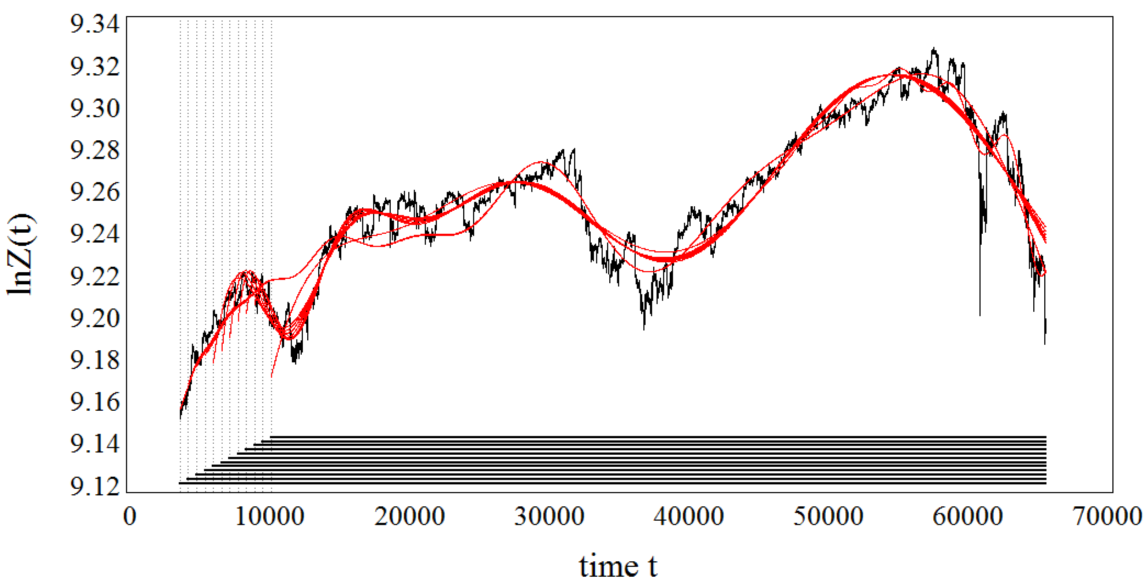

Figure 3. Overlaid fits. The size of the fit intervals were changed in steps of 592 minutes by squeezing the windows using a fixed end time at 10:09 am of May 25, 2010, and a starting time increasing from 3:09 pm of September 18, 2009 to 1:48 pm of October 26, 2009. Twelve different time windows were considered, beginning with the longest of 61,984 minutes, then the subsequent one of 61,392, and so on, following steps of 592 minutes each. Horizontal solid lines represent the range of variation of the time windows whereas vertical dotted lines represent the starting time of each fit. The original fit seems stable.

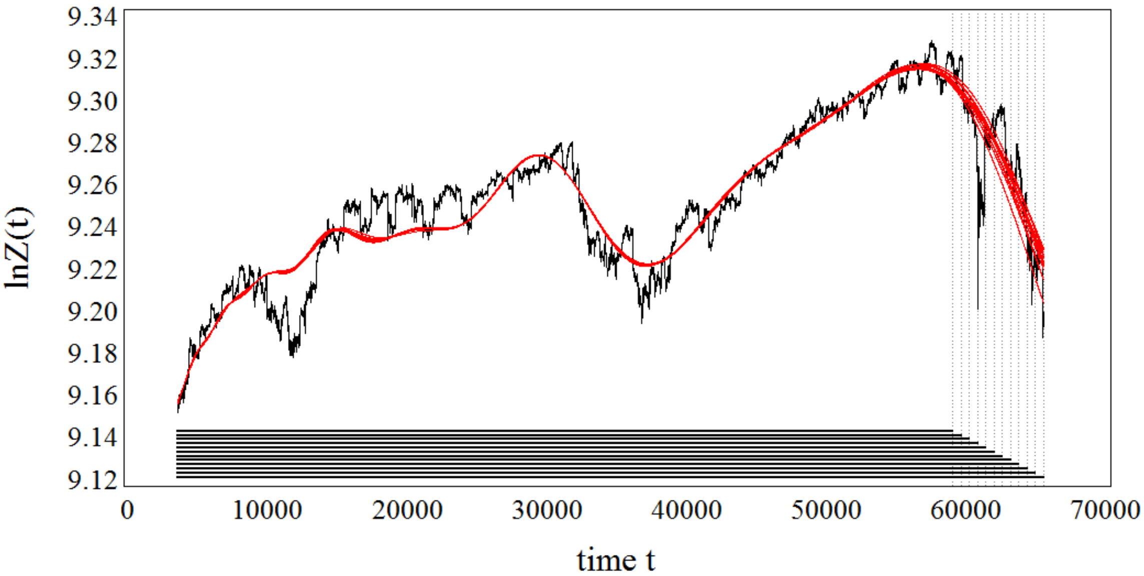

Figure 4. Overlaid fits. The size of the fit intervals were changed in steps of 592 minutes by stretching the windows considering a fixed starting time at 3:09 pm of September 18, 2009, and the end point increasing from 10:06 am of April 19, 2010 to 10:09 am of May 25, 2010. Twelve different time windows were considered, beginning with the shortest of 54,880 minutes, then the subsequent one of 55,472, and so one, following steps of 592 minutes each. The values at the end of the range were extrapolated until 10:09 am of May 25, 2010 on the basis of the adjusted parameters. Horizontal solid lines represent the range of variation of the time windows whereas vertical dotted lines represent the ending time of each fit. The original fit seems stable.

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