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A Note on Shock Persistence in Total Factor Productivity Growth

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Abstract

We study implications of persistence of shocks in total factor productivity (TFP) growth under Bayesian framework for a set of African countries over the period 1970-2003. Contrary to convention, we find that stochastic unit root is present for most of the African countries and that there is time-varying dependence structure in the underlying processes. The implication of our finding is that the persistence process governing TFP series is non-linear, stationary for some period and (mildly) explosive for others pointing to the fact that linear policy rules to counteract stochastic shocks in TFP may not prove useful. The repeat of TFP cycles is traced to this behaviour.

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1. Introduction

Theories and empirics related to economic growth (fluctuations) are so vast and debatable that it is hard to pinpoint a single theory that serve as sole explanation of the phenomenon.[1] There is, however, a common point of agreement in that total factor productivity (TFP) growth – that is the portion of output unexplained by the amount of inputs used in production – plays pivotal role in explaining cross-country income differences and volatility.[2] A shock to TFP and its persistence can, therefore, have important implications for economic growth fluctuations. Following real business cycle theory (RBC, initiated by Kydland and Prescott, 1982), such shocks are propagated by pro-cyclical labor supply and investment, thereby generating fluctuations in output (Comin, 2006). A positive shock to TFP (in the form of high investment in scientific infrastructure, or sudden scientific discoveries, etc.) has growth-enhancing attributes. Similarly, a negative shock to TFP (in the form of chaotic political economic milieu, decelerated investment in science and infrastructure, etc.) can be growth retarding.

The nature of such shocks varies according to country settings. For instance, in a recent study (viz., Barrios et al., 2009), TFP was shown to be highly correlated with changes in weather condition (especially in agriculture-dependent countries, such as Asian and African countries) where the extent of rainfall determines agricultural productivity growth. In either case, shock persistence in TFP has direct implications for growth fluctuations. The extent of persistence, however, will depend on whether TFP is endogenous or exogenous. If a large portion of TFP growth is caused by endogenous innovations and age-structured human capital growth, the measurement of persistence becomes complicated due to own and cross-interactions mechanism. Persistence is reflected by the slow-convergent pattern of TFP shocks. However, if TFP growth is exogenous, shocks to TFP taper off pretty quickly. Comin and Gertler (2006) demonstrate that low persistence and non-technological shocks generate pro-cyclical fluctuations in the market value of innovations. Importantly, by linking a component of TFP to innovation, TFP becomes a mechanism that propagates low-persistence shocks, thus increasing its persistence than its disturbances, as in standard RBC models.

Analyzing from micro-economic perspective, recent research have demonstrated that the degree of shock persistence in TFP is contingent upon the type of market structure considered. Under imperfect competition, persistence of shocks in TFP is argued to be fairly greater than under perfect competition (Martin, 2008). This conclusion reflects well the general perception of unit root persistence in output: Durlauf (1989) and others argue that incomplete information, imperfect market structure and consequent coordination failures are fundamental causes of existence of unit root in output. The history dependent nature of TFP can be identified with the degree of imperfection of the economy because convergence speeds of shocks are likely to be contingent upon the degree of imperfection of the economic system. If dependence between past and present values of TFP is small it would reflect the economy's re-adjustive, re-generative and forward looking capability. In

[1] Reasons of economic growth fluctuations are often traced to technological change and innovation following neoclassical tradition, human capital growth and distribution following modern growth tradition, environmental and demographic changes following recent research. The core of economic fluctuations is rather complex and may contain elements intertwined for a unique interaction from each of theories mentioned above.

[2] TFP is often seen as the real driver of growth within an economy and studies reveal that whilst labour and investment are important contributors, TFP may account for up to 60% of growth within economies.

light of the implications of possible persistence of shocks in TFP, in this paper we test for the persistence and employ Bayesian framework for the purpose.

Research pertaining to test of persistence of TFP is rather sparse. In an exceptional work Gil-Alana and Mendi (2005) utilizes classical method and study the stochastic properties of different measures of TFP for USA using fractional integration procedure. The authors showed that the stochastic structure of TFP is more complicated and that it is formed by the interaction of various seasonal and non-seasonal unit (or fractional) processes. Whether one uses a classical or Bayesian framework for analysis of an economic event, it is important that the method identifies (or gives a hint of) the economic structure that generates this economic event. In an interesting paper, Durlauf (1989) argues that both exact and near unit root cases cannot identify economic structure mainly due to the way the method incorporates uncertainty in the model. Bayesian paradigm is relevant in this context. Instead of testing whether there is a unit root or not or in the fractional context testing whether the event is a unit root or a fractional process, in Bayesian framework one evaluates the probability of the existence of a unit root or a fractional process. Model uncertainty in fact is better understood in a Bayesian setting than under classical paradigm. Recent literature (Sala-i-Martin et al., 2004) points to the capital importance of Bayesian implementation while determining long-term economic growth using a large set of possible explanatory variables.

While Bayesian test is claimed to perform better while dealing with and understanding uncertainty, classical procedure of testing unit root is very much complementary. However, due to the perceived advantage of Bayesian framework with respect to uncertainty modelling in the autoregressive dependence structure of a time series (to be delineated shortly in the following section), we employ Bayesian procedure in this paper to test for persistence in TFP growth. The rest of the paper is organized as follows. Section 2 describes Bayesian approach to testing unit root and provides insight into the time varying character of autoregressive structure of TFP. Section 3 describes data and estimation issues. Empirical results are analyzed in Section 4 and finally Section 5 concludes with implications of our results.

2. Defining TFP persistence and testing via Bayesian route

2.1 How does persistence occur?

Assume that TFP at time t is denoted by z_t. Also denote the shock to z_t by $\varepsilon(z_t)$. The duration that a shock to TFP survives to time t describes persistence character and which is given by error duration structure.[3] Let $\{\varepsilon_t$, t=1,2,...$\}$ be a series of i.i.d. shocks with mean zero and finite variance σ^2. The error ε_s has a stochastic duration $\eta_s \geq 0$, surviving from period s until period $s+\eta_s$. Let $g_{s,t}$ be an indicator function for the event that error ε_s

[3] The corresponding conventional method is based on time series, such as the class of autoregressive fractionally integrated moving average models (ARFIMA) model. ARFIMA models are criticized on theoretical grounds that they do not identify the source of persistence and it is difficult to know what generated persistence in TFP series for instance. Error duration model overcomes this short-coming. However, both error duration and ARFIMA class suffers from the assumed knowledge of the parameters, which in contrast to Bayesian setting are assigned some probability values. Since most of the properties of error duration model contains characteristics of ARFIMA class of persistence, it is worthwhile to describe persistence in TFP with error duration structure.

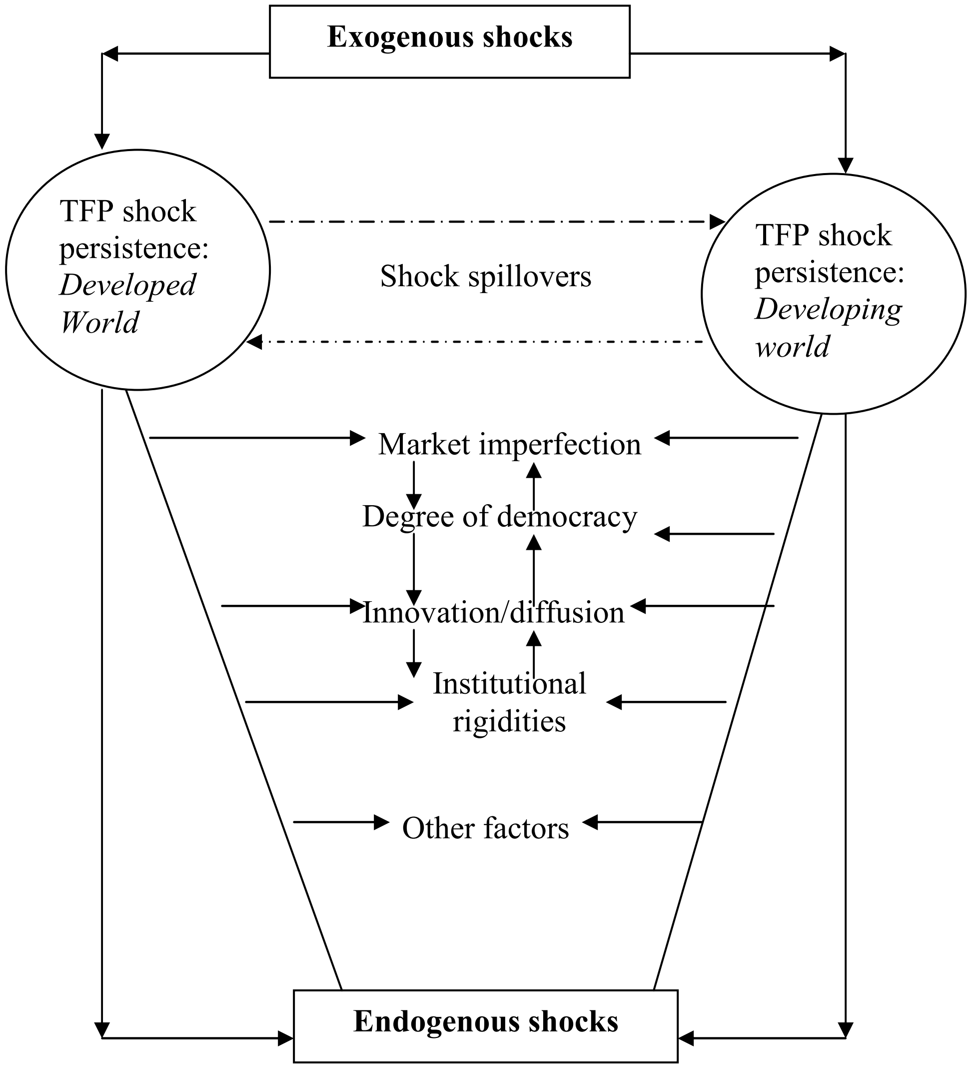
survives to period *t*. That is, $g_{s,t} = 1$ for $t \leq s + \eta_s$ and $g_{s,t} = 0$ for $t \geq s + \eta_s$. Let p_k be the probability of the event that ε_s survives until period *s*+*k*. That is, $p_k = P(g_{s,s+k} = 1)$. Assume that $p_0 = 1$ and that the sequence of probabilities $\{p_k, k=0,1,2,...\}$ is monotone non-increasing. The realization z_t is the sum of all errors $\varepsilon_{t-i}, i = 0,1,2,...$ that survive until period t is given by $z_t = \sum_{s=-\infty}^{t} g_{s,t}\varepsilon_s$. The survival probabilities $\{p_0, p_1, p_2, ...\}$ are the fundamental parameters of this error duration representation of z_t.

Whether a shock to z_t survives and assumes a long life or persistence depends on both micro and macroeconomic factors, viz., market imperfection, incomplete information, institutional rigidities, stable government, innovation and diffusion rates, and the exogenous factors such as rainfall, agricultural productivity, etc. Persistence of shocks in TFP is therefore inherently complex and does not lend to easy and direct modelling convenience. The interpretations and implications of shocks persistence in TFP on business cycle behaviour and regenerative and re-adjustive capacity of the economy are therefore varied. Empirical scrutiny of TFP shock persistence weigh varying importance for different country settings: determinants of TFP shocks for Africa and some Asian countries are predominantly influenced by rainfall and natural impediments including some degree of institutional imperfections. For developed economies, TFP shock persistence is mainly determined by innovation and diffusion rates of new ideas and in some degree by democratic setting and market imperfections. The survival probabilities of shocks converge slowly to zero in case of high persistence of shocks and this is clearly defined by the above factors for different country settings.[4] If that is the case, we would expect a fairly rapid increase in innovation and diffusion and the high growth momentum of national output.

Chart 1 below outlines the various determinants of TFP shock persistence for developing and developed world. Due to high degree of economic integration through trade of goods and services, TFP shocks can be correlated across countries even when TFP growth is affected by exogenous (viz., rainfall, agricultural productivity, natural disaster like earthquake, etc.) and endogenous shocks (viz., innovation and diffusion rate, democratic setting reflecting stability, institutional rigidities including government's action/inactions, and market imperfections). The endogenous and exogenous nature of shocks is important for defining the degree of shock persistence although there is no clearly differentiated stream of literature to support this claim. Irrespective of their nature, shocks must survive certain period in order to be persistent which implicitly reflects the nature of uncertainty, imperfection and incompleteness of the economic system.

[4] Parke (1999) developed a duration dependent model where he showed that a long-memory persistence of shock can be modelled as the slow convergence of survival probabilities of shocks to one.

Chart 1: Role of exogenous and endogenous shocks in TFP persistence



2.2 The construct

To define TFP let's assume the standard neoclassical production technology with constant returns to scale:

$$Y_t = AK_t^{\alpha} L_t^{\beta} \qquad (1)$$

where Y is output, physical capital (*K*) and labour (*L*). In (1) α represents the share of capital in the production of one unit of output, *Y*. Labor's share is presented by β. Various degrees of returns to scale occur when the combined value of $\alpha + \beta$ exceeds, is less than or equal to unity. For instance, decreasing returns to scale to labor and capital occurs when the inputs' marginal productivities decline over time in the absence of any qualitative improvement of their efficiencies. In the wake of endogenous growth theory's emphasis on the centrality of human capital in ensuring increasing returns to scale in production, recent studies in TFP prefer to utilize human capital (*H*) instead of labor (L) in (1).

$$Y_t = AK_t^{\alpha} H_t^{\beta} \qquad (1')$$

Efficiency enhancement in inputs gives rise to increasing returns to scale. In the absence of *K* and *H*, output *Y* grows due to the *A*, the TFP or broadly due to innovation. Simple algebraic manipulation leads to the following TFP growth equation which is in line with Solow[5] (1956):

$$\frac{\dot{A}}{A} = \frac{\dot{Y}_t}{Y_t} - \alpha \frac{\dot{K}_t}{K_t} - \beta \frac{\dot{H}_t}{H_t} \qquad (2)$$

Let's denote by z_t, the TFP (i.e., $\dot{A}/A$) at time t. The evolutionary path of z_t is governed by how a shock imparted to z_t evolves over time. Denote by ε_t, a shock at time t. Then a negative shock to TFP in the form of natural disaster, political turmoil, etc., will give rise to decelerated growth while a positive shock in terms of innovation and diffusion, and good social development will accelerate economic growth. Perpetuation of business cycles will be directly influenced by whether there is a persistent negative/positive TFP shocks. While it is required that at least a constant or increasing returns to scale should exist in the production technology to generate persistence in output (*Y*), such requirements are not necessary for enabling TFP persistence as this is determined outside the economic system.[6]

Recall that shocks to TFP can take the form of both growth-enhancing and retarding effects. TFP series can be persistent, if shocks imparted to the series take long time to

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[5] Solow (1956) showed that long-run growth in income per capita in an economy must be driven by growth in TFP. That is, assuming standard neoclassical production function where if inputs are measured correctly, then the net increase in output growth or its fluctuations are accounted for by TFP growth. From broader perspective, TFP growth is associated with technological advances and innovations, the growth of which is highly correlated with labor productivity and investment in education, infrastructure and innovation projects.

[6] This is the case of exogenous TFP. Even if TFP is endogenous, such requirements are obsolete in this case as endogeneity of TFP depends on policies than on input use.

converge or forever drift away from the (long-run) mean. If innovation is diffused widely and equally valued, the long-run TFP fulfils the following relation:

$$z_t = \xi t \tag{3}$$

where t is time unit and ξ defines the growth. The TFP growth is deterministic to the extent that each innovation or technological development leads to the same economy-wide implementation. However, TFP growth will be stochastic if there are multiple equilibria for the implementation of each technology, and that these equilibria endogenously evolve in response to various random events in the economy (Cordoba and Ripoll, 2008). In this case, the realized TFP will be the sum of random variables, ς such that

$$z_t = \sum_{i=0}^{t} \xi_i \tag{4}$$

In (4) the TFP process contains an exact unit root. To understand its implication, let's describe the evolution of z_t by an autoregressive (AR) moving average (MA) specification (ARMA). The endogenous or exogenous nature of TFP is characterized by whether z_t is a pure AR or a pure MA process. For our purpose, we assume that z_t follows a history dependent structure such that the evolutionary path of z_t is provided by the following $AR(1)$ process without constant term,

$$z_t = \rho z_{t-1} + \varepsilon_t \tag{5}$$

where in (5) it is further assumed that z_0 is a known constant, ε_t are *i.i.d* normally distributed with mean zero and unknown variance, σ^2, and $\rho \in S \cup \{1\}; S = \{\rho \mid -1 < l \leq \rho < 1\}$, l is the lower bound which determines the specification of the prior for ρ. We are interested in discriminating between a stationary model $(\rho < 1)$ and the nonstationary model with $\rho = 1$, *i.e*, a random walk. The assumption of known z_0 points to the dependence of analysis on initial observations as the treatment of such condition will differ between stationary and non-stationary regions (Sims, 1988; Sims and Uhlig, 1991).

There is a sharp distinction between the testing procedure of existence of unit root between classical and Bayesian models. While a knife-edge distinction is made between the presence and absence of a unit root in the form of testing whether $\rho = 1$ or $\rho < 1$ in classical Dickey-Fuller (1979) and its subsequent extensions, Bayesian mechanism asks how probable is the hypothesis that $\rho = 1$ against $\rho < 1$. This is because Bayesians are uncomfortable with testing a point hypothesis since it is not natural to compare an interval that receives a positive probability (the composite alternative $H_1 : \rho < 1$) with a point null hypothesis of zero mass (the null hypothesis $H_0 : \rho = 1$). It is argued that the classical econometricians cannot provide probability that a hypothesis holds. What they can tell us is whether a hypothesis is rejected or not rejected (Koop, 1992). Moreover, classical test procedure is also criticized very strongly on the ground that it uses information that is not contained in the likelihood function which violates the likelihood principle[7] (Bauwens et al. 1999).

Sims (1988) argues that the classical tests for unit root possess the unusual nature of asymptotic theory leading to disconnected confidence intervals and the lack of power in small samples. In a simple AR (1) process as in (5), Sims and Uhlig (1992) show that (flat-prior) Bayesian theory produces symmetric posterior distributions centred on the true value of ρ, even when ρ equals 1. Thus, larger t-values are required to reject the null hypothesis of a unit root. Sims and Uhlig (1992) show that classical unit root tests implicitly place higher probability on values of ρ above 1.0 than those below. To interpret it implies that "naïve use of classical tests' p-values not only gives special prior weight to $\rho = 1$, it implies a prior belief that a ρ of 1.05 is more likely than a ρ of 0.95" (Sims and Uhlig, 1992). Interpreted in line with the argument of the existence of incomplete information and imperfect market structure, then classical unit root test will systematically place higher weight on the existence of such character to high degree than to its lesser degree. Test of fractional integration as first proposed by Granger (1980) and Hosking (1981) and later modified by a series of papers (e.g., see Kim and Phillips, 2002; Davidson and Sibbersten, 2008) overcomes some of the problems of unit root persistence, but the fundamental problem remains: that it is necessary to know the probability of existence of a unit root/the value from a fractional estimation. The Bayesian unit root test procedure seems to overcome the problem. The method is summarized below.

2.3 Bayesian unit root test

Recall that equation 5 reflects that TFP at t, i.e., z_t depends on its past value as well as on the stochastic error term, ε_t. History is shown to affect the evolution of z_t and as long as ε_t is an *iid* process, the evolutionary path of z_t will be solely determined by its past, z_{t-1} and the coefficient determining the extent of dependence is ρ. Question may arise then what is the probability that a particular value of ρ will occur given the value of z_t, i.e, one needs to find, $\Pr(\rho \mid z_t)$. This is arrived at by using Bayes theorem which amount to evaluating the product of the likelihood of $\Pr(z_t \mid \theta)$ and a prior probability $p(\theta)$, where $\theta = \{\rho, \sigma\}$. That is, the posterior information on ρ given the evolution pattern of z_t can be described by: $\Pr(\rho \mid z_t) \propto p(\theta).\Pr(z_t \mid \theta)$. Zellner (1971) proposes the following posterior odds ratio test to compare a sharp null hypothesis with a composite alternative hypothesis,

$$M_1 = M_0 \frac{\int_0^{\infty} p(\sigma) L(z \mid \rho = 1, \sigma, z_0) d\sigma}{\int_S \int_0^{\infty} p(\sigma) p(\rho) L(z \mid \rho, \sigma, z_0) d\sigma d\rho} = \frac{\Pr(\rho = 1 \mid Z)}{\Pr(\rho \in S \mid Z)} \tag{6}$$

where M_0 is the prior odds in favour of the hypothesis $\rho = 1$, M_1 is the posterior odds in favour of the hypothesis $\rho = 1$, $p(\rho)$ is the prior density of $\rho \in S$, $p(\sigma)$ is the prior density of σ, $L(z \mid .)$ is the likelihood function of the observed TFP data $z = (z_1, ... z_T)'$,

[7] This principle makes explicit the notion that only the observed data should be relevant to the inference about the parameter. This lies at the heart of the Bayesian inference.

and finally, $Z = (z_0, z')'$, all observed data. The posterior odds M_1 are equal to the prior odds M_0 times the Bayes factor. The Bayes factor is the ratio of the marginal posterior density of ρ under the null hypothesis $\rho = 1$ to a weighted average of the marginal posterior under the alternative using the prior density of ρ as a weight function. The specification of marginal prior of ρ and σ are assumed as: $\Pr(\rho = 1) = \omega = M_0 /(1 + M_0)$, $p(\rho \mid \rho \in S) = 1/(1 - a)$, and $p(\sigma) \propto 1/\sigma$. The prior on ρ is uniform and has a discrete probability ω that $\rho = 1$. The prior on σ is diffuse, and corresponds to a uniform prior on $\ln\sigma$.

Testing for unit root under Bayesian setting was proposed by Sims (1988) who utilized a flat prior on the AR parameter. The idea lies in discriminating between a stationary $(\rho < 1)$ and a nonstationary ($\rho = 1$) model in (5). We initially put probability α on the interval (0,1) for ρ, probability $1-\alpha$ on $\rho = 1$, and independently a flat prior on $\ln\sigma^2$. The likelihood then assumes a normal inverse-gamma shape, conditional on the initial observations. For large T the odds ratio in favour of the $\rho = 1$ null hypothesis is: $\dfrac{(1-\alpha)\phi(\tau)}{\sigma_\rho \{\alpha\Phi(\tau)\}}$ where it is assumed that the posterior probability on $\rho < 0$ turns out to be negligible. The criterion then compares: τ^2(the square of conventional *t*-statistics) to $2\log(1-\alpha)/\alpha - \log(\sigma_p^2) + 2\log(1 - 2^{-1/s}) - 2\log\Phi(\tau)$ (the Schwarz value which has has an asymptotic Bayesian justification and is considered as the asymptotic Bayesian critical value). $\sigma_p^2 = \sigma^2 \sum z_{t-1}^2$, σ^2 is the variance of ε_t. $s = 1$ for annual data. If $t^2 > Schwarz$ limit, we reject the null hypothesis of a unit root.

Sims (1988) notes that it may not be reasonable to treat the prior as uniform over (0,1). Instead, we are interested in the case when the likelihood is concentrated somewhere near one. A lower limit for the stationary part of the prior is also specified such that the prior for ρ is flat on the interval (lower limit, 1.0). The concentration of the prior around 1 increases with the frequency of the data. If the prior is concentrated on (0.5, 1) for annual data, then for monthly data it is on (0.94, 1) where $0.94=0.5^{1/12}$. Following Sims (1988), α = 0.8 is a reasonable choice since for this level the odds between stationarity and the presence of a unit root are approximately even.

2.4 Posterior modelling of AR parameter of TFP

Given that TFP growth is described by an AR process (as in (5)), it is interesting to study how the posterior value of ρ in (5) changes given the prior information we have on the data. Persistence character is essentially posterior information and this is affected by initial value of the series as well as the prior information. Marginal posterior distributions of ρ are then derived to show how z is sensitive to the changes in the information in the parameters, ρ and σ as well as the initial value, z_0. Conditioning on z_0, the Gaussian likelihood of TFP growth follows from the density:

$$f(z \mid \rho, \sigma, z_0) = (2\pi)^{-T/2} \sigma^{-T} \exp\left\{ -(1/2)\sigma^{-2} \sum_{t=1}^{T} (z_t - z_{t-1})^2 \right\} \qquad (7)$$

A natural question is how to express ignorance about the parameters of interest? The first method to do this is to use a uniform or a flat prior. No prior is uniformly best when the initial condition is modelled.[8] The alternative solution is suggested by Phillips (1991) who suggested the use of a linear model without modelling the initial condition together with the complete Jeffreys' prior. This is because, as Phillips puts, the impact of the initial condition is negligible if the magnitude is not too large. Bauwens et al (1999) compare the risk function of this procedure to the risk function of the criticized flat prior for ρ combined with the marginal exact likelihood function[9] and conclude that Jeffreys' prior used by Phillips is uniformly dominated. The authors conclude that the flat prior is by no means responsible for the statistical paradox. The sensitivity of different priors can be tested and this has been provided by Bauwens et al. (1999). For the purpose of the study, we utilize flat prior for $(\rho, \log\sigma)$ which gives rise to the uninformative prior for (ρ,σ) so that

$$p(\rho,\sigma) \propto 1/\sigma \tag{8}$$

The joint distribution is:

$$p(\rho,\sigma \mid z, z_0) = \sigma^{-T} \exp\{-(1/2)\sigma^{-2}[B(\hat{\varepsilon}) + (\rho-\hat{\rho})^2 B(z)]\} \tag{9}$$

where $\hat{\rho} = \dfrac{\sum z_t z_{t-1}}{\sum z_{t-1}^2}$, $B(\hat{\varepsilon}) = \sum \hat{\varepsilon}_t^2$, $B(z) = \sum z_{t-1}^2$

The corresponding marginal posteriors are:

$$p_1(\rho \mid z, z_0) \propto [B(\hat{\varepsilon}) + (\rho-\hat{\rho})^2 B(z)]^{-T/2} \tag{10}$$

$$p_2(\sigma \mid z, z_0) \propto \sigma^{-T} \exp\{-(1/2)\sigma^{-2} B(\hat{\varepsilon})\} \tag{11}$$

Thornber (1967) and Zellner (1971, Ch. VII) both used this framework and emphasized its applicability for stationary and nonstationary cases. Geweke (1988) used the same approach in a cross-country applied study but used a restricted domain in addition to the flat prior. Sims (1988) and Sims and Uhlig (1988/1991) also utilize this framework. Schotman and van Dijk (1991) employ a similar approach in studying real exchange rate data. However, since their objective is to perform a posterior odds analysis of the unit root hypothesis, they modify (8) by truncating the domain over which p has a flat prior to a proper subset of the stationary interval and they assign a discrete prior probability mass to p = 1 (values of p in the explosive range being excluded).

In (10) and (11), we would like the marginal posteriors to be finite and integrable over [0,1] when combined with the prior density. To derive marginal posteriors it is necessary to implement numerical integration method and in this case we utilize Simpson's rule of integration, which is based on the method of interpolation. This rule is by far the most

[8] Bauwens et al (1999) examines this in case of flat prior , Uhlig, Lubrano, Berger and Yang, and Phillips prior.
[9] The authors use the likelihood function of the non-linear model with a constant term.

frequently used in obtaining approximate integrals (the detailed derivation is provided in the appendix).

2.5 Is the AR parameter of TFP changing over time?

An important issue of persistence test is whether autoregressive parameter itself is changing over time. In a historical time series like TFP, it is possible that the economic structure governing the AR process is time varying and that several epochs may well exist in the series which have implications for persistence profile and business cycles. In light of this importance, we next test whether the AR structure of z_t is time varying. We will focus on the question of whether the basic structure of the time series model driving the total factor productivity growth is changing over time. To this end we consider an AR(p) model with time varying coefficients in z_t:

$$z_t = \mu_{0t} + \mu_{1t} z_{t-1} + ... + \mu_{pt} z_{t-p} + \varepsilon_t \tag{12}$$

where an AR(p) structure of z_t is assumed. In (11) for $i = 0,..., p$

$$\mu_{it+1} = \mu_{i,t} + u_{it} \tag{13}$$

Independence of errors is assumed in such that $\varepsilon_t \sim iid\ N(0, h^{-1})$ and $u_{it} \sim iid\ N(0, \lambda h^{-1})$ where h is the variance. Equation (12) describes is an AR model for TFP with time varying coefficients including the intercept. We further assume that the coefficients are gradually changing over time. From economic growth theoretic perspective, this lends interesting insights as TFP growth change is most probable in the face of rapid scientific development, governance change and so forth. The most likely effect of such changes would reflect on the AR coefficients of TFP. If a change in the intercept is observed over time, this would imply a perceived change in the economic structure itself. Similary, if a change in AR lag structure is observed over time, the TFP series history dependence structure governed by innovation and diffusion, etc., can be perceived. Koop (2003) provides a detailed description of the Bayesian analysis of the test of time-varying AR structure. The author suggests the use of an informative prior for the parameters h and λ_i. Significance of values of λ for both intercept and lagged coeffients are then checked to comment on the stability of the AR conefficients and provide implications for the presence of stochastic unit root.

3. Data and estimation issues

In this section, we briefly discuss the data issues related to TFP calculation and estimation issues related to posterior estimation. Our sample covers three decades (1970-2003). Physical capital stocks were calculated according to the method used in Klenow and Rodriguez-Clare (1997). Initial capital stocks are calculated according to the formula,

$$\frac{K}{Y_{1970}} = \frac{I/Y}{\delta + \eta} \tag{14}$$

where (I/Y) is the average share of physical investment in output from 1970 through 2003, Y represents the average rate of growth of output per capita over that period, η represents the average rate of population growth over that period, and δ represents the rate of depreciation, which is set equal to 0.03. Given the initial capital stock, the capital stock of country i in period t is calculated by the perpetual inventory method:

$$K_{it} = \sum_{j=0}^{\infty}(1-\delta)^{t-j} I_{ij} (1-\delta)^{t} K_{1970} \qquad (15)$$

TFP is then calculated as,

$$TFP_{it} = y_{it} - (1/3)k_{it} - (2/3)h_{it} \qquad (16)$$

where the lower case letters for K and H represent $ln(K)$ and $ln(H)$. The global share of labor and capital in the Cobb-Douglas production technology has been assumed to be approximately (1/3) and (2/3) respectively where a constant returns to scale is allowed in the aggregate growth of all inputs together. The real GDP per capita series, measured in thousand constant dollars in 2000 international prices, are extracted from the Penn World Table Version 6.1 (Summer and Heston, 2005), while the age-structured human capital data is obtained from IIASA-VID (see Lutz et al. 2007).

The new data set on human capital comprises educational attainment by age groups for most countries in the world at five-years intervals for the period 1970-2003. Demographic back-projection methods were used in order to recover the age/education pyramid of each country, taking into account differential mortality and migration by both age groups and educational attainment. The back-projection exercise was carried out as a joint effort by the International Institute for Applied Systems Analysis (IIASA) and the Vienna Institute of Demography (VID) of the Austrian Academy of Sciences, so we will refer to this dataset as the IIASA-VID data. Lutz et al. (2007) provide a detailed account of all the specific assumptions that had to be made as part of this reconstruction exercise, discuss their plausibility and provide sensitivity analysis.[10]

We have estimated the posterior density of AR parameter in (5) and performed Bayesian unit root test for 22 African economies' TFP data for the period 1970-2003.[11] Under non-stationarity, estimation of posterior density is not straightforward as it involves lot of computational problems. Especially, it is required to solve a high-dimensional integral to integrate out the posterior function. Among several approaches to solve this problem, Simpson's integration rule is easy to use, at least when a flat prior is used for defining the posterior, which is the case with our specification. For details on Simpson's and other rules, the readers are referred to Bauwens et al. (1999).

[10] This new dataset allows us to assess the importance of the interaction of the demographic and educational characteristics of a society on income growth at the macroeconomic level. The results of Crespo Cuaresma and Lutz (2007) and Crespo Cuaresma and Mishra (2007) point at a capital importance of assessing the demographic dimension of education data when explaining cross-country differences in income, income growth and economic growth externalities.

[11] The choice of the countries is mainly based on data availability. Especially the age-structured human capital data is not available for all African countries consistently for all the years.

4. Empirical results

4.1 Bayesian unit root test results

Before analysing the results of the posterior density, we first interpret the results of Bayesian unit root which utilizes the posterior odds ratio test (in 4) with flat prior. Bayesian unit root test results are presented in Table 1. Note here that α gives the prior probability on the stationary ρ; the remaining probability is concentrated on $\rho = 1$. 'Marginal α' is the value for alpha at which the posterior odds for and against the unit root are even. A higher value of 'marginal α' favours the presence of unit root. Similarly, if $t^2 > Schwarz$ (asymptotic Bayesian) limit, we reject the null hypothesis of a unit root. From Table 1 it can be observed that $t^2 <$ Schwarz (asymptotic Bayesian) limit for 14 out of 22 countries. That is, unit root cannot be rejected for fourteen countries while for eight countries there is no unit root persistence in TFP series. If we examine the marginal α for each country's TFP, it provides evidence of the estimated probability of the existence of unit root persistence for respective countries. Among 22 countries, the marginal α is highest for Egypt (0.908) and lowest for Morocco (0.000). In other cases, when t^2 exceeds Schwarz limit, but have small values of marginal alpha (less than 0.5), it indicates that only a very strong prior on the unit root will overcome the data evidence against it.

Table 1: Bayesian unit root test for Total Factor Productivity for Africa (1970-2003)

Variables	Squared t (t^2)	Scwarz limit	Marignal α
Benin	2.304	4.772	0.406
Burkina Faso	13.240	5.056	0.003
Chad	3.064	5.637	0.419
Cote d'Ivoire	9.449	5.711	0.029
Egypt	0.034	7.851	0.908
Gambia	8.048	5.763	0.059
Ghana	8.603	5.047	0.032
Guinea	4.247	6.130	0.338
Kenya	6.119	4.834	0.094
Madagascar	0.304	6.544	0.818
Malawi	3.376	4.829	0.292
Mali	0.860	5.147	0.629
Mauritius	0.522	7.718	0.879
Morocco	31.199	6.813	0.000
Mozambique	2.219	5.240	0.474
Niger	0.498	6.404	0.792
Nigeria	3.900	5.554	0.313
South Africa	7.623	6.167	0.087
Togo	0.077	6.624	0.840
Uganda	0.892	5.776	0.696
Zambia	6.720	5.150	0.083
Zimbabwe	3.879	5.068	0.265

15 out 22 countries with t^2 < Schwarz limit.

4.1 Posterior analysis

The posterior distribution of the parameters, e.g., the *AR* parameter (ρ) have been obtained using Simpson's rule of integration and the estimation model of Zivot and Phillips (1994). The model is described in the appendix A where we have used an ADF type of specification with augmented lags for TFP for defining a history dependence character of shocks. The results of posterior ρ, its standard deviation and the corresponding range of integration are presented in Table 2. The range of integration is adjusted so as to achieve a normal distribution of ρ. The estimated values of ρ (column one) of Table 2 reflects on our expectations of the possible non-stationary or stationary value of ρ conditional on the available set of information on total factor productivity data over three decades (1970-2000) for each country. Statistically, this is given by $\Pr(\rho \mid z_t)$. From Table 2, it is evident that the posterior value of ρ is greater than 0.5 for all countries under examination. This is highest for Burkina Faso (0.925) and Uganda (0.931) whereas for Kenya (0.514) and Mali (0.547), the posterior ρ is the lowest. The range of integration for all countries shows that they swing widely between stationary and non-stationary regions.

Table 2: Comparison of posterior mode for ρ Africa (1970-2000)

Countries (TFP)	ρ	σ_ρ	Range of Integration
Benin	0.680	0.139	0.20-1.20
Burkina Faso	0.925	0.140	0.40-1.50
Chad	0.813	0.088	0.50-1.10
Cote d'Ivoire	0.626	0.134	0.10-1.10
Egypt	0.784	0.137	0.30-1.30
Gambia	0.823	0.082	0.50-1.10
Ghana	0.685	0.147	0.15-1.20
Guinea	0.923	0.079	0.60-1.20
Kenya	0.514	0.137	0.05-1.00
Madagascar	0.646	0.147	0.15-1.15
Malawi	0.656	0.153	0.10-1.18
Mali	0.547	0.116	0.10-0.95
Mauritius	0.587	0.083	0.30-0.90
Morocco	0.572	0.134	0.10-1.05
Mozambique	0.785	0.130	0.30-1.30
Niger	0.572	0.182	0.00-1.20
Nigeria	0.847	0.096	0.50-1.20
South Africa	0.806	0.113	0.40-1.20
Togo	0.821	0.071	0.50-1.10
Uganda	0.931	0.067	0.70-1.20
Zambia	0.642	0.114	0.20-1.10
Zimbabwe	0.774	0.133	0.30-1.30

To summarize, the derived posterior values of ρ indicate in our case that the TFP series has high persistent character and that the history dependence feature reflected by ρ conditional on the initial and past information about the data is very high. The posterior plots of the respective countries are presented in Figures 1 through 22, which basically reflect the posterior distribution of ρ and its range of integration to achieve normal shape.

Table 3 contains posterior results for the state space model using the TFP data and prior discussed above. We employed Gibbs sampling method which was run for 21000 replications with 1000 burn-in replications discarded and 20 000 replications retained. Posterior means and standard deviations for λ_0 and λ_1 indicate that a substantial amount of parameter variation has occurred both in the intercept and the AR(1) coefficient. This implies that in addition to the presence of a stochastic trend in TFP, the AR process itself is changing over time.

Table 3: Posterior results of state-space model for testing change in AR coefficients for z_t

Countries (TFP)	h (Mean)	Standard Dev.	λ_0 (Mean)	Standard Dev.	λ_1 (Mean)	Standard Dev.
Benin	4.198	2.792	1.056	0.476	0.232	0.095
Burkina Faso	4.266	2.937	1.050	0.459	0.236	0.094
Chad	4.221	2.819	1.052	0.459	0.240	0.091
Cote d'Ivoire	3.639	2.643	1.061	0.474	0.205	0.086
Egypt	3.622	2.683	1.064	0.474	0.201	0.085
Gabon	2.871	2.313	1.073	0.478	0.159	0.071
Ghana	4.118	2.756	1.054	0.463	0.262	0.113
Guinea	3.661	2.660	1.061	0.465	0.203	0.087
Kenya	4.274	2.805	1.054	0.466	0.234	0.093
Madagascar	3.905	2.656	1.058	0.467	0.222	0.091
Malawi	4.495	2.847	1.050	0.457	0.253	0.102
Mali	4.278	2.853	1.053	0.463	0.244	0.097
Mauritius	3.178	2.481	1.067	0.473	0.179	0.079
Morocco	3.557	2.507	1.062	0.463	0.202	0.085
Mozambique	3.899	2.729	1.059	0.467	0.212	0.088
Niger	4.001	2.811	1.058	0.471	0.226	0.094
Nigeria	3.958	2.667	1.055	0.465	0.223	0.090
South Africa	3.082	2.435	1.063	0.467	0.172	0.075
Togo	4.279	2.810	1.052	0.462	0.241	0.099
Uganda	3.906	2.691	1.059	0.472	0.216	0.088
Zambia	4.853	2.984	1.043	0.458	0.275	0.110
Zimbabwe	3.557	2.557	1.058	0.463	0.204	0.085

5. Implications and conclusion

This paper attempted to characterize the nature of shock persistence in TFP under Bayesian framework for the case of Africa. The contribution of the paper is thus two-fold: First, instead of simply testing for unit root in TFP for Africa, we concentrated on near-unit root situation, which is very similar to long-memory test in time series. It appears that stochastic unit root tests do indeed appear to provide an efficacious diagnostic for understanding the persistence behaviour of shocks. The use of stochastic unit root instead of the conventional knife-edge unit root procedure enabled us to endogenize both stationary and explosive trend behaviour of TFP series. In case of Africa, the TFP growth over the past decades have been observed to be volatile, that is remaining stationary for some period but highly volatile in other periods. This mixture of stationarity and non-stationarity could not be modelled using conventional unit root test. Employing stochastic unit root procedure has improved our understanding of TFP growth in Africa and its evolutionary pattern by the time varying estimation. The use of Bayesian mechanism has dealt with issues of model uncertainty by estimating the likelihood of stochastic unit root in TFP processes of these countries using non-informative prior on the parameter estimates.

Second, our consideration of Bayesian perspective for investigating shock persistence in TFP has several merits. Indeed, the implications of persistence under Bayesian and classical setting widely differ, at least while lending tractable economic theoretic reasons of imperfect market structure and incomplete information leading to a unit root kind of behaviour in TFP. Our test for a set of African countries for the period 1970-2003 confirmed found the TFP series inherited high persistent character as reflected by the estimated posterior value of the autoregressive parameter. The results of Bayesian unit root test (using posterior odds ratio) also confirms the above conclusion. As such, the Bayesian test provided a realistic check of the probability of occurrence of TFP value in non-stationary region, which is in contrast to the classical test of unit root, a typical knife-edge test. High range of integration of posterior density in Table 2 indicates the nature of volatitlity of TFP for the examined period. It also reflects on the type of economic structure which is identified under the frequentist approach. Additionally, it could be argued that while the issue of the presence of an exact unit root in the classical sense fail to identify economic structure (Durlauf, 1989), Bayesian analysis could provide some intuition about the behaviour of the parameter and their relation with the structure of the economy.

Finally, a note on implication of the existence of stochastic unit root in TFP for Africa is in order. The presence of stochastic unit root in TFP broadly implies that the innovation process for African economies are subject to time varying volatile shocks and no linear prediction about the nature of such shocks can be made. The low-growth momentum of African economies can be explained, at least partially, by the time-varying volatility in innovation, which can be further explained by a multitude of factors, such as rainfall, social disintegration, human capital formation, etc. In developed countries, unit root in TFP is often explicated within defined and structured market conditions and degree of incompleteness of market. In Africa and in some other transition economies, the problem points more to the combination of socio-economic, natural and political factors than are otherwise defined in developed country TFP growth processes. The finding of stochastic unit root in Africa also implies that the process governing TFP series is not linear and therefore linear policy rule to counteract the effect of stochastic shock may not prove beneficial.

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Appendix

(A) Estimation model

We utilize Zivot and Phillips (1994) idea of a stochastic nonstationary framework for Bayesian analysis of persistence. The authors employ a modified information matrix-based prior that accommodates stochastic nonstationarity and takes into account the interactions between long-run and short-run dynamics and controls for the degree of permitted stochastic nonstationarity. With augmented lags, the autoregressive dependence structure of TFP is described as:

$$z_t = \mu + \beta t + \rho z_{t-1} + \sum_{1}^{k-1} \varphi_i \ \Delta z_{t-i} + u_t. \quad (17)$$

Let $\theta' \in R^{k+2} \times R_+$ be the vector of parameters (μ, ρ, φ', β, σ) in our model;
$Z' = (z_1, \ldots, z_T)$ denote T $\times$ 1 vector of sample observations;
$\tau'_0 = (z_0, \ldots, z_{-k+1})$ denote K $\times$ 1 vector of initial values.

Then $f(z \mid \theta, \tau_0)$ is the joint probability density function (pdf) of the sample given the parameter vector θ and the initial values τ_0. Here $f(z \mid \theta, \tau_0)$ is the likelihood $L(\theta \mid z, \tau_0)$. The prior and posterior *pdf* in this case are $\pi(\theta)$ and $P(\theta \mid z, \tau_0)$ where $P(\theta \mid z, \tau_0) \propto \pi(\theta) \,.\, L(\theta \mid z, \tau_0)$. To derive the posterior, we assume that the vector of parameter in our model, $\theta' \in R^{k+2} \times R_+$: (μ, ρ, φ', β, σ). The derivation of marginal posterior pdf's for ρ would then involve extracting the following integral:

$$\begin{aligned} P(\rho \mid z, \tau_0) &\propto \int \ldots \int \ P(\theta \mid z, \tau_0)\, d\mu\, d\beta\, d\varphi\, d\sigma \\ &= \int \ldots \int \ \pi(\theta) \,.\, L(\theta \mid z, \tau_0)\ d\mu\, d\beta\, d\varphi\, d\sigma \end{aligned} \quad (18)$$

The marginal posteriors are then derived using Simpson's rule of integration.

(B) Simpson's rule for numerical integration

We are interested in computing the posterior moment:

$E[g(\theta)] = \int g(\theta)\varphi(\theta) d\theta$. Now if we write $h(\theta) = g(\theta)\varphi(\theta)$

then $h(\theta) \cong \sum_{j=1}^{n} w_j h(\theta_j)$ where w_j are positive weights and sum to one. Let's approximate $h(\theta)$ by a polynomial $p(\theta)$ of order three matching the values of h at three points 0, 0.5, and 1. Thus $\int h(\theta) d\theta$ is approximated by $\int p(\theta) d\theta$, i.e.,

$\int_0^1 h(\theta) d\theta \cong [h(0) + 4h(0.5) + h(1)]$. This approximation is exact if $h(\theta)$ is quadratic, i.e., of order 2 at the most. This rule is most frequently applied in its extended or *compound (or composite form as some authors refer it)* form. We can split the [0,1] interval into subintervals or panels and Simpson's rule is applied to each subinterval because one can miss important regions of variation of *h.* Thus, with 2*n* intervals of equal length $d = \theta_j - \theta_{j-1} = 1/2n$ based on (2*n*+1) points $\theta_0 (= 0), \theta_1, \ldots, \theta_{2n} (= 1)$, one can obtain the extended Simpson's rule (Bauwens et al. 1999):

$$\int_0^1 h(\theta) \cong (d/3) \left\{ h(\theta_0) + 4[h(\theta_1) + h(\theta_3) + \ldots + h(\theta_{2n-1})] + 2[h(\theta_2) + h(\theta_4) + \ldots + h(\theta_{2n-2})] + h(\theta_{2n}) \right\}$$

It is intuitive to note that using more points increases the quality of approximation which we follow in the estimation.

Figure 1: Posterior of ρ for Zimbabwe

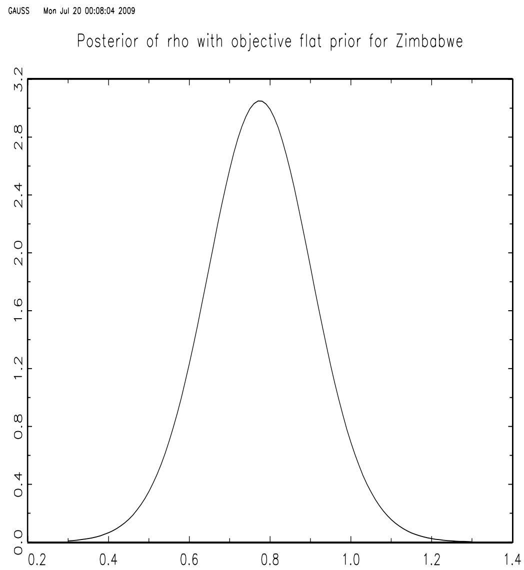

Figure 2: Posterior of ρ for Benin

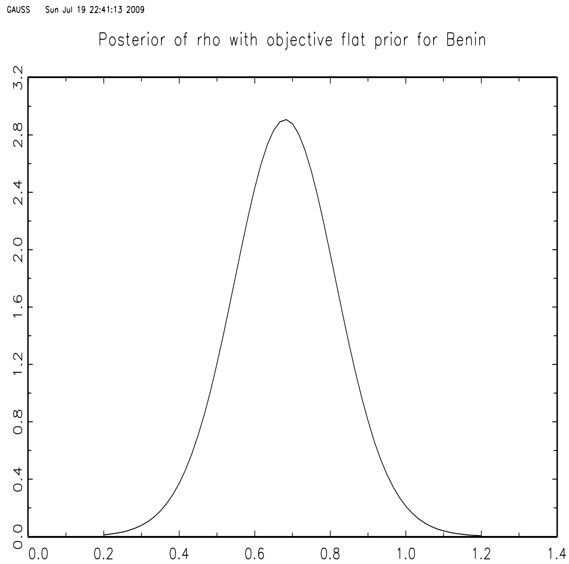

Figure 3: Posterior of ρ for Burkina Faso

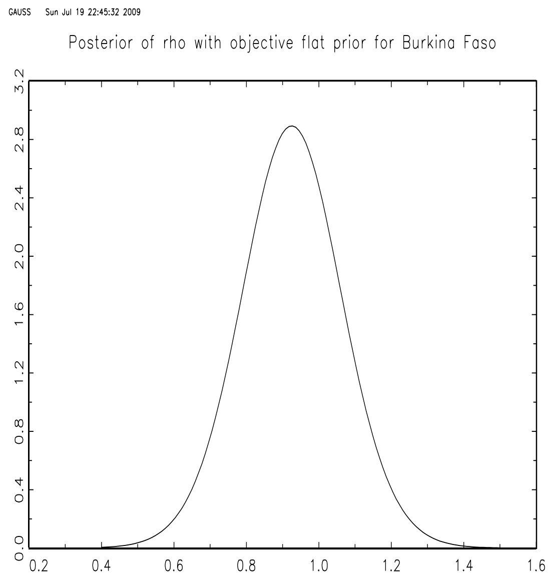

Figure 4: Posterior of ρ for Cambodia

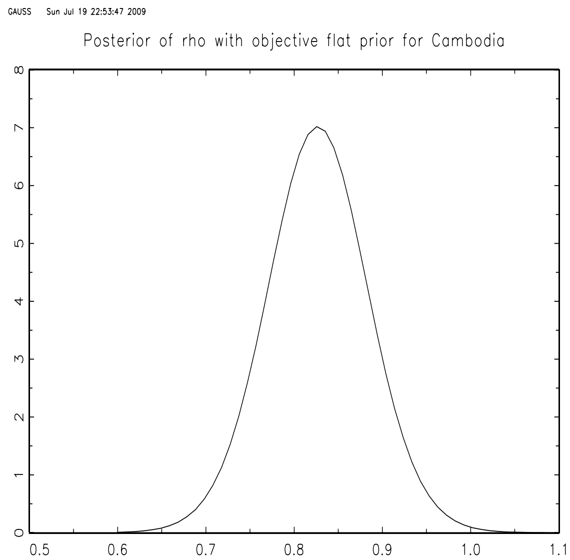

Figure 5: Posterior of ρ for Chad

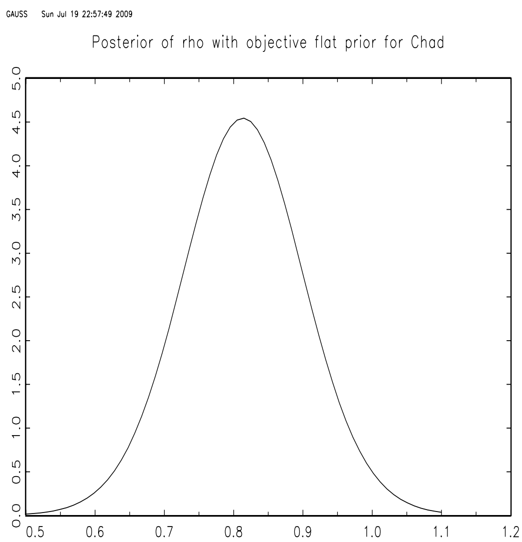

Figure 6: Posterior of ρ for Coe-d'Ivore

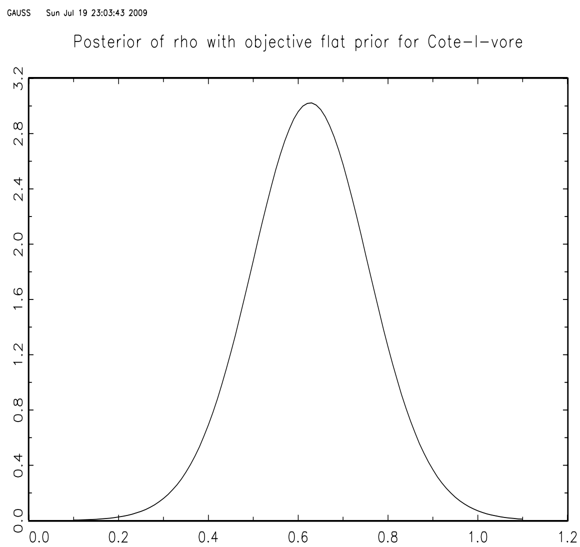

Figure 7: Posterior of ρ for Gambia

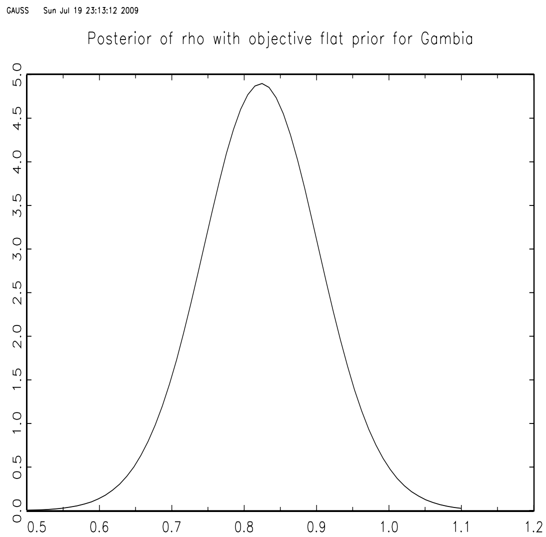

Figure 8: Posterior of ρ for Ghana

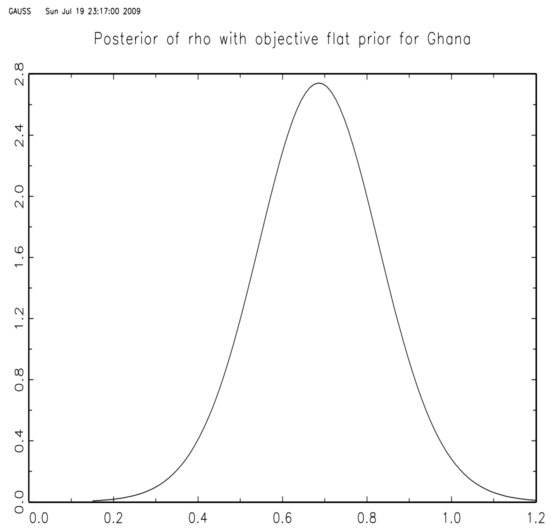

Figure 9: Posterior of ρ for Madagascar

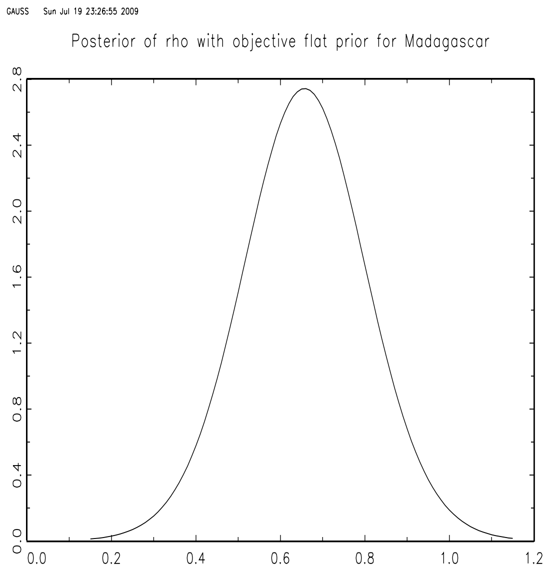

Figure 10: Posterior of ρ for Guinea

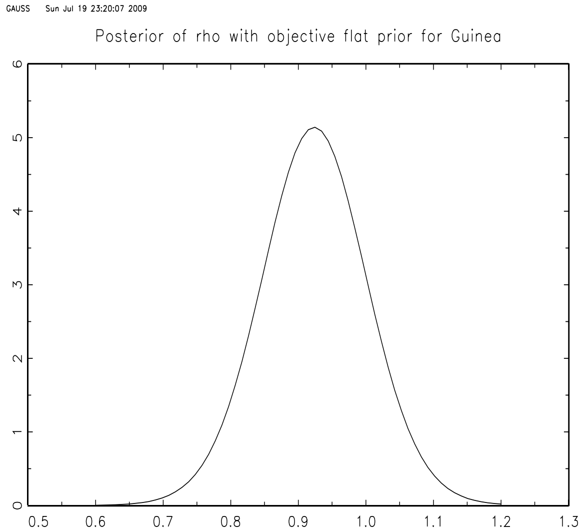

Figure 11: Posterior of ρ for Kenya

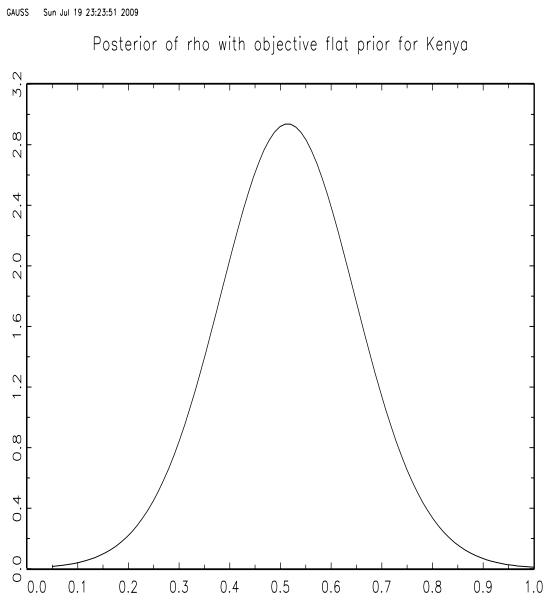

Figure 12: Posterior of ρ for Malawi

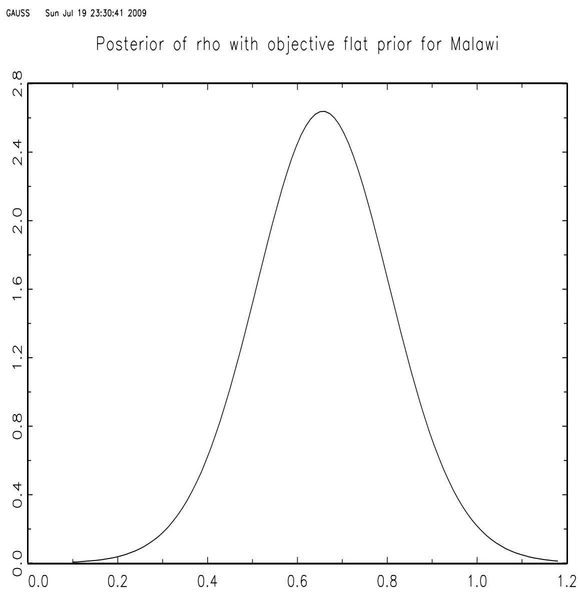

Figure 13: Posterior of ρ for Mali

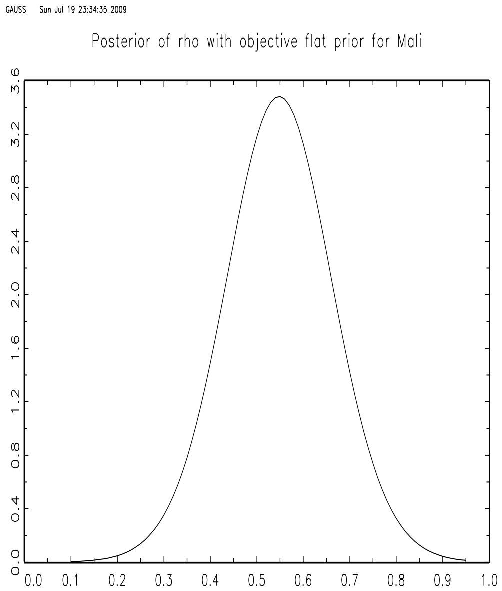

Figure 14: Posterior of ρ for Mauritius

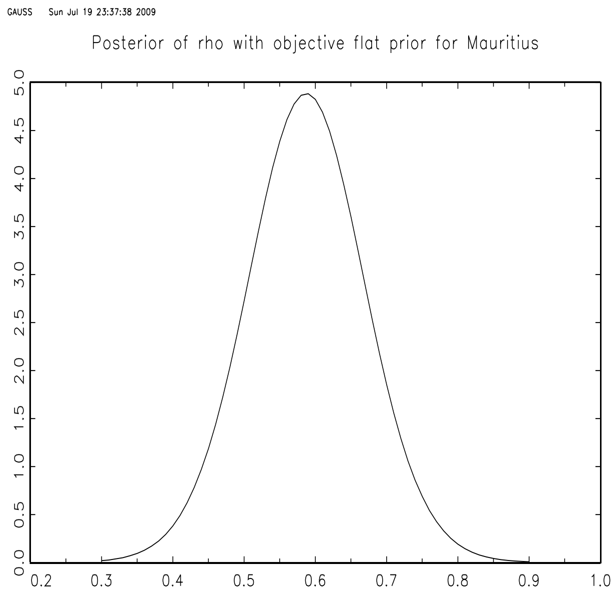

Figure 15: Posterior of ρ for Morocco

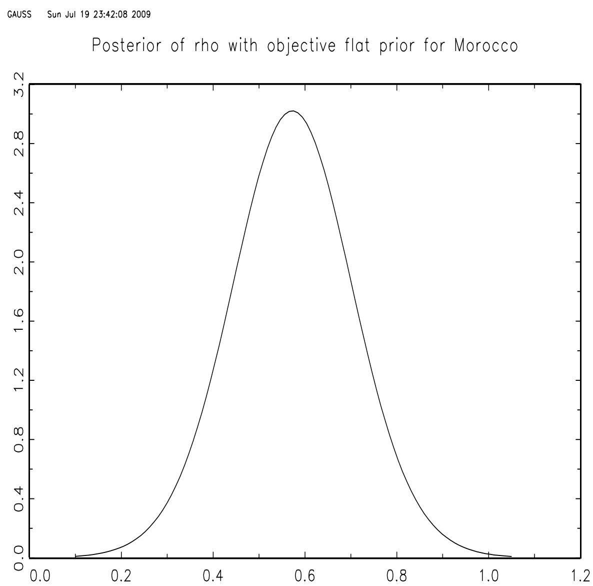

Figure 16: Posterior of ρ for Mozambique

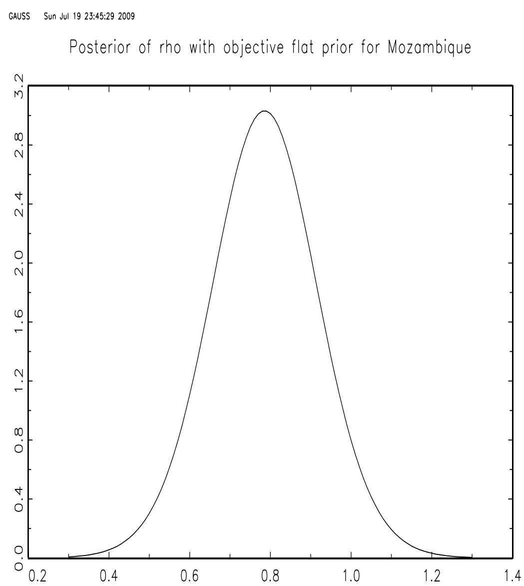

Figure 17: Posterior of ρ for Niger

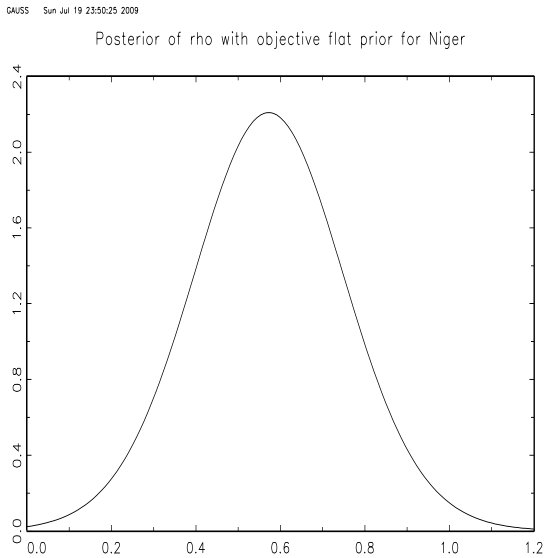

Figure 18: Posterior of ρ for Nigeria

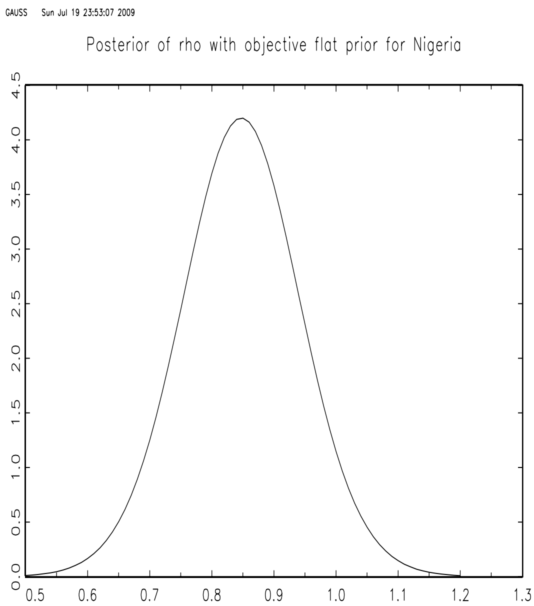

Figure 19: Posterior of ρ for South Africa

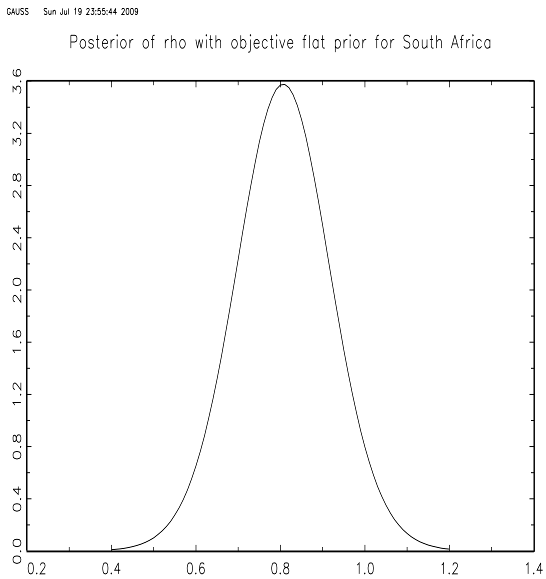

Figure 20: Posterior of ρ for Togo

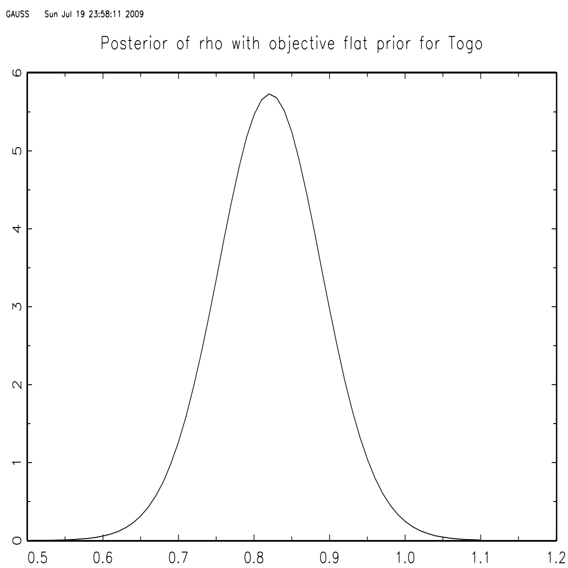

Figure 21: Posterior of ρ for Uganda

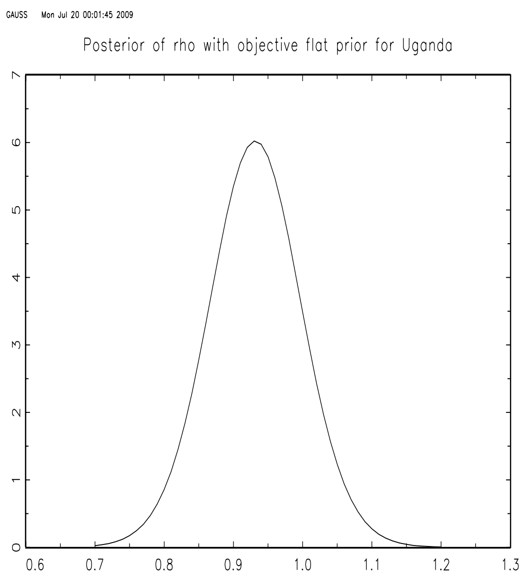

Figure 22: Posterior of ρ for Zambia