The Impact of Differential Capital Income Taxation on the Value of Risky Projects

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Abstract

We analyze the impact of differential capital income taxation on the value of risky investment under irreversibility. Under a uniform tax rate, raising the tax rate can either increase or reduce the value of a risky project. Many countries have introduced a separate flat tax on capital income. In contrast to uniform taxation, differential capital income taxation crowds out risky real investment. This dysfunctional effect can neither be corrected by generous depreciation schedules nor by increasing the flat tax rate. This tax discrimination of risky real investment might have contributed to the current crisis.
1. Introduction

The impact of income taxes on the value of risky real investment has been analyzed in valuation literature at least since Domar and Musgrave (1944). Utility function-based approaches have been extended with respect to progressive income taxes and information asymmetry by Fellingham and Wolfson (1985), for example. In recent years, the tax planning and risk-taking literature has been enriched by real option theory (see Altug et al. 2001, Niemann and Sureth 2004, Klassen and Sansing 2006, Alvarez and Koskela 2008, Agliardi and Agliardi 2009). Accounting for conditions of uncertainty and flexibility, real option theory permits an analysis of the effects of taxation on risk-taking for irreversible investment decisions. Integrating option values into the decision calculus shows that even risk-neutral investors tend to delay risky projects.

Since the 1990s, many countries have introduced separate flat tax rates on capital income. Tax systems like the Nordic dual income tax levy a low proportional tax rate on all kinds of capital income (Nielsen and Sørensen, 1997). Moreover, abstracting from flexibility differential tax rates on different assets depending on their risk exposure are examined (Bulow and Summers 1984, Gordon 1985, Weisbach 2004, Niemann 2008). Weisbach (2004) finds that investors may eliminate the tax on risk inherent in such tax systems by costly portfolio adjustments.

Other schedular taxes with differential taxation of selected types of capital income (e.g., dividends and interest income) were subsequently implemented in Austria, Belgium, Italy, Germany, Greece, and the Netherlands (Genser and Reutter, 2007). Although these schedular tax systems with differential taxation of different kinds of capital income have gained considerable importance throughout Europe, the degree to which they affect the value of risky projects and hence encourage risk-taking has not yet been sufficiently analyzed. In a real options context differential taxation of different kinds of capital income has not been analyzed at all until now.

To close this research gap, we model a tax system that differentiates between capital income from real investment and from financial investment. We integrate this system into a real option framework. Whereas we find ambiguous effects of a uniform capital income tax on risk-taking, differential taxation substantially decreases the willingness to realize risky projects.

2. Model setup

We use the Dixit-Pindyck (1994) paradigm and consider an option to invest in a project with stochastic cash flows. The risk-neutral individual investor faces the decision between investing either immediately or at a later point of time. If the tax system favors earlier investment, it implicitly encourages risk-taking.1

We assume a simple income tax system in continuous time with a profit tax base equal to cash flow π less depreciation allowances d. Cash flows follow a geometric Brownian motion

\[ d\pi = \mu \pi dt + \sigma \pi dW \]

1This effect is important to be considered as Sgroi (2003) shows that the attempt to gather more information to overcome uncertainty at least partially will generate additional delays.
with \( \frac{d\pi}{\pi} = \alpha \, dt + \sigma \, dz \), where \( \alpha \) and \( \sigma \) denote the drift and volatility parameters. The tax rate \( \tau \) on profits from real investment is assumed to be deterministic and constant. Under an immediate loss-offset, the after-tax cash flow \( \pi_\tau \) is defined as \( \pi_\tau = (1 - \tau) \, \pi + \tau \, d \). As long as the option to invest is not exercised, available funds yield the risk-free constant capital market rate \( \omega \), which is subject to the tax rate \( \omega \). \( \omega \) is a separate tax rate levied on income from financial investment, such as interest income or dividends, which might differ from the ordinary tax rate \( \tau \). Hence, the risk-free after-tax interest rate \( r_\omega \) is given by \( r_\omega = (1 - \omega) \, r \).

To exclude arbitrage opportunities, we assume \( r_\omega > \alpha \).

To derive a rule for optimal investment timing, first we have to assess the value of the underlying project. If the project is in place, its economic value consists solely of its expected future cash flows. Assuming risk neutrality, the after-tax project value \( V \) is determined by its expected present value:

\[
V = E \left[ \int_t^\infty \pi_\tau \, e^{-r_\omega (\xi - t)} \, d\xi \right] = \frac{(1 - \tau) \, \pi}{r_\omega - \alpha} + \tau \, E \left[ D \right] \, e^{-r_\omega (\xi - t)} \, d\xi,
\]

with \( D \), the present value of depreciation deductions.

Given \( V \), the value of the option to invest can be determined. As long as the option to invest is not exercised, its only payoff is the expected appreciation. Determining the post-tax option value \( F \) requires an instantaneous return that in equilibrium equals the after-tax risk-free rate: \( r_\omega \, F = \frac{1}{2} \, E \left[ dF \right] \). The application of Itô’s lemma to the stochastic differential \( dF \) yields the partial differential equation \( \frac{\partial F}{\partial \tau} + \frac{1}{2} \sigma^2 \pi^2 \frac{\partial^2 F}{\partial \pi^2} \, + \, \alpha \, \pi \, \frac{\partial F}{\partial \pi} \, - \, r_\omega \, F = 0 \). Assuming a perpetual real option, the time derivative \( \frac{\partial F}{\partial \tau} \) vanishes, leading to the ordinary differential equation \( \frac{1}{2} \sigma^2 \pi^2 \frac{d^2 F}{d\pi^2} + \alpha \pi \, \frac{dF}{d\pi} - r_\omega \, F = 0 \) with the general solution

\[
F(\pi) = A \, \pi^\lambda, \quad A > 0, \quad \lambda = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2 \, r_\omega}{\sigma^2}} > 1,
\]

where \( A \) is a constant to be determined. The well-known boundary conditions are \( F(0) = 0 \), \( F(\pi^*) = V(\pi^*) - I_0 \), with \( I_0 = 1 \) as the initial investment, and \( \frac{dF(\pi^*)}{d\pi} = \frac{dV(\pi^*)}{d\pi} \). Finally, we obtain the critical investment threshold \( \pi^* \):

\[
\pi^* = \frac{\lambda}{\lambda - 1} \, \frac{r_\omega - \alpha}{1 - \tau} \, (I_0 - \tau \, D).
\]

The critical value \( \pi^* \) indicates whether investment should be delayed or not. If the currently observed realization \( \pi \) is higher than the critical value \( \pi^* \), the investment should be carried out immediately, otherwise it must be postponed until \( \pi^* \) is reached.

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2By contrast, the effects of asymmetric taxation in a real options setting are investigated by Panteghini (2001) and Panteghini (2005).

3For an economic interpretation of the boundary conditions see Dixit and Pindyck (1994), p. 141.
3. Tax rate differentials and the value of risky projects

From $\pi^*$, it is not obvious how taxes affect risk-taking – whether they delay or foster risky projects.\(^4\) In the case of a comprehensive income tax ($\omega = \tau$) the partial derivatives with respect to the tax rate may take either algebraic sign. For given tax parameters $D$, $\tau$ and $\omega$ higher volatility $\sigma$ delays investment and lowers risk-taking. Typically, the functional relation of risk and taxes, i.e., $\sigma$, $\tau$, and $\omega$, cannot be derived analytically. Numerical simulations are necessary to get an impression of the dominant forces.

It is well known that introducing uniform tax rates on the return of both real and financial investment can increase or decrease the investment threshold for low depreciation deductions. By contrast, given sufficiently high depreciation allowances, realistic uniform tax rates may easily halve the critical threshold.\(^5\)

To find how differential capital income taxation affects risk-taking we model two separate tax rates. Further, we account for different degrees of risk involved in the real investment. We study various combinations of the tax rate $\tau$ on profits from real investment and volatility $\sigma$, each inducing identical investment thresholds $\pi^*$.

Formally, this means that the critical threshold’s total differential $d\pi^*$ taking into account the variables $\tau$ and $\sigma$ is set to zero, whereas the other determinants are kept constant.

\[
\begin{align*}
\frac{d\pi^*}{d\tau} &= \frac{\partial \pi^*}{\partial \tau} d\tau + \frac{\partial \pi^*}{\partial \sigma} d\sigma = 0 \\
\frac{d\sigma}{d\tau} &= -\frac{\partial \pi^*}{\partial \sigma} 
\end{align*}
\]

With respect to $\omega$ we analyze two different alternatives: Under the first assumption, both tax rates always coincide ($\tau = \omega$), which means that potential variations of $\tau$ are accompanied by identical variations of the financial tax rate: $d\omega = d\tau$. In the second alternative, we keep $\omega$ constant: $d\omega = 0$.

As a result, $\frac{d\sigma}{d\tau}$ represents the slope of “investment threshold isoquants” and is defined as the negative ratio of two partial derivatives. This slope indicates whether higher tax rates $\tau$ on the return from the real investment encourage (increasing isoquants) or discourage risk-taking (decreasing isoquants).\(^6\) We determine these isoquants for different settings of $\alpha$, $r$, and $D$ while keeping the tax rate on income from financial investment $\omega$ constant. Hence, we study a schedular tax system that has gained importance in many countries.\(^7\) The pre-tax case can be observed for $\tau = 0$ and serves simultaneously as the reference case for neutral taxation. Thus, neutral tax systems are characterized by horizontal isoquants.\(^8\)

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\(^4\)This well-known result is also true for tax-free models and under certain assumption for irreversible investment with financial constraints. See, e.g., Caggese (2007) and Holt (2007).

\(^5\)See Niemann and Sureth (2004).

\(^6\)Alvarez and Koskela (2008) show that for sufficiently high volatilities, the investment threshold decreases with increasing tax rates under progressive taxation. Further, Altug et al. (2001) study the implications of tax risk an irreversible investment decisions and provide sufficient conditions for increases in tax risk to reduce investment.

\(^7\)See OECD (2009) and the Excel files provided there for an overview of time series of ordinary tax rates and special tax rates on capital income.

\(^8\)Niemann and Sureth (2004) derive neutral tax systems under risk aversion and irreversibility.
Figure 1: $\tau$-$\sigma$-isouquants of the critical investment threshold $\pi^*$

The left graph of figure 1 displays the ambiguous effect of a uniform tax rate for all types of capital income.\(^9\) Increasing the tax rate may either encourage (for sufficiently low values of $\sigma$) or discourage risk-taking (for high values of $\sigma$). This effect can be explained analytically by the signs of the partial derivatives in eq. (4). Whereas $\frac{\partial \pi^*}{\partial \sigma}$ may take either sign for $\omega = \tau$, depending on the level of depreciation allowances, increasing the volatility always increases this critical investment threshold: $\frac{\partial \pi^*}{\partial \sigma} > 0$. Hence, $\frac{d\sigma}{d\tau} \in \mathbb{R}$.

By contrast, differential taxation of real and financial investment delays real investment and hence discourages risk-taking. This effect can be alleviated by higher depreciation rates. However, even an (almost) immediate write-off cannot overcompensate this disincentive, as shown in the right graph. Holding the tax rate on capital income $\omega$ constant, increasing the tax rate $\tau$ always reduces the project value\(^10\) and hence $\frac{\partial \pi^*}{\partial \tau} > 0$. Since $\frac{\partial \pi^*}{\partial \tau} > 0$, the slope of the isoquant must be negative. Further, risk-taking is penalized even if $\omega > \tau$. Qualitatively, the “risk-taking penalty” emerges regardless of the level of capital income taxation. Increasing $\tau$ reduces the value of real investment projects, but does not affect alternative financial investments. Consequently, the option to wait becomes relatively more attractive and risky projects will be delayed.

The effects of varying both tax rates $\tau$ and $\omega$ simultaneously can also be analyzed by setting the total differential of the critical investment threshold with respect to the variables $\tau$ and $\omega$ equal zero while keeping the other determinants constant:

$$
\frac{d\pi^*}{d\tau} = \frac{\partial \pi^*}{\partial \tau} d\tau + \frac{\partial \pi^*}{\partial \omega} d\omega \overset{!}{=} 0
$$

$$
\frac{d\omega}{d\tau} = -\frac{\partial \pi^*}{\frac{\partial \pi^*}{\partial \omega}} \quad \text{for} \quad d\pi^* = d\alpha = dr = dd = d\sigma = 0. \quad (5)
$$

\(^9\)Using various models, Näslund (1968) also finds ambiguous effects of taxes on risk-taking.

\(^{10}\)For a given investment threshold $\pi^*$ the investment project always yields a positive net present value. Even if losses may occur during some time intervals, the resulting tax reimbursement cannot be the dominant value driver, so that increasing the tax rate $\tau$ tends to postpone rather than accelerate the investment decision.
Again, $\frac{d\omega}{d\tau}$ represents the slope of “investment threshold isoquants”, i.e., the slope of identical investment thresholds for the plotted combinations of the tax rates $\tau$ and $\omega$. As above, the tax system would be neutral for horizontal isoquants. Examples are illustrated in figure 2 for different levels of depreciation allowances:

![Figure 2: $\tau$-$\omega$-isoquants of the critical investment threshold $\pi^*$](image)

$r = 0.1, \delta = 0.01, \alpha = 0.02, \sigma = 0.2$

$r = 0.1, \delta = 0.2, \alpha = 0.02, \sigma = 0.2$

Obviously, the isoquants are always increasing. This means that an increase in the investment tax rate $\tau$ must be accompanied by a corresponding increase in the capital income tax rate $\omega$ in order to keep the investment threshold constant. If both tax rates are independent, the partial derivatives have opposing algebraic signs. As mentioned above, increasing the investment tax rate $\tau$ increases the critical investment threshold: $\frac{\partial \pi^*}{\partial \tau} > 0$. By contrast, increasing the capital income tax rate $\omega$ reduces the discount rate as well as the option value and hence accelerates investment: $\frac{\partial \pi^*}{\partial \omega} < 0$. As can be observed by comparing the left hand side and the right hand side of figure 2, the isoquants may be concave or convex, and the slope may exceed or fall short of unity. The higher the depreciation allowances, the smaller is the partial derivative $\frac{\partial \pi^*}{\partial \tau}$ and with it the slope of the isoquants.

In other words, a reduction of the capital income tax rate $\omega$ must come along with a corresponding reduction in the investment tax rate $\tau$. Otherwise, real investment will be replaced by financial investment. In recent tax policy developments in Europe only capital income tax rates were reduced, increasing the likelihood to crowd out risky real investment projects.

4. Conclusions

In recent years, schedular taxation with flat tax rates on financial capital income have been introduced by many jurisdictions without considering the negative implications for
risk-taking. While the tax legislators intended tax simplification, our model indicates that a flat tax is likely to crowd out risky real investments. Even extremely generous depreciation schedules or increasing the flat tax rate on financial income will be insufficient to correct this dysfunctional effect. It remains an open question whether the recently introduced preferential taxation of financial capital income in many countries contributed to the current crisis. If tax politicians want to foster risky real investment by adjusting tax rates, our results indicate they have to abolish the differential taxation of different types of capital income. The dual income tax with its uniform tax rate on capital income could serve as a starting point for such a reform.

References


