Abstract

This paper investigates whether unobserved asymmetries can account for irregularities in the Fisher effect for the exclusive case of South Africa. This objective is attained by investigating unit roots within a threshold auto-regressive (TAR) models and estimating a threshold vector error correction (TVEC) models for the data. The empirical analysis depicts significant long-run Fisher effects whereas such effects are deficient with regards to the short-run. These results improve on those obtained in preceding studies for South Africa, in the sense of being closely emulated with the original hypothesis as presented by Fisher (1907).
1. INTRODUCTION

Ever since the South African Reserve Bank’s (SARB) adoption of an inflation targeting regime, the comprehension of interest rate movements with respect to inflationary behaviour has played a pivotal role in the conduct of monetary policy. In this regard, the Fisher effect provides a hypothetical rationale for keeping monetary authorities concerned with managing inflation expectations as a means of stabilizing real interest rates. This in turn, would bear a positive influence on saving-investment decisions in the economy. Specifically, a full Fisher effect would have nominal interest rates reflect movements in the expected rate of inflation in a proportionate ratio of one-to-one without exerting any direct influence on real interest rates. Such a described hypothesis has generally been met with varying degrees of success in the empirical literature. With direct reference to South African case studies, a full Fisher effect has, however, not been successfully established. This conclusion is deduced based on a review of the works presented by Mitchell-Innes, Aziakpono and Faure (2007) and Alangideal and Panagiotidis (2010).

Taking into consideration a real world with no market rigidities, homogenous behaviour of the agents and opportunist behaviour of monetary authorities, it would be irrational to expect a perfect fit for the Fisher effect. Given no evidence of any of these theoretical conditions being empirically fulfilled, a considerable amount of energy has been devoted towards providing systematic reasoning as to why the relationship between the nominal interest rate and inflation expectations may only be approximate in the real world. Inclusive of credible attempts in accounting for such potential stochastic heterogeneities in the Fisher effect is the recently-popularized threshold cointegration approach. Generally, studies which employ such asymmetric frameworks tend to generate more satisfactory results in comparison to studies which adopt linear frameworks. For instance, Million (2004) and Ahmed (2010) are able to account for significant Fisher effects by employing threshold autoregressive models (TAR) and smooth transition autoregressive (STAR) econometric models in their respective studies. The aforementioned studies investigate the Fisher hypothesis on the basis of real interest rate equilibrium adjustments whilst bearing no regards for co-equilibrium adjustments between nominal interest rates and expected inflation rates. In this sense, the use of a threshold vector error correction (TVEC) model, as introduced by Blake and Fombly (1997), holds a certain appeal towards establishing cointegration asymmetries in the Fisher
effect. To the best of our knowledge, asymmetries in the cointegration relation between nominal interest rates and the expected rate of inflation within a TVEC framework has not been effectively captured in previous studies (see Bajo-Rubio, Diaz-Roldan and Esteve (2005) and Dutt and Ghosh (2006) for practical examples).

All in all, asymmetries in the cointegration relation between nominal interest rates and the expected rate of inflation in South Africa have not been investigated in any manner with regards to previous case studies. Our study is concerned with filling the existing void in the literature which can be encompassed by examining the asymmetric relationship between nominal interest rates and inflation within the context of TAR and TVEC econometric models for the exclusive case of South Africa. The remainder of the paper is structured as follows. The following section lays forth the empirical foundations to the study. The third section of the paper formulates the data and presents the empirical analysis. Section four concludes the overall study.

2. EMPIRICAL FOUNDATIONS OF THE STUDY

Due to the strong presumption of the presence of stochastic trends in both the nominal interest rates and the inflation rate, it has been viewed as necessary to facilitate the Fisher effect within a cointegration framework. The standard procedure in empirically testing for the Fisher effect is by means of a bivariate cointegrating regression of nominal interest rate \( i_t \) on a constant plus the expected rate of inflation \( \pi^e_t \):

\[
i_t = \alpha + \beta \pi^e_t \tag{1}
\]

Nominal interest rates \( i_t \) and expected inflation \( \pi^e_t \) are regarded as reflecting a full Fisher effect if the above regression satisfies the condition of \( \beta = 1 \). An alternative method of testing for the validity of the Fisher effect, involves testing whether the real interest rate i.e. \( r_t = i_t - \pi^e_t \), evolves as a stationary process. To test for stationarity, the real interest rate can be placed subject to the following generalized autoregression:

\[
r_t = \phi r_{t-1} + \epsilon_t \tag{2}
\]

Where \( \phi \) is the least squares estimate and \( \epsilon_t \) is an iid error process meeting the requirement of \( \epsilon_t \sim (0, \sigma^2) \). For the Fisher effect to be valid, the hypothesis of \( |\phi| < 1 \) should not be capable of being rejected such that \( r_t \) can be modelled as a mean reverting autoregressive process with a finite variance. In scope of a
cointegration system of variables in context of Engle and Granger’s (1987) definition, if the real rate of interest ($r_t$) is a stationary I(0) process, then the nominal interest rate ($i_t$) and the expected inflation rate ($\pi^e_t$) can be expected to be cointegrated under the restriction of both variables retaining stationarity in their first differences. According to Engle and Granger (1987) if two economic time series are integrated of similar order I(1), then there exists an error correction mechanism governing the equilibrium dynamics of the system which can be directly derived from the first differences of the linear combination of the observed I(1) variables. For the case of the Fisher equation, the error correction mechanism ($\zeta_{t-1}$) can be depicted in the following bivariate cointegration system of nominal interest rates and expected inflation:

$$
\Delta i_t = \alpha_{i0} + \alpha_{i1} \zeta_{t-1}(\beta) + \alpha_{i2} \Delta i_{t-1} + \alpha_{i3} \Delta \pi^e_t + \epsilon_{i1}
$$

(3.1)

$$
\Delta \pi^e_t = \alpha_{\pi0} + \alpha_{\pi1} \zeta_{t-1}(\beta) + \alpha_{\pi2} \Delta i_{t-1} + \epsilon_{\pi1}
$$

(3.2)

The error correction coefficients $\alpha_{i1}$ and $\alpha_{\pi1}$ respectively capture the dynamics of how $i_t$ and $\pi^e_t$ respond to deviations from the equilibrium relationship. Only if the condition of $\alpha_{i1} < 0$ and/or $\alpha_{\pi1} < 0$ are satisfied, can $i_t$ and $\pi^e_t$ be deemed as converging towards a unique equilibrium described by a singular cointegration vector relation $[1, \beta]$.

As highlighted in the introductory section, this study is concerned with shifting focus of methodology by estimating asymmetric versions of the above-described Fisher cointegration systems. Firstly, the examination of asymmetric effects in the unit root process of real interest rates is attained through the use of Kapetanois and Shin (2006) nonlinear unit root tests. These tests are based on Hansen’s (2000) three-regime TAR model:

$$
r_t = \alpha_{r0} r_{t-1} I(r_{t-1} < \gamma_\ell) + \alpha_{r1} r_{t-1} I(\gamma_\ell \leq r_{t-1} \leq \gamma_2) + \alpha_{r2} r_{t-1} I(r_{t-1} > \gamma_2) + \epsilon_t
$$

(4)

From which asymmetric unit root testing procedures are derived within the following auxiliary three-regime TAR regression:

$$
\Delta r_t = \psi_1 r_{t-1} I(r_{t-1} < \gamma_\ell) + \psi_0 r_{t-1} I(\gamma_\ell \leq r_{t-1} \leq \gamma_2) + \psi_2 r_{t-1} I(r_{t-1} > \gamma_2) + \epsilon_t
$$

(5)

Under the null hypothesis i.e. $H_0$: $\psi_0=1$, $\psi_1=\psi_2=0$, regression (5) reduces to a unit root process in the corridor regime:

$$
\Delta r_t = r_t - r_{t-1} = \epsilon_t
$$

(6)
Whereas under the alternative hypothesis i.e. $H_1: \psi_0 = 0$, $|\psi_1|<0$, $|\psi_2|<0$, the regression reduces to a globally stationary three-regime TAR process:

$$\Delta r_t = \psi_{r_{t,1}} I(r_{t,1} \leq \gamma_1) + \psi_{r_{t,1}} I(r_{t,1} > \gamma_2) + \epsilon_t$$

An appropriate test of the joint null hypothesis of a unit root against an alternative of a threshold stationary process is achieved through the standard Wald statistic. However, due to inference complexities associated with the unidentified threshold parameters under the null hypothesis, Kapetanos and Shin (2006) opt to derive asymptotically valid distributions from Supremum, average and exponential average-based tests of the Wald statistics. These statistics are respectively defined as:

$$KSW_{\text{SUP}} = \text{SUP}_{(i \in \Gamma^*)} W_{(\gamma_1, \gamma_2)}$$

$$KSW_{\text{AVE}} = \frac{1}{\#\Gamma} \sum_{i=1}^{\#\Gamma} W_{(\gamma_1, \gamma_2)}$$

$$KSW_{\text{EXP}} = \frac{1}{\#\Gamma} \sum_{i=1}^{\#\Gamma} W_{(\gamma_1, \gamma_2)}$$

The optimal values of the threshold parameters $\gamma_1$ and $\gamma_2$ are obtained by maximizing the above-defined Wald statistics over the selection grid, $\Gamma^*$, and summary statistics are then constructed for these estimates. To ensure that the thresholds estimates are optimally selected whilst simultaneously maintaining a finite width in the corridor regime under both the null and alternative hypotheses, the threshold parameters contained within the grid $\Gamma^*$ are bound by the conditions:

$$\gamma_1 = \bar{\gamma} + 3/\sqrt{T}, \quad \gamma_2 = \bar{\gamma} - 3/\sqrt{T}$$

Where $\bar{\gamma}$ denotes the sample quantile corresponding to zero and the sample size is given by $T$.

The second examination in respect of investigating asymmetric Fisher correlations, involves an extension of the linear cointegration models (3.1) and (3.2) to include asymmetries in the adjustment process of the error correction model. As described in Blake and Fombly (1997), this can be depicted in the following threshold vector error correction (TVEC) regression:

$$\Delta Y_t = \Theta_1 \Delta X_{t-1} I\{\zeta_{t-1}(\beta) \leq \zeta_{t-1}^*\} + \Theta_2 \Delta X_{t-1} I\{\zeta_{t-1}(\beta) > \zeta_{t-1}^*\} + \epsilon_t$$

In applying Fisher’s equation to the TVEC regression (10), the following parametric representations are specified:
\[ \Delta Y_t = \begin{bmatrix} \Delta i \\ \Delta \pi_t \end{bmatrix}, \]
\[ \Delta X_{t-1} = \begin{bmatrix} 1 \\ \varepsilon_{t-1} / \Delta i_{t-1} \\ \Delta \pi_{t-1} \end{bmatrix}, \]
\[ \Theta_1 = \begin{bmatrix} \alpha_{10} & 0 & 0 & 0 \\ 0 & \alpha_{11} & 0 & 0 \\ 0 & 0 & \alpha_{12} & 0 \\ 0 & 0 & 0 & \alpha_{13} \end{bmatrix}, \]
\[ \Theta_2 = \begin{bmatrix} \alpha_{20} & 0 & 0 & 0 \\ 0 & \alpha_{21} & 0 & 0 \\ 0 & 0 & \alpha_{22} & 0 \\ 0 & 0 & 0 & \alpha_{23} \end{bmatrix}. \]

The estimation of the TVEC equation, as suggested by Hansen and Seo (2002), is prompted by considering the following Gaussian likelihood function:

\[ L_n(\Theta_1, \Theta_2, \varepsilon, \beta, \zeta^{*}_{t-1}) = -n/2 \log -\frac{1}{2} \sum u_t (\Theta_1, \Theta_2, \varepsilon, \beta, \zeta^{*}_{t-1})' E^{-1} u_t (\Theta_1, \Theta_2, \varepsilon, \beta, \zeta^{*}_{t-1}) \]  \hspace{1cm} (11)

The maximization of above likelihood function is feasible via quasi-maximum likelihood estimates (MLE). This procedure is instigated by holding \((\Theta_1, \Theta_1, \varepsilon)\) fixed and concentrating out \((\beta, \zeta^{*}_{t-1})\) from which the following concentrated likelihood function is yielded:

\[ L_n(\beta, \zeta^{*}_{t-1}) = -n/2 \log \left| E(\beta, \zeta^{*}_{t-1}) \right| - np/2 \]  \hspace{1cm} (12)

The above function serves as a foundation in obtaining the true values of \(\beta\) and \(\zeta^{*}_{t-1}\), from which the remainder of the parameters in the TVEC specification are estimated via backward substitution. Denoting ‘n’ as the trimming parameter of the data which is set at 0.05 (5%), the MLE of the cointegration vector \(\beta\) and the threshold parameter \(\zeta^{*}_{t-1}\) are obtained through a two-dimensional grid search as the values that minimize \(\log \left| E(\beta, \zeta^{*}_{t-1}) \right| \) subject to the constraint:

\[ n \leq n^{-1} \sum I(x_i, \beta \leq \zeta^{*}_{t-1}) \leq 1 - n \]  \hspace{1cm} (13)

Testing for significant threshold cointegration effects is conducted via a two-staged testing procedure. In the first stage, Hansen and Seo’s (2002) supremum LM test \(^{HS}LM_{sup}\) is used in testing the null hypothesis of linear cointegration (i.e. \(\Theta_1 = \Theta_2 \neq 0\)), against the alternative hypothesis of threshold cointegration (i.e. \(\Theta_1 \neq \Theta_2 \neq 0\)). Given that the test of Seo and Hansen (2002) exempts the
possibility of testing for no cointegration effects within the TVEC system, the second stage of the testing procedure relies on Seo’s (2006) supremum Wald test ($W_{sup}$) to test the null hypothesis of no cointegration (i.e. $\Theta_1 = \Theta_2 = 0$) against the alternative of threshold cointegration (i.e. $\Theta_1 \neq \Theta_2 \neq 0$). The critical values and p-values for the test statistics are computed through the use of a residual bootstrap method as suggested by each of the aforementioned authors. In both of the described threshold tests, the alternative hypothesis of threshold cointegration can only be accepted if the test statistics exceed their critical values.

3. DATA AND EMPIRICAL ANALYSIS

The data used in the study is available from the SARB website (http://www.resbank.co.za/Research/Statistics/Pages/OnlineDownloadFacility.aspx). The empirical analysis uses seasonally adjusted, monthly time-series data obtained for periods between January 1980 and April 2011. The dataset consists of the three-month banker’s acceptance ($i_{ba}$) and the 10-year yield on government bonds ($i_{govbond}$) which are used as proxies for short term and long-term nominal interest rates, respectively. As is the norm in empirical studies, the actual inflation in total consumer prices is used as a proxy for inflation expectations (see Bajo-Rubio, Diaz-Roldan and Esteve (2005) and Dutt and Ghosh (2006), Alangideal and Panagiotidis (2010)). By further adopting Fisher’s (1930) real interest rate definition of $r_t = i_t - \pi_e$, two additional time series are formulated to represent the short-run real interest rate ($r_{ba} = i_{ba} - \pi_e$) and the long-run real interest rate ($r_{govbond} = i_{govbond} - \pi_e$).

As mentioned in the previous section, Fisher’s hypothesis is in part crucially dependent on the integration and stationary properties of the real rate of interest. In this regard, Kapetanos and Shin’s (2006) nonlinear unit root test are performed on the formulated short-run and long-run real interest rate data with the results reported in Table 1.
In their levels, both long-run and short-run real interest rates contain a unit root in the corridor regime whilst retaining stationary threshold processes in their first differences at 10% significance levels. What is most commendable about Kapetanois and Shin’s (2006) unit root testing procedure in application to examining Fisher effects is its ability to define a specific range at which real interest rates contain a unit root. This range is defined by the threshold estimate points which are 0.10 and 4.92 for the long-run real interest rate, whereas for the short-run data the obtained estimates are 1.15 and 2.88. Interpretively, these estimates determine the range of short-run and long-run real interest rates at which potential Fisher effects become invalid. However, this analysis is incomplete without establishing cointegration effects between the alternative definition of a Fisher correlation as described by co-movements between the nominal rate of interest and inflation expectations.

Table 2 below presents the threshold cointegration tests on designated pairs of variables representing the short-run and long-run Fisher effects. The short-run Fisher effect is represented by pairing the variables \((i_{ba}, \pi^e_t)\) and the long-run Fisher relation is defined by the pairing of \((i_{govbond}, \pi^e_t)\). Both Hansen and Seo (2002) and Seo (2006) threshold tests fail to reject the alternative hypothesis of threshold cointegration for short-run and long-run Fisher effects. With the exception of Seo’s test on short-run effects being significant up to a critical level of 5%, all other results are verified at all significance levels.
In view of significant cointegration effects being established, the estimation of the TVEC models for both short-run and long-run Fisher effects is implemented. In examining the significance of Fisher effects within the TVEC model, two conditions are taken into consideration. Firstly, the threshold error correction term \( \zeta^*_{t-1} \) must be of a negative value to ensure the possibility of convergence in both regimes. If \( \zeta^*_{t-1} \) is found to be a positive integer, then equilibrium convergence is only possible in the lower regime of the TVEC. Secondly, there must be at least one significantly negative error correction term \( \zeta_{t-1} \) associated with the nominal interest rate or/and the inflation expectations equations under regimes encompassed by negative values of the threshold parameter, \( \zeta^*_{t-1} \). Given negative \( \zeta^*_{t-1} \) estimates of -1.51 for the short-run and -0.34 for long-run Fisher effects as is respectively shown in Tables 3 and 4, implies the possibility of convergence towards equilibrium in both the upper and lower regimes of the TVEC models. This result bears full satisfaction to the first condition. However, significant negative error correction terms i.e. \( \zeta_{t-1} \) are only established in regimes associated with the long-run nominal interest rate equations and not with the inflation expectations equations. Therefore the paper concludes on significant Fisher effects existing solely in the long-run with inflation expectations being weakly exogenous within the cointegration system i.e. inflation expectations granger causes nominal interest rates. This result emulates the original hypothesis as presented by Fisher (1907) in which changes in inflation
expectations are expected to granger-cause changes in the long-term nominal rate of interest. In view of a cointegration vector of $[1, -1.19]$ established for the long-run Fisher effect, Crowder and Wohar (1999) have suggested that cointegration relations of between the ratios of $[1, -1.1]$ and $[1, -1.7]$ may be delegated towards tax effects for Fisher elasticities that are found to be of a ratio greater than unity. Since our study does not account for such tax effects in nominal interest rates, this is rendered as a plausible explanation for our obtained results. In comparison to the Fisher ratios of $[1, -0.23]$ and $[1, -2.27]$ depicted in the respective works of Mitchell-Innes, Aziakpono and Faure (2007) and Alangideal and Panagiotidis (2010), the overall results presented in our study prove to be a positive development in the academic literature.

**Table 3: TVEC Estimates For Short-Run Fisher Effect**

<table>
<thead>
<tr>
<th>LOWER REGIME</th>
<th>UPPER REGIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\zeta_t^{\prime}(\beta) \leq \zeta_t^{*})$</td>
<td>$(\zeta_t^{\prime}(\beta) &gt; \zeta_t^{*})$</td>
</tr>
<tr>
<td>NOMINAL INTEREST RATE $(\Delta i_t)$</td>
<td>INFLATION EXPECTATIONS $(\Delta \pi_t^e)$</td>
</tr>
<tr>
<td>CONSTANT $(\alpha_i)$</td>
<td>4.83 (0.00)***</td>
</tr>
<tr>
<td>ECT $(\zeta_t)$</td>
<td>0.61 (0.00)***</td>
</tr>
<tr>
<td>NOMINAL INTEREST RATE $(\Delta i_{t-1})$</td>
<td>0.03 (0.57)</td>
</tr>
<tr>
<td>INFLATION EXPECTATIONS $(\Delta \pi_{t-1}^e)$</td>
<td>-1.29 (0.24)</td>
</tr>
</tbody>
</table>

COINTEGRATION VECTOR $(\beta)$: $[1, -1.20]$

THRESHOLD ECT $(\zeta_t^{*})$: -1.51

"***", "**" and "*" denote the 1%, 5% and 10% significance levels respectively. P-values are reported in ().
TABLE 4: TVEC ESTIMATES FOR LONG-RUN FISHER EFFECT

<table>
<thead>
<tr>
<th>LOWER REGIME</th>
<th>UPPER REGIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\zeta_{t+1} (\beta) \leq \zeta^*_{t+1})$</td>
<td>$(\zeta_{t+1} (\beta) &gt; \zeta^*_{t+1})$</td>
</tr>
<tr>
<td>NOMINAL INTEREST RATE $(\Delta i_t)$</td>
<td>NOMINAL INTEREST RATE $(\Delta i_{t-1})$</td>
</tr>
<tr>
<td>CONSTANT $(\alpha_i)$</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>(0.07)*</td>
</tr>
<tr>
<td>ECM $(\zeta_t)$</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(0.05)*</td>
</tr>
<tr>
<td>NOMINAL INTEREST RATE $(\Delta i_{t-1})$</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>(0.00)***</td>
</tr>
<tr>
<td>INFLATION EXPECTATIONS $(\Delta \pi_{t-1})$</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
</tr>
<tr>
<td>COINTEGRATION VECTOR $(\beta)$</td>
<td>[1, 1.19]</td>
</tr>
<tr>
<td>THRESHOLD ECM $(\zeta^*_{t+1})$</td>
<td>-0.34</td>
</tr>
</tbody>
</table>

***”, **” and ‘*’ denote the 1%, 5% and 10% significance levels respectively.

P-values are reported in (). 

4. CONCLUSION

Despite the increasing surge of interest found in empirical literature opting to rectify Fisher’s hypothesis through threshold cointegration techniques, such econometric frameworks have not been employed within the context of the South African economy. This paper contributes to the literature by demonstrating how significant long-run Fisher effects for South African data are more effectively captured by introducing asymmetries into the empirical framework. These results are not surprising considering that the period span of the employed data covers consecutive regime shifts in the conduct of monetary policy. Notwithstanding the encouraging results derived from this study, further developments in our research will focus on estimating the Fisher effect in South Africa by using empirically derived proxies of the inflation expectation variable in the econometric analysis.

REFERENCES


