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Equilibrium, Adverse Selection, and Statistical Distributions

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Abstract

This paper addresses the problem of multiple equilibria in markets with adverse selection. Akerlof (1970) identified an unique equilibrium of the total market failure under adverse selection. Posterioly, Wilson (1979, 1980) argued that the presence of adverse selection may lead to multiple equilibria. In particular, this paper extends the work of Rose (1993), who stated that the existence of multiple equilibria depends on the distribution of quality. Rose found that multiple equilibria are highly unlikely for most standard probability distributions. This work considers additional statistical distributions for quality. The simulation results suggest the existence of multiple equilibria when the quality follows a beta normal distribution.

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1. Introduction

The Akerlof's 1970 essay (Akerlof 1970), The 'Market for lemons', introduces a formal analysis of market with asymmetric information known as adverse selection. In a market with adverse selection, the seller has more information than the buyer concerning the quality of the product. Akerlof uses the market of cars to illustrate his idea, where a potential buyer can not distinguish if he is buying a good car or a lemon (defective old car). Akerlof shows that this information problem can either cause an entire market breakdown or drive out high-quality goods from the market. Although many goods are traded in a market with different prices and qualities, it is not obvious if high price itself can be a signal of high quality. Examples of signaling device may be found in Spence (1973) and Milgrom & Roberts (1986). Wilson (1979, 1980) shows that in a lemons market, i.e. a market with adverse selection, if the expected quality is an increasing function of the price, then there may be multiple equilibria among which high-quality goods can be sold with positive probability. Multiple equilibria in those cases can be ranked according to the Pareto criterion with higher price equilibria generating higher welfare, that is, when the equilibrium price equals the marginal buyer's willingness-to-pay for the expected level of quality, total expected consumer surplus does not decline at higher prices, and sellers obviously prefer a higher to a lower price (Poutvaara & Wagener 2004).

Rose (1993) indicated that the existence of multiple equilibria under adverse selection depends critically on the distribution of quality, and that multiple equilibria are highly unlikely if quality follows some standard distributions. The Rose's result indicated a unique equilibrium. This paper extends the work of Rose assuming different distributions (in addition to the normal distribution) for quality, among them: exponential power, Kumaraswamy, wakeby, and beta normal. Carrying out simulations and using the R Statistical Computing Environment, it is evidenced that when the quality follows a beta normal distribution, then multiple equilibria are obtained.

The remainder of the paper is organized as follows: the next Section introduces mathematically the Akerlof-Wilson model. Section 3 presents the statistical distributions assumed for quality and illustrates graphically the results. Finally, Section 4 is dedicated to the concluding remarks.

2. The Akerlof-Wilson model

The Akerlof's model approaches the problem of asymmetric quality information, i.e. individuals buy new goods (used cars) without knowing whether the goods they buy will be good or lemon (low quality), whereas each seller knows the quality of his own car. In this model it is assumed the absence of signaling and search. Each agent in the economy has an identical utility function given by

$$U = U(c, n|t, q) = c + tqn, \tag{1}$$

where c is consumption of other goods, $q \in [q_0, q_1]$ is the quality of car he consumes with density $f(q)$, $t \in (t_0, t_1)$ is a parameter equal to his marginal rate of substitution of car quality for consumption with density $h(t)$, and n is a discrete binary variable representing consumption of used cars ($n = 0$ or $n = 1$). Let p be the price of used cars, and let the price of other goods be unity.

The supply side

Following Wilson (1979, 1980), all agents (sellers and owners) have the same utility index, $t \in (t_0, t_1)$, denoted here by $\tilde{t}$. Sellers differ only in the quality of cars they own, which has density $f(q)$ on $[q_0, q_1]$ with $q_0 > 0$. The distribution function is given by $F(q) = \int_{q_0}^{q} f(x)dx$. Notice that $F(q)$ indicates the number of sellers with cars of quality less than q. A seller will offer his car if and only if $q \leq p/\tilde{t}$, which implies that at any price the supply of cars may be considered as a proportion of cars in which the inequality holds, i.e.

$$S(p) = \text{prob}\left(q \leq \frac{p}{\tilde{t}}\right) = \begin{cases} \int_{q_0}^{p/\tilde{t}} f(q)dq, & \text{for} \quad p > \tilde{t}q_0 \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Wilson (1980) shows that the average quality of cars at price p is given by

$$q^a(p) = E\left(q|q \leq \frac{p}{\tilde{t}}\right) = \frac{\int_{q_0}^{p/\tilde{t}} qf(q)dq}{S(p)}, \quad \text{for} \quad p > \tilde{t}q_0, \tag{3}$$

where E denotes the expectation operator.

The demand side

The utility index $t \in (t_0, t_1)$ has density $h(t)$ defined on $[t_0, t_1]$, where $t_0 > 0$. The implied distribution function is $H(t) = \int_{t_0}^{t} h(x)dx$, which can be interpreted as the number of buyers with utility index less than t. The buyer will demand a car if and only if $tq_a \geq p$. Similarly to the supply case, the demand for cars may be computed as the proportion of buyers for which the inequality holds, i.e.

$$D(p) = \text{prob}\left(t \geq \frac{p}{q^a(p)}\right) = \begin{cases} \int_{p/q^a(p)}^{t_1} h(t)dt, & \text{for} \quad p < t_1 q^a(p) \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Multiple equilibria

The existence of multiple equilibria depends on an upward-sloping segment in the demand curve, since Equation (2) (supply curve) is monotonically increasing in price. Rose (1993) derives from Equation (2) the condition for an upward-sloping demand curve, i.e.

$$\frac{dD(p)}{dp} > 0 \quad \text{if and only if} \quad \frac{d[p/q^a(p)]}{dp} = \frac{1}{q^a}\left[1 - \frac{dq^a(p)}{dp}\frac{p}{q^a(p)}\right] < 0. \tag{5}$$

One can note that the last term of Equation (5) is negative, that is, the requirement for an upward-sloping demand curve is satisfied if

$$\epsilon = \frac{dq^a(p)}{dp}\frac{p}{q^a(p)} > 1, \tag{6}$$

where ϵ denotes the price elasticity of average quality. In short, the conditions can be summarized as follows:

$$\frac{dD(p)}{dp} \gtreqqless 0 \quad \text{if and only if} \quad \epsilon \gtreqqless 1, \tag{7}$$

where

$$\epsilon = \frac{\frac{p}{\tilde{t}} f\left(\frac{p}{\tilde{t}}\right)}{\int_{q_0}^{p/\tilde{t}} f(q)dq} \left[\frac{p/\tilde{t}}{q^a(p)} - 1\right]. \tag{8}$$

Rose (1993) points out that Equation (8) tells us nothing whether $\epsilon \gtreqqless 1$. Hence, he applied numerical techniques and noted that the existence of multiple equilibria will depend on the distribution of quality $f(q)$. The next section provides simulations to evaluate the existence of multiple equilibria.

3. Statistical distributions and multiple equilibria

This section assumes that the distribution of quality is well approximated by five[1] probability density functions[2]: normal, exponential power, Kumaraswamy, wakeby, and beta normal. A brief mathematical review of each distribution is presented, and simulations are performed using the R Statistical Computing Environment in order to verify the existence of multiple equilibria.

3.1. Normal distribution

The normal (Gaussian) distribution is the most common and important distribution in statistics. The probability density function is given by

$$f(q) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{q-\mu}{\sigma}\right)^2}, \tag{9}$$

where $\mu > 0$ is a location parameter, equal to the mean, and $\sigma^2 > 0$ the variance. If $\mu = 0$ and $\sigma = 1$, a standard normal distribution is obtained. The normal distribution is distributed over $-\infty < q < \infty$, then a truncated normal distribution, with truncation at zero, is used. Figure 1 presents the plot of ϵ versus p for the normal distribution, with its probability density function. Similarly to Rose (1993), the normal distribution yields $\epsilon > 1$ for a low range of prices $p \in (0, p^*)$, and $\epsilon < 1$ for $p \in (p^*, \infty)$. Note that $p^* \approx 3$. This result may imply multiple equilibria, i.e. a demand curve with both downward and upward sloping segments.

As pointed out anteriorly, the results indicate that when $f(q)$ follows a normal distribution multiple equilibria may arise. However, Figure 2 shows that multiple equilibria can not be generated, since supply is a monotonically increasing function of price and demand (hump-shaped) must exceed supply at $p = 0$. The distribution of preferences $h(t)$ is uniform with minimum and maximum values given by 1 and 3, respectively.

[1]Beyond these five distributions, this work extended the analysis to the following distributions: Rice, Levy, Nakagami, Pareto, Chi-squared, Rayleigh, Frechet, Generalized extreme value, and Beta. However, no relevant result was obtained.

[2]It is emphasized that four of these distributions are different from those analyzed in Rose (1993), i.e. excluding the normal distribution.

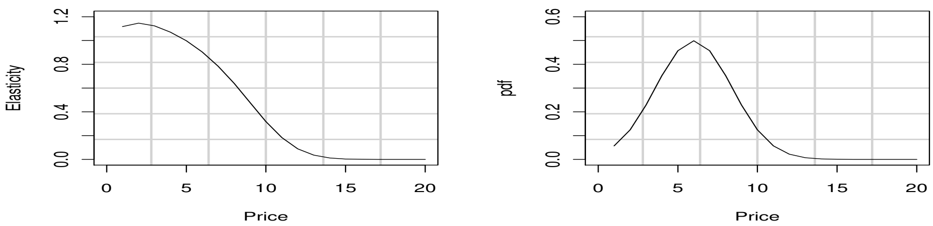

Figure 1. Elasticity versus price and probability density function: normal distribution ($\mu = 2$, $\sigma = 0.8$, $\tilde{t} = 3$)

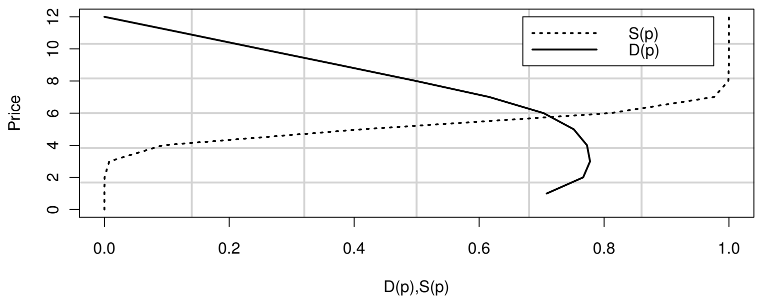

Figure 2. Demand and supply curves: normal distribution

3.2. Exponential power distribution

The first version of the exponential power distribution can be ascribed to Subbotin (1923). The probability density function of the exponential power is given by

$$f(q) = \frac{1}{\phi\Gamma\left(1+\frac{1}{2\beta}\right)2^{1+\frac{1}{2\beta}}} \exp\left\{-\frac{1}{2}\left|\frac{q-\mu}{\phi}\right|^{2\beta}\right\}, \tag{10}$$

where $\mu \in \mathbb{R}$ and $\phi, \beta > 0$ are location, scale, and kurtosis parameters, respectively. If one changes the shape parameter β, the exponential power distribution describes both leptokurtic $0 < \beta < 2$ and platikurtic $p > 2$ distributions. Particularly, for $\beta = 1$ one obtains the the Laplace distribution, the normal distribution for $\beta = 2$, and the uniform distribution for $\beta \to \infty$. Figure 3 shows the plot of ϵ versus p for the exponential power distribution, with its probability density function. The range of prices in which $\epsilon > 1$ is similar to the normal distribution, that is, for $p \in (0, p^*)$, where $p^* \approx 3$, and $\epsilon < 1$ for $p \in (p^*, \infty)$. Thus, the exponential power distribution may also generate multiple equilibria, since the demand curve may have both downward and upward sloping segments.

Figure 4 shows the computer-generated supply and demand curves for the case where the distribution of quality $f(q)$ is exponential power, and the distribution of preferences $h(t)$ is uniform with minimum and maximum values given by 1 and 3, respectively. One can note similar results between normal and exponential power distributions, hence multiple equilibria can not be generated.

3.3. Kumaraswamy distribution

The Kumaraswamy distribution (Kumaraswamy 1980) was introduced for double bounded random processes with hydrological applications. This Kumaraswamy distribu-

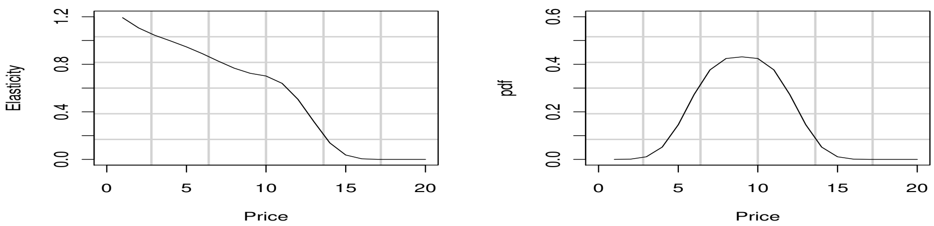

Figure 3. Elasticity versus price and probability density function: exponential power distribution ($\mu = 3$**,** $\phi = 0.9$**,** $\beta = 3$**,** $\tilde{t} = 3$**)**

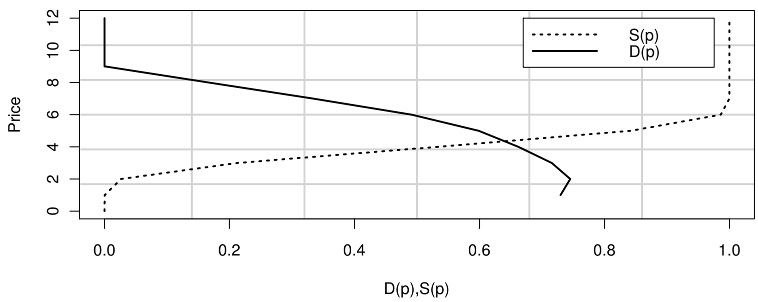

Figure 4. Demand and supply curves: exponential power distribution

tion is similar to the beta distribution and has the following probability density function

$$f(q) = \alpha_1 \alpha_2 q^{\alpha_1 - 1}(1 - q^{\alpha_1})^{\alpha_2 - 1}, \tag{11}$$

where $\alpha_1, \alpha_2 > 0$ are shape parameters. The Kumaraswamy distribution is bounded at zero and 1 and can take a wide variety of shapes. Figure 5 reports an upward sloping for the whole range of prices, i.e. $\epsilon > 1$. However, it excludes the possibility of multiple equilibria, since only a upward sloping segment exists.

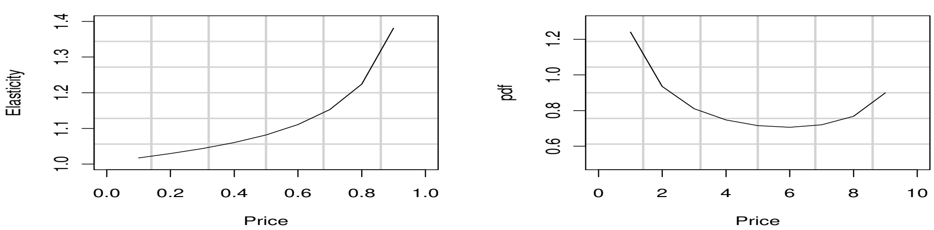

Figure 5. Elasticity versus price and probability density function: Kumaraswamy distribution ($\alpha_1 = 0.5$**,** $\alpha_1 = 0.7$**,** $\tilde{t} = 1$**)**

The computed-generated supply and demand curves when $f(q)$ follows a Kumaraswamy distribution, and when distribution of preferences $h(t)$ is uniform between 1 and 3, are shown in Figure 6. The Figure 5 showed that $\epsilon > 1$, i.e. the demand curve is a monotonically increasing function of price, which in turn rules out the possibility of multiple equilibria. This result is confirmed in Figure 6, where even a single equilibrium seems unlikely.

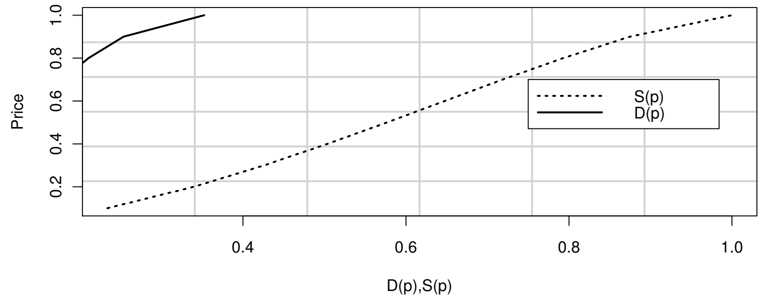

Figure 6. Demand and supply curves: Kumaraswamy distribution

3.4. Wakeby distribution

The Wakeby distribution is defined only with the quantile estimation equation, i.e.

$$x(F) = \epsilon + \frac{\alpha}{\beta}[1 - (1 - F)^{\beta}] - \frac{\gamma}{\delta}[1 - (1 - F)^{-\delta}], \tag{12}$$

where ϵ is a location parameters, and α, β, γ, and δ are always positive parameters. The scale of the variable is related to both $\alpha - \beta$ and $\gamma - \delta$, and the shape of the quantile function are determined by both β and δ. Note that Figure 7 presents a result similar to the Kumaraswamy distribution, i.e. $\epsilon > 1$ for the whole range of prices. Thus, it rules out the possibility of multiple equilibria.

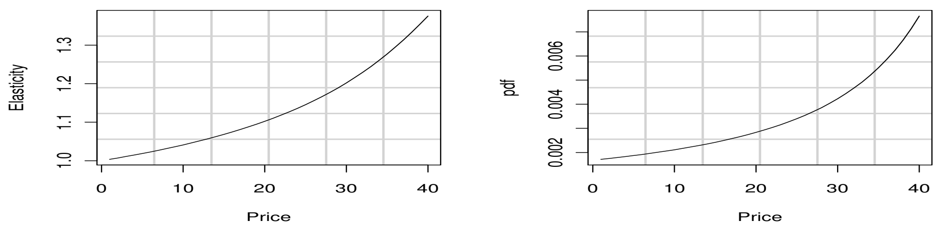

Figure 7. Elasticity versus price and probability density function: Wakeby distribution ($\epsilon = 123$**,** $\alpha = 34$**,** $\beta = 4$**,** $\gamma = 654$**, and** $\delta = 37$**,** $\tilde{t} = 2$**)**

From Figure 8 one can see that similarly to the normal distribution, when $f(q)$ follows a wakeby distribution, multiple equilibria can not be generated, since demand (hump-shaped) must not exceed supply at $p = 0$.

3.5. The beta normal distribution

The beta normal distribution was introduced by Eugene *et al.* (2002). They introduced a general class of distributions generated from the logit of the beta random variable, in which the beta normal distribution is a particular case. Let $\Phi(x)$ and $\phi(x)$ be the cumulative and the density function of the standard normal distribution, respectively. Then the density function of the beta normal is given by

$$f(q) = \frac{1}{\sigma B(a, b)} \left[\Phi\left(\frac{q-\mu}{\sigma}\right)\right]^{a-1} \left[1 - \Phi\left(\frac{q-\mu}{\sigma}\right)\right]^{b-1} \phi\left(\frac{q-\mu}{\sigma}\right), \tag{13}$$

where $a > 0$, $b > 0$, $\sigma > 0$, $\mu \in \mathbb{R}$ and $q \in \mathbb{R}$. Since the measure of quality can not be negative, the truncated (at zero) beta normal distribution is used. The shape parameters

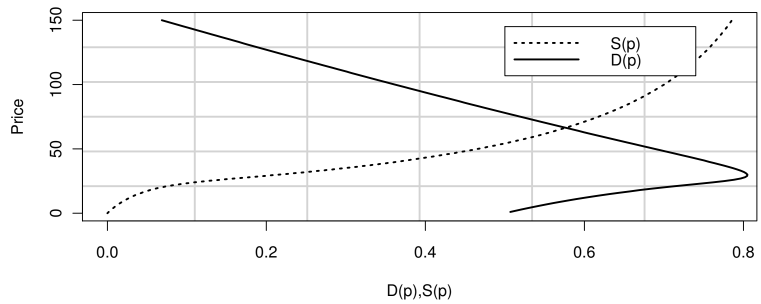

Figure 8. Demand and supply curves: wakeby distribution

a and b model the skewness, kurtosis, and bimodality of the distribution. The parameter μ is a location parameter and σ is a scale parameter that stretches out or shrinks the distribution.

Figure 9 presents the plot of ϵ versus p for the beta normal distribution, with its probability density function. This distribution yields $\epsilon > 1$ for a certain range of prices (approximately between 6 and 11), and $\epsilon < 1$ for the remainder. This result may imply multiple equilibria, i.e. a demand curve with both downward and upward sloping segments.

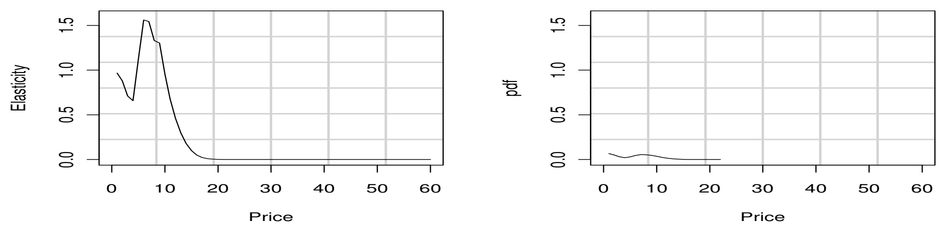

Figure 9. Elasticity versus price and probability density function: beta normal distribution ($\mu = 4$, $\sigma = 0.5$, $\tilde{t} = 1$, $a = 0.01$, $b = 0.02$)

Figure 10 shows the computer-generated supply and demand curves, where the distribution of preferences $h(t)$ is uniform with minimum and maximum values given by 1 and 3, respectively. Note that Figure 10 corroborates the assumption of multiple equilibria, that is, the supply curve crosses the demand curve in at least three different points.

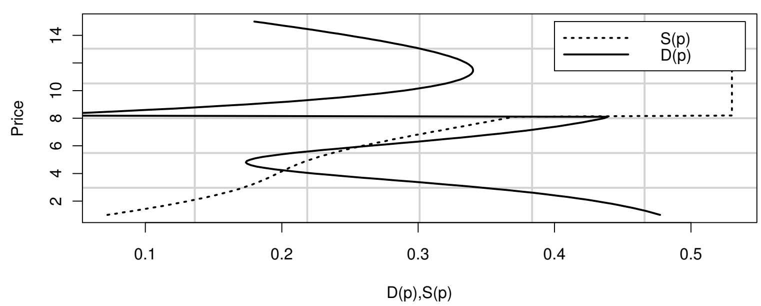

Figure 10. Demand and supply curves: beta normal distribution

4. Concluding remarks

This work had studied the implementation of five statistical distributions for the distribution of quality, i.e. normal, exponential power, Kumaraswamy, wakeby, and beta normal distributions, in order to assess the likely nature of multiple equilibria under adverse selection. Simulations were carried out using the statistical software R and evidence of multiple equilibria was obtained when the quality follows a beta normal distribution. This result supports empirically Wilson's argument (Wilson 1979, Wilson 1980) in which markets with adverse selection may be characterized by multiple equilibra. In particular, the Rose's statement (Rose 1993) of no evidence to cause one to change the status quo of the unique equilibrium with adverse selection can be relaxed.

References

Akerlof, G. (1970), 'The market for lemons: quality uncertainty and the market mechanism', *Quarterly Journal of Economics* **84**, 488–500.

Eugene, N., Lee, C. & Famoye, F. (2002), 'Beta-normal distribution and its applications', *Communications in Statistics. Theory and Methods* **31**, 497–512.

Kumaraswamy, P. (1980), 'A generalized probability density function for double-bounded random processes', *Journal of Hydrology* **46**, 79–88.

Milgrom, P. & Roberts, J. (1986), 'Advertising signals of product quality', *The Journal of Political Economy* **94(4)**, 796–821.

Poutvaara, P. & Wagener, A. (2004), The invisible hand plays dice: Eventualities in religious markets, Others, EconWPA.

Rose, C. (1993), 'Equilibrium and adverse selection', *RAND Journal of Economics* **24(4)**, 559–569.

Spence, M. (1973), 'Job market signaling', *The Quarterly Journal of Economics* **87(3)**, 355–374.

Subbotin, M. T. (1923), 'On the law of frequency of errors', *Matematicheskii Stornik* **31**, 296–301.

Wilson, C. A. (1979), 'Equilibrium and adverse selection', *American Economic Review* **69**, 313–317.

Wilson, C. A. (1980), 'The nature of equilibrium in markets with adverse selection', *Bell Journal of Economics* **11**, 108–130.