Gender based intra-household inequality of opportunity in academic skills among Indian children

Ashish Singh
Indira Gandhi Institute of Development Research

Abstract

Using households with a pair of male-female siblings (aged 8-11 years) from a nationally representative survey, the paper estimates gender based intra-household inequality of opportunity in academic skills by comparing test scores of the siblings in reading and mathematics skills within each household. The study finds substantial level of gender based intra-household inequality in both the skills. The paper also estimates household fixed-effects models for reading and mathematics skills, and finds significant difference between male and female children with female children at a disadvantaged position. Further support for gender differential (bias against female children) is provided by the analysis of the expenses incurred by households on the education of their children, which shows that the educational expenditure on female children is substantially lower than that on male children.

I thank the associate editor of the journal, John P. Conley and an anonymous referee for their valuable comments and suggestions.

Citation: Ashish Singh, (2011) "Gender based intra-household inequality of opportunity in academic skills among Indian children", Economics Bulletin, Vol. 31 no.3 pp. 2333-2346.
1. Introduction

Sen (1992) talks about 41 million “missing women” in India – women and girls who died prematurely due to mistreatment. There is evidence that the bias towards males has only increased in the last two decades, mainly due to sex-selective abortions which is being more widely used to avoid female births (Jha et al. 2006). In general, girls in India are discriminated against in different aspects of life including (but not limited to) education, nutrition, healthcare and immunization (Basu 1989; Borooah 2004; Das Gupta 1987; Griffiths et al. 2002; Kishor 1993; Mishra et al. 2004; Pande 2003; Sen 1988; Singh, Hazra and Ram 2008).

The discrimination against girls becomes even more prominent when seen in the light of literature on inequality of opportunity. Inequality of opportunity which is a relatively new concept is defined as the part of an outcome inequality which can be associated to circumstances beyond the control of an individual. For example, inequality of opportunity in income is the part of total income inequality which is attributed to circumstances (such as race, ethnicity, religion, gender, parental characteristics, place of birth and so on) of individuals, factors lying outside the sphere of their control. Though, in the last decade or so a number of papers (Barros et al. 2009; Bourguignon et al. 2007; Checchi and Peragine 2010; Checchi, Peragine and Serlenaga 2010; Cogneau et al. 2006; Dias 2009; Ferreira and Gignoux 2008; Lefranc et al. 2008; Pistolesi 2009; Singh 2010a, 2010b, 2011a, 2011b; Trannoy et al. 2010; Zhang and Eriksson 2009) have been written on inequality of opportunity, majority of them have focused on parental characteristics as the circumstance variable and a few of them include race, ethnicity (or caste), religion, gender and place of birth in the list of circumstance variables. In case of India, there are limited studies (Asadullah and Yalonetzky 2010; Singh 2010a, 2010b, 2011a, 2011b) which empirically investigate the extent of inequality of opportunity in various economic and non-economic outcomes but none of them explicitly explore the role of gender when it comes to inequality of opportunity.

Role of gender becomes especially important when the fact that discrimination due to gender takes place within the households in addition to taking place outside the house. Circumstances like caste (or ethnicity), religion or parental characteristics result in discrimination outside the household but there is sufficient evidence of discrimination against female children within the households, particularly in India.

Given this context, I focus on within household inequalities and estimate gender based intra-household inequality of opportunity in academic skills in Indian children (aged 8-11 years) based on their test scores in reading and mathematics skills. Academic skills have been chosen as the outcome measures because of two reasons: first, they are important indicators and second they have never been investigated upon in Indian context. Simple but innovative inequality decomposition technique has been used to carry out the decomposition of overall inequality in academic skills (measured by scores) into intra-household and inter-household components. Before, carrying out the actual decomposition, I have estimated household fixed effects linear probability models to show that girls in the households are at a disadvantaged position when it comes to academic skills. The paper finds substantial gender based inequality of opportunity in academic skills in Indian children. It also finds that when it comes to expenditure on education of children, Indian households spend substantially more on the education of boys than girls.

The next section describes the data used for the analysis. It is followed by a section on empirical analysis and the main results of the study; the results are further discussed in the section on discussion and conclusion which concludes the study.
2. Data Description

I use the publicly available data from the Indian Human Development Survey (IHDS), conducted by National Council of Applied Economic Research, New Delhi, India in collaboration with the University of Maryland, in 2004-05. The survey is a micro unit recorded, nationally representative survey based on a stratified multistage sampling procedure. The survey was spread over 33 states and union territories of India and covers 26,734 households (143,374 individuals) in rural areas and 14,820 households (72,380 individuals) in urban areas. This survey is unique in the sense that it was designed to measure different dimensions of human development with modules on education, health, employment, income, and gender empowerment.

A major contribution of this survey was the administration of education module which assesses reading, mathematics and writing skills for children aged 8 to 11 years. A major focus was to measure basic skills by tests that can be administered relatively easily and with low anxiety levels on the part of children. Also, it was administered at home in order not to miss children who are absent from school. The tests were simple, intuitive and were translated into 13 languages in addition to English and the children were asked to take the test in whichever language they were most comfortable in (Desai et al., 2010). Interviewers were trained using specifically developed films so that they could differentiate between a child’s shyness and inability to read. They were also taught how to develop rapport with children.

The focus was on children aged 8 to 11 years because “all of these children should have acquired the basic skills” (Desai et al. 2010, p.79) which are the outcome variables in this study. Children’s reading skills are divided into five categories: (i) cannot read at all; (ii) can read letters but not form words; (iii) can put letters together to read words but not read whole sentences; (iv) can read a short paragraph for 2-3 sentences but not fluent enough to read a whole page; and (v) can read a one page short story. Mathematics skills are divided into four categories: (i) cannot read numbers above 10; (ii) can read numbers between 10 and 99 but not able to do more complex number manipulation; (iii) can subtract a two digit number from another; (iv) Can divide a number between 100 and 999 by another number between 1 and 9.

Since, the interest of the study is in gender based intra-household inequalities, the eligible sample comprises of those households which have at least one pair of male-female children in the age group 8-11 years. The total number of households with at least a male-female pair of children was 1068 (i.e., the eligible sample). Of these there were 1010 households with exactly one male-female pair of children. These households which comprise of more than 94% of the eligible sample are used in the analysis. This has been done for a meaningful interpretation of the results.

Table 1 reports the age wise summary statistics of reading and mathematics scores of the children in the sample. It can be seen from the table that except for the reading scores in the children aged 9 years, the mean reading and mathematics scores of boys is greater than that of girls. To check for whether girls are at a disadvantaged position compared to boys, I run a household fixed effects linear probability model for each of the academic skills. The details of the analysis are presented in the next section on empirical analysis and results.
Table 1 Summary Statistics: Reading and Mathematics Scores of children by Age

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Girls</th>
<th>Boys</th>
<th>Girls</th>
<th>Boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1.992</td>
<td>2.043</td>
<td>1.124</td>
<td>1.188</td>
</tr>
<tr>
<td></td>
<td>1.311</td>
<td>1.329</td>
<td>0.864</td>
<td>0.966</td>
</tr>
<tr>
<td></td>
<td>249</td>
<td>329</td>
<td>249</td>
<td>329</td>
</tr>
<tr>
<td>9</td>
<td>2.585</td>
<td>2.469</td>
<td>1.355</td>
<td>1.488</td>
</tr>
<tr>
<td></td>
<td>1.173</td>
<td>1.344</td>
<td>0.926</td>
<td>0.960</td>
</tr>
<tr>
<td></td>
<td>183</td>
<td>213</td>
<td>183</td>
<td>213</td>
</tr>
<tr>
<td>10</td>
<td>2.751</td>
<td>2.934</td>
<td>1.617</td>
<td>1.832</td>
</tr>
<tr>
<td></td>
<td>1.304</td>
<td>1.236</td>
<td>1.021</td>
<td>1.047</td>
</tr>
<tr>
<td></td>
<td>321</td>
<td>256</td>
<td>321</td>
<td>256</td>
</tr>
<tr>
<td>11</td>
<td>3.004</td>
<td>3.113</td>
<td>1.868</td>
<td>1.986</td>
</tr>
<tr>
<td></td>
<td>1.252</td>
<td>1.142</td>
<td>1.056</td>
<td>1.009</td>
</tr>
<tr>
<td></td>
<td>257</td>
<td>212</td>
<td>257</td>
<td>212</td>
</tr>
</tbody>
</table>

Notes: (1). First row: mean; second row: standard deviation; third row: number of observations.

(2). Reading scores: 0 = cannot read at all; 1 = can read letters but not form words; 2 = can put letters together to read words but not read whole sentences; 3 = can read a short paragraph for 2-3 sentences but not fluent enough to read a whole page; and 4 = can read a one page short story.

(3). Mathematics scores: 0 = cannot read numbers above 10; 1 = can read numbers between 10 and 99 but not able to do more complex number manipulation; 2 = can subtract a two digit number from another; 3 = can divide a number between 100 and 999 by another number between 1 and 9.

3. Empirical Analysis and Results

One of the main objectives of the paper is to see whether female child is at a disadvantaged position vis-a-vis the male child within the household as far as academic skills are concerned. The academic skills (measured by reading and mathematics scores) of a child depends upon his/her personal characteristics (such as gender and age) and the characteristics of the household that s/he resides in (for example, parental education, ethnicity, religion etc.). Some of these household characteristics might be observed while the others may not. Use of the household fixed effects makes it possible to control for all observed and unobserved household-level variables. As the dependent variables (reading and mathematics scores) are categorical, I use linear probability models (LPM) with household fixed effects. Since, it is not possible to apply linear probability modeling with a dependent variable with multiple outcomes, the reading and mathematics scores have been dichotomized (only for household fixed effects analysis). Reading score = 1 if a child can read a paragraph or a short story and 0 otherwise. Similarly, mathematics score = 1 if a child can perform mathematical operation of subtraction or division and 0 otherwise. Formally the model can be written as:

$$Score\ (reading/mathematics)_{ij} = a + b\text{Female}_{ij} + c\text{Age}_{ij} + h_j + e_{ij}$$  \hspace{1cm} (1)
where, \(i\) stands for the male (= 0) or female (= 1) child within the household and \(j\) stands for the household. “Female” stands for the dummy for the gender (male as reference) of the child; “Age” for age of the child and “h” stands for household fixed effects. It can be noted that “Age\(^2\)” is not included in the model as the variable itself has been treated as a categorical (non-linear) variable for the regression analysis. Similar kind of modeling have been used in past studies (refer to Motiram and Osberg 2010; Chudgar 2011).

It may also be noted that there are two problems associated with LPM; first, by construction it produces heteroskedasticity in the residual variance and second, in many cases the predicted probability of dependent variable (=1) turn out to be either below 0 or above 1 which indicates that the probabilities cannot be linearly related to the independent variables for all their possible values. The first problem is taken care by using the option “robust” in STATA (package used for regression analysis), whereas the second problem doesn’t arise in our estimation which will become clear when the post estimation checks will be discussed.

Table 2 presents the estimates from the household fixed-effects linear probability models.

Table 2 Linear Probability Estimates (household fixed effects) of a child’s academic scores

<table>
<thead>
<tr>
<th></th>
<th>Reading</th>
<th>Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.423***</td>
<td>0.395***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>Female (reference: male)</td>
<td>-0.042**</td>
<td>-0.054***</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>Age (reference: age 8 years)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy for age 9</td>
<td>0.126***</td>
<td>0.109***</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>Dummy for age 10</td>
<td>0.324***</td>
<td>0.290***</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Dummy for age 11</td>
<td>0.352***</td>
<td>0.308***</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Sample size</td>
<td>2020</td>
<td>2020</td>
</tr>
<tr>
<td>R(^2) within</td>
<td>0.209</td>
<td>0.170</td>
</tr>
<tr>
<td>R(^2) between</td>
<td>0.025</td>
<td>0.016</td>
</tr>
<tr>
<td>R(^2) overall</td>
<td>0.089</td>
<td>0.065</td>
</tr>
<tr>
<td>Post estimation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Predicted probability, (y=1) (min.)</td>
<td>0.339</td>
<td>0.287</td>
</tr>
<tr>
<td>Predicted probability, (y=1) (max.)</td>
<td>0.733</td>
<td>0.649</td>
</tr>
</tbody>
</table>

Notes (1). For reading, \(y=1\) if the child can read a paragraph or a short story, 0 otherwise; For mathematics, \(y=1\) if the child can perform subtraction or division, 0 otherwise.
(2). ***significant at 1%, **significant at 5%.
(3). Figures in parentheses are robust standard errors. Since the model is linear probability model, robust option has been used to take care of heteroskedasticity.

It can be observed that for both the reading as well as mathematics skills, the coefficients are negative and highly significant. It shows that the probability of reading a paragraph or a short story (and the probability of performing mathematical operation of subtraction or division) for a female child is significantly lower than the male child from the same household. Also the
probability of achieving the above mentioned scores increases with the age of a child. It makes sense because the same set of tests were administered to the children of all ages.

It may be noted (post estimation in Table 2) that the predicted probability of reading score (or mathematics score) = 1 is never below 0 or above 1 (that is, it lies in the interval 0-1). This shows that the model used in the analysis is robust as the probabilities can be linearly related to the independent variables for all their possible values.

The primary focus of this study is to estimate the extent of gender based intra-household inequality of opportunity in reading and mathematics skills. To this end, the study uses a simple but innovative technique whose basic intuition lies in the fact that the difference between the academic skills of male and female siblings (within a household) is due to either gender of the children or their age. If the scores can be corrected for age, then the sole difference in academic skills of children within household can be attributed to the gender of children. The other household factors are same for both the children of the household. One implicit assumption here is that a child at such a tender age cannot be held responsible for putting efforts by himself/herself. If a child of a particular gender in a particular household is investing more efforts (in terms of doing homework), then it is taken as an indication that the parents are motivating (investing in) him/her more than the other child. Similarly, if parents are selectively sending a child of a particular gender to a better school, then it is also taken as a consequence of the gender of the child itself. So, if the academic score in a particular skill is corrected for the age of the children and then the overall inequality in the corrected academic scores (reading or mathematics) can be decomposed into within household and between household components, then the within household component can be attributed to the gender based (intra-household) inequality of opportunity in that particular academic skill (reading or mathematics). A ratio of within household inequality to the overall inequality will give the inequality of opportunity in the particular academic skill as a fraction of total inequality in the same academic skill. It is deemed inequality of opportunity because it can be associated to the gender of children which is clearly a circumstance beyond the control of a child.

As the academic skills of a child within a household vary according to his/her age, that needs to be controlled, I have regressed actual observed scores on age of the child and have taken the residuals from this regression. This variable represents the scores obtained by a child of an “average” age. The details have been presented subsequently. These corrected scores have been used in the inequality decomposition exercise. The exact mechanism for carrying out the decomposition is discussed below:

The decomposition of overall inequality into within household (intra-household) and between household (inter-household) has been carried out separately for the two academic skills (reading and mathematics). For ease of explaining, I will take one of them as example and elaborate the decomposition procedure followed in this analysis. Let us take the example of reading skills. First, the total sample is partitioned into groups based on household itself. That is each household is considered as a group in itself. So, there are totally 1010 groups (as there are 1010 households). Each group (household) contains the reading scores (corrected) of the male-female pair of children present in the group (household). With such a partitioning, the difference in the reading scores within a group (household) can be considered as the result of gender of the children in the household. Now, the overall inequality in reading scores is decomposed into within-group (within-household) and between-group (between-household) components. The resulting within-group component in this decomposition is nothing but the gender based intra-
household (or within-household) inequality of opportunity (IOp) in reading scores (i.e., reading skills). The same procedure has been carried out for mathematics skills as well.

The overall inequality in reading (and mathematics) scores is decomposed into the above-mentioned components using mean log deviation (for similar decompositions, refer to Checchi and Peragine 2010, Singh 2010a and Singh 2011b). Mean log deviation is chosen because it is the only measure which satisfies six axioms or properties which comprise of the four standard axioms of (i) anonymity or symmetry; (ii) population replication or replication invariance; (iii) mean independence or scale invariance; (iv) Pigou-Dalton principle of transfers and the additional axioms of (v) additive subgroup decomposability and (vi) path independence. The additional properties of additive subgroup decomposability and path independence are particularly important for the present study. The additive subgroup decomposability is important because the study primarily decomposes the total reading (and mathematics) scores inequality into within-group and between-group components. Since the interest is in within-group component, the property of path independence is also required in the sense that the decomposition must yield the same result or the decomposition is invariant to whether between-group inequality is eliminated first and the within-group component computed second, or the reverse. The results of the decomposition of the overall reading (and mathematics) scores inequality into within-group component (gender based intra-household inequality of opportunity) and between-group component have been presented in Table 3.

Table 3 Gender Based Intrahousehold inequality of opportunity in academic skills

<table>
<thead>
<tr>
<th></th>
<th>Within group inequality (A)</th>
<th>Between group inequality (B)</th>
<th>Total inequality (C)</th>
<th>Gender based Intrahousehold IOp (%) (D) = (A)/(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading Scores</td>
<td>0.059</td>
<td>0.106</td>
<td>0.165</td>
<td>35.7</td>
</tr>
<tr>
<td>Mathematics</td>
<td>0.067</td>
<td>0.146</td>
<td>0.214</td>
<td>31.5</td>
</tr>
</tbody>
</table>

Notes: 1. Groups are based on households, every household is a group in itself; Mean Log Deviation has been used for carrying out the inequality decompositions.
2. The inequality has been estimated on academic scores corrected for age of children.
   That is, the residuals from the following regressions:
   (a) Reading = 2.021 + 0.502.dummy<sub>age 9</sub> + 0.811.dummy<sub>age 10</sub> + 1.033.dummy<sub>age 11</sub>
   Since the residuals are centered around zero, they have been added a constant (2.591) in order to match the actual series.
   (b) Mathematics = 1.161 + 0.266.dummy<sub>age 9</sub> + 0.551.dummy<sub>age 10</sub> + 0.760.dummy<sub>age 11</sub>
   Since the residuals are centered around zero, they have been added a constant (1.547) in order to match the actual series.

1 These properties have been described in Appendix I. Since these properties are standard properties generally associated with commonly used inequality measures, the description has been kept to a minimum. Readers may refer to Shorrocks (1980), Foster and Shneyerov (1999, 2000), Shorrocks and Wan (2005), and Ferreira and Gignoux (2008) for a detailed discussion.
2 For interested readers, the decomposition procedure using mean log deviation as the inequality measure has been presented in Appendix II.
It can be noted from Table 3 that when it comes to reading skills, nearly 36% of overall inequality can be attributed to gender based intra-household inequality of opportunity. The same figure for mathematics skills stand at approximately 32%. The estimates are substantial as almost one-third of the overall inequality can be associated to the difference in gender of the children within the households. These results have been discussed further in the next section which concludes the present study.

4. Discussion and Conclusion

The study was initiated with a primary objective of enquiring into within household gender based differences in academic skills of children. The analysis provides compelling evidence that within households, it is the female child which is at a disadvantaged position compared to the male child. The study also finds substantial level of gender based intra-household opportunity share in the overall inequality in academic skills. To further explore the possible reasons for the lower level of academic scores (skills) of the female children compared to their male counterparts, I examine the expenses incurred by the households on the education of their children. Table 4 reports the mean annual expenditure by households on various aspects of their children’s education.

### Table 4 Mean annual expenditure by households on children’s education

<table>
<thead>
<tr>
<th>Age</th>
<th>School Fees (INR)</th>
<th>Books, Uniform, Transportation and other material (INR)</th>
<th>Private Tuition (INR)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td>Diff.</td>
</tr>
<tr>
<td>Overall</td>
<td>526.23</td>
<td>478.02</td>
<td>48.21</td>
</tr>
<tr>
<td>8</td>
<td>450.78</td>
<td>425.35</td>
<td>25.43</td>
</tr>
<tr>
<td>9</td>
<td>521.22</td>
<td>457.38</td>
<td>63.84</td>
</tr>
<tr>
<td>10</td>
<td>502.66</td>
<td>475.44</td>
<td>27.22</td>
</tr>
<tr>
<td>11</td>
<td>676.25</td>
<td>546.69</td>
<td>129.56</td>
</tr>
</tbody>
</table>

Note: 1. INR stands for Indian Rupees
2. Diff. stands for the difference between the mean expenditures on male and female children.
3. **Significant at 5%; based on two-sample t test with unequal variances.

It can be observed from the Table that, barring an exception of educational expenditure on books, uniforms and other material on the children aged 11 years, the mean expenses on every other category at every age is more for the male children compared to the female counterparts. It may also be noted that the difference in expenses on private tuition between male and female children is starker than the other educational heads. For the overall sample, the expenditure on private tuitions on male children is significantly higher than that of female children. This should
be seen in the light of the fact that, private tuitions are optional and households can decide to choose which children they want to provide this facility and to whom not. Unlike expenses on other items, for example, school fees, which are relatively fixed, the expenses on private tuitions are discretionary which the households are using more in favor of male children.

As a concluding remark it can be said that inequality of opportunity is fast becoming the centre of debate among the researchers and policy makers working in areas related to the welfare of the population. An increasing part of academia is feeling the need for policies which can reduce circumstances based inequality in various economic and non-economic outcomes in a society. Special mention can be made to World Development Report 2006 (World Bank, 2006), which suggests that, on the day of their birth, children cannot be held responsible for their circumstances, despite the fact that these circumstances such as race, gender, parent’s income and education, and urban or rural location will make major differences in the lives they lead. Given this context, focusing on children helps put inequality of opportunity at the centre of the policy debate. Governments world-wide are focusing on eradicating any sort of circumstance based discrimination in access to governmental and non-governmental services among the populace but the major challenge lies in countering the discrimination based on gender within the households as there is always a high chance that it remains undetected. Observing the scenario in India, where there is substantial evidence of discrimination against the female children within the households, it is high time for Government of India to start programmes which can tackle this social evil.
References

Asadullah, N. and Yalonetzky, G. (2010) “Inequality of educational opportunity in India: Changes over time and across states” IZA discussion paper number 5146.


Borooah, V. (2004) “Gender bias among children in India in their diet and immunization against disease” Social Science and Medicine 58, 1719- 1731


Appendix I Details of the properties satisfied by mean log deviation as listed in section 3 (Empirical Analysis and Results).

The description of the properties has been provided in a generalized form in the context of income for which mean log deviation (MLD) and other common inequality measures are generally used. The same properties can be thought of in the context of academic scores by replacing incomes with academic scores. Since these are standard properties which have been fairly developed and described in the literature related to inequality measures, only an intuitive description has been provided here. For greater details please refer to Shorrocks (1980), Foster and Shneyerov (1999, 2000), Shorrocks and Wan (2005), and Ferreira and Gignoux (2008).

Consider a population of individuals represented by $N = (1, 2, \ldots, n)$, with $y = (y_1, \ldots, y_n)$ as the income vector. The mean income is denoted by $\mu$. Inequality in income distribution is captured by an index, $I(y)$.

Property 1. Anonymity (Symmetry)

$I(y_1, y_2, \ldots, y_n)$ is invariant to permutations of $(y_1, y_2, \ldots, y_n)$. That is, $I(y) = I(x)$ whenever $x$ is obtained from $y$ by a permutation. In simple terms only the income distribution matters and not the individuals who are earning them.

Property 2. Population Replication (Replication Invariance)

$I(y_1, y_2, \ldots, y_n) = I(y_1, y_2, \ldots, y_n; y_1, y_2, \ldots, y_n)$ or in general $I(y) = I(x)$ whenever $x$ is obtained from $y$ by a replication, that is, incomes in $x$ are simply the incomes in $y$ repeated a finite number of times. Simply put, cloning the whole income distribution doesn’t affect the inequality measure.

Property 3. Mean Independence (Scale Invariance)

$I(y_1, y_2, \ldots, y_n) = I(\delta y_1, \delta y_2, \ldots, \delta y_n) \forall \delta > 0$; that is $I(y) = I(x)$ whenever $x$ is obtained from $y$ by a scalar multiple. The inequality measure doesn’t change if income of every individual in the population is scaled up or down by the same multiple.

Property 4. Pigou-Dalton Transfer Principle

$I(y_1, y_2, \ldots, y_i - \lambda, \ldots, y_j + \lambda, \ldots, y_n) > I(y_1, y_2, \ldots, y_i, \ldots, y_j, \ldots, y_n)$ if $\lambda > 0$ and $y_i < y_j$. In simple terms, if income is transferred from a poorer individual to a richer individual (regressive transfer), the inequality measure increases. Analogous definition can be mentioned for progressive transfers also, where the inequality measure should decrease, in case income is transferred from a richer individual to a poorer individual.

The details presented in this appendix have been derived from the referred studies. Also, some standard notations are retained in order to maintain coherence.
Property 5. Additive Decomposability

Consider that the individuals, \( N \), are partitioned into \( m \) proper subgroups \( N_k \) \((k = 1, 2, \ldots, m)\) based on some criteria, with respective income vectors \( y^k \), mean incomes \( \mu_k \), population sizes \( n_k \), and population shares \( v_k = \frac{n_k}{n} \). Also, let \( \bar{y}^k \) denote the distribution obtained by replacing each income in the vector \( y^k \) with the subgroup mean, \( \mu_k \). Then (following Shorrocks and Wan 2005), for MLD as the inequality index,

\[
I(y) = I(y^1, y^2, \ldots, y^m) = \frac{1}{n} \sum_{k=1}^{m} \sum_{i \in N_k} \ln \frac{\mu_k}{y_i}
\]

\[
= \sum_{k=1}^{m} \frac{n_k}{n} \sum_{i \in N_k} \ln \frac{\mu_k}{y_i} + \frac{1}{n} \sum_{k=1}^{m} \sum_{i \in N_k} \ln \frac{\mu}{\mu_k}
\]

\[
= \sum_{k=1}^{m} v_k I(y^k) + \sum_{k=1}^{m} v_k \ln \frac{\mu}{\mu_k}
\]

\[
= W + B
\]

where \( W \) is the within-group inequality and \( B \) represents the between-group component. \( W \) is nothing but a weighted average of subgroup inequality values and \( B \) is the between-group contribution to inequality, representing the level of inequality obtained by replacing the income of each individual with the mean income of their respective subgroup.

Therefore for MLD, the overall level of inequality for the population can be expressed in an intuitively appealing manner as an exact sum of the average inequality within groups and the inequality due purely to differences in average incomes between groups (Shorrocks 1980; Shorrocks and Wan 2005). Any inequality measure is said to be additively decomposable when it can be decomposed in this way.

Property 6. Path Independence

Consider an inequality measure which satisfies the above decomposability property and that we are interested in obtaining \( W \), which is the within-group component. It can be directly obtained as follows: replace the individual incomes, \( y^k_i \), in every group with \( \frac{y^k_i \mu}{\mu_k} \) (where \( \mu \) is the overall mean for the population). This operation will suppress all between-group inequality, leaving only inequality within groups. If the considered inequality measure is now applied on this “standardized” distribution, it will give the within-group component directly.

Instead, if we replace the individual incomes, \( y^k_i \), in every group with the group-specific mean (\( \mu_k \)), then all the within-group inequality will be eliminated, and the resulting “smoothed” distribution will have only the between-group component. The within-group component \( W \) can now be obtained (indirectly) from subtracting the inequality (using the considered inequality measure) in above “smoothed” distribution from the overall inequality (using the same inequality measure) in the actual distribution. If the within-group component obtained from the two processes is same, then the inequality measure is considered to be path independent (Ferreira and Gignoux 2008, p.9).
Appendix II Details of decomposition method used for analysis in section 3 (Empirical Analysis and Results)

The decomposition of overall academic skills (reading and mathematics) inequality into within-group and between-group (each household is taken as a group) has been carried out separately for each skill using mean-log deviation. The exact decomposition procedure is as follows:

Let the index (mean log deviation) be represented by $M$, and suppose that the children, $N$, are partitioned into $m$ (=1010) proper subgroups $N_k$ ($k = 1,2, ..., m$) based on their household, with respective academic score vectors $y^k$, mean scores $\mu_k$, population sizes $n_k$ (in the present case there are two children in every group), and population shares $v_k = \frac{n_k}{n}$. Also, let $\bar{y}^k$ denote the distribution obtained by replacing each score in the vector $y^k$ with the subgroup mean, $\mu_k$. Then,

$$M(y) = M(y^1, y^2, ..., y^m) = \frac{1}{n} \sum_{k=1}^{m} \sum_{i \in N_k} \ln \frac{\mu}{y_i}$$

$$= \sum_{k=1}^{m} \frac{n_k}{n} \sum_{i \in N_k} \ln \frac{\mu_k}{y_i} + \frac{1}{n} \sum_{k=1}^{m} \sum_{i \in N_k} \ln \frac{\mu}{\mu_k}$$

$$= \sum_{k=1}^{m} v_k M(y^k) + \sum_{k=1}^{m} v_k \ln \frac{\mu}{\mu_k}$$

$$= W + B$$

where $W$ is the within group inequality (intra-household inequality of opportunity) and $B$ represents the between group component (refer to Shorrocks and Wan 2005 for greater details).