Economics Bulletin

Volume 31, Issue 3

The Hedonic Price Function in a Matching Model of Housing Market

Gaetano Lisi
University of Cassino

Abstract

This paper develops a theoretical model in which the matching framework à la Pissarides (2000) extended to the housing market is integrated with the hedonic price theory. Market tightness and selling price collectively determine the long-run equilibrium of the economic system in which a seller can become a buyer, and vice versa. As a result, the house price depends not only on the housing characteristics but also on the market tensions and bargaining power of the parties. This integration allows to overcome a major drawback of the hedonic price theory, namely the assumption of perfect competition.

Citation: Gaetano Lisi, (2011) "The Hedonic Price Function in a Matching Model of Housing Market", *Economics Bulletin*, Vol. 31 no.3 pp. 2347-2355.
Submitted: Jun 23 2011. **Published:** August 21, 2011.

1. Introduction

The hedonic price theory, traced back to the papers of Lancaster (1966), Rosen (1974) and Epple (1987), is the theory of reference for the housing market.[1] Due to the distinctive characteristics of the good taken into consideration, hedonic models differ from all other economic models and their formulation poses additional difficulties (Brown and Ethridge, 1995). The price of a composite asset such as a home, in fact, depends heavily on its intrinsic and extrinsic characteristics, whose prices (being implicit or hedonic) are not observable.

As shown by the rigorous theoretical analysis proposed by Rosen (1974), closed-form analytical solutions to the hedonic model are not always possible. This is due to the fact that the hedonic price function (which defines the budget constraint of households and the market value of the composite good, and from which the hedonic or implicit prices of each integrated characteristic are derived) is non linear.[2] For this reason, a strictly empirical approach is required and the marginal implicit prices are "revealed" by estimating the coefficients of a multiple regression model in which the sale price is a function of the home's characteristics.

However, the use of a particular empirical model rather than another should be indicated by economic theory (Stock and Watson, 2003). Theoretical models are in fact critical in determining an accurate and consistent econometric model. Empirical analysis alone cannot replace conceptual reasoning when estimating relationships of most economic phenomena (Brown and Ethridge, 1995).

Recently, from a theoretical point of view, there has been much focus on formulising the housing market through the use of search & matching models (Wheaton, 1990; Albrecht *et al.* 2007; Caplin and Leahy, 2008; Genesove and Han, 2010). It has, in fact, been acknowledged that housing markets clear not only through price but also through the time that a buyer and a seller spend on the market. Consequently, the search & matching approach is completely consistent even with this type of market. Furthermore, also in housing markets the price is substantially determined by a deal between the parties (Quan and Quigley, 1991; Habito *et al.*, 2010). Indeed, a major drawback of the hedonic pricing theory is that bargaining has no impact on price because this theory assumes perfect

[1] For an exhaustive overview see Sheppard (1999) and Malpezzi (2003).

[2] According to Rosen (1974), there is no reason for the hedonic price function to be linear; in fact, the linearity of the hedonic price function is unlikely as long as the marginal cost of attributes increases for sellers and it is not possible to untie packages.

competition (Harding *et al.*, 2003a; Harding *et al.*, 2003b; Cotteleer and Gardebroek, 2006).

The main aim of this paper is to develop a long-run equilibrium model for the housing market that brings together the hedonic price theory with the matching framework. Basically, the two theoretical approaches pursue two different goals: the determination of hedonic or implicit prices (hedonic price theory) and the determination of the natural rate of vacancies (search and matching theory).

In particular, we develop a decentralised long-run equilibrium model based on the costly search activity that characterises the housing market. In fact, sellers and buyers spend time and money – for advertising vacancies and making the effort to visit the greatest number of house – before concluding the deal. The proposed work takes the distinctive features of the considered market into account, where the formal distinction between buyer and seller becomes very subtle. In the model, in fact, a seller can become a buyer and *vice versa*. Indeed, most houses are bought by those who already own one, and most houses are sold by those wanting to buy another house (Janssen *et al.*, 1994). Finally, the selling price of a house depends not only on the housing characteristics but also on the market tensions and bargaining power of the parties.

This model can be used as theoretical groundwork for any empirical specification selected to estimate hedonic (or implicit) prices, and it allows to overcome a major drawback of the hedonic pricing theory: the assumption of perfect competition.

The rest of the paper is organised as follows: section 2 presents the housing market matching model; while section 3 concludes.

2. The model

2.1 The hypotheses of the model

As we are interested in determining the sale price, the market of reference is the 'homeownership market' rather than the 'rental market'. Hence, if a contract is legally binding (as hypothesised) it is no longer possible to return to the circumstances preceding the bill of sale, unless a new and distinct contractual relationship is set up.[3]

[3] In matching model jargon this means that the destruction rate of a specific buyer-seller match does not exist.

Buyers (*b*) expend costly search effort to find a new and better house (as in Wheaton (1990) there are no homeless buyers),[4] while sellers (*s*) hold *n* houses (with $n \geq 2$) of which $n - 1$ are on the market, i.e. vacancies (*v*) are simply given by $(n - 1) \cdot s$. It is therefore possible that a buyer can become a seller and that a seller can re-enter the market as a buyer. The expected values of a (house) vacancy *V* and of finding a new and better house *B* are represented as: [5]

$$rV = -a + q(\theta) \cdot G(\eta, y) \cdot [P - V] \quad [1]$$

$$rB = -e + g(\theta) \cdot G(\eta, y) \cdot [x - B - P] \quad [2]$$

where $\theta \equiv v / b$ is the housing market tightness from the sellers' standpoint,[6] while $q(\theta)$ and $g(\theta)$ are, respectively, the (instantaneous) probability of finding a buyer and of finding a vacancy. The properties of these functions are intuitive and straightforward: $q'(\theta)<0$ and $g'(\theta)>0$.[7] Furthermore, the customary hypothesis of constant returns to scale in the matching function $m=m(v,b)$ is adopted (Pissarides, 2000; Petrongolo and Pissarides, 2001) . $G(\eta, y) \in (0,1)$ is the acceptance rate which depends on the housing preferences η and income y of the buyer; buyers are therefore assumed to be fully characterized by an income y and parameters vector η, with distribution over possible values described by the joint probability $G(\eta, y)$. Sellers are instead characterized by their price bargaining power $0 < \gamma \leq 1$ (see later).

If a contract is stipulated, the buyer gets a benefit *x* from the property (abandoning the home searching value) and pays the sale price *P* to the seller (who abandons the value of finding another buyer). Finally, *a* and *e* represent, respectively, the costs sustained by sellers for the advertisement of vacancies and the effort (in monetary terms) made by buyers to find and visit the largest possible number of houses.

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[4] According to Wheaton (p. 1274, 1990): «*Homelessness (equivalent to unemployment) is relatively inconsequential in the housing market, and so moves are like voluntary "quits" in the labor market. Furthermore, moves involve some spell in which the household owns two units, whereas even voluntary job transitions usually carry some period of unemployment. More important, the causes of housing mobility are usually different from those generating job mobility*».

[5] Time is continuous, and individuals are risk neutral, live infinitely and discount the future at the rate *r*. As usual in matching-type models, the analysis is restricted to the stationary state.

[6] By definition, markets with frictions require that: $0 < \theta < \infty$.

[7] Standard technical assumptions are assumed, i.e. $\lim_{\theta \to 0} q(\theta) = \lim_{\theta \to \infty} g(\theta) = \infty$, and $\lim_{\theta \to 0} g(\theta) = \lim_{\theta \to \infty} q(\theta) = 0$.

2.2 Equilibrium

The endogenous variables that are determined simultaneously at equilibrium are market tightness (θ) and sale price (P).

The customary (long term) "zero profit" equilibrium condition normally used by matching models (Pissarides, 2000) yields the first key relationship, in which market tensions are a positive function of price. Using the condition $V = 0$ in [1], we obtain:

$$\frac{a}{P \cdot G(\eta, y)} = q(\theta) \qquad [3]$$

with $\partial\theta / \partial P > 0$ since $q'(\theta)<0$.

The (generalized) Nash bargaining solution, usually used for decentralised markets, allows the sale price P to be obtained through the optimal subdivision of surplus (S) deriving from a successful match:[8]

$$S = \underbrace{(P - V)}_{\text{capital gain of seller}} + \underbrace{(x - B - P)}_{\text{capital gain of buyer}}$$

$$\Rightarrow S = x - B \qquad [4]$$

The price is obtained by solving the following optimisation condition:

$$P = argmax\left\{(P - V)^{\gamma} \cdot (x - B - P)^{1-\gamma}\right\} \Rightarrow P = \frac{\gamma}{(1-\gamma)} \cdot (x - B - P)$$

knowing that $(x - B - P) = \frac{(1-\gamma)}{\gamma} \cdot P$, eventually we get:

$$P = \gamma \cdot (x - B)$$

$$\Rightarrow P = \frac{\gamma \cdot (rx + e)}{r + g(\theta) \cdot G(\eta, y) \cdot (1-\gamma)} \qquad [5]$$

since $g'(\theta)>0$, as market tensions increase, the sale price decreases; hence, we obtain the second key relationship of the model: $\partial P / \partial\theta < 0$. The selling price is a positive function of the buyer's benefit. Indeed, the selling price can be higher or lower than x depending on the bargaining power of the seller. In fact,

$$\gamma \to 0 \Rightarrow P = 0$$

$$\gamma \to 1 \Rightarrow P = x + \frac{e}{r} > x$$

since the price can never be negative or null, $0 < \gamma \leq 1$. Therefore, as usual for heterogeneous goods, bargaining plays a key role in the price formation process

[8] Entering into a contractual agreement obviously implies that $S > 0$, i.e. $x > B$, $\forall\theta$. This realistic condition ensures that the price is positive.

(Quan and Quigley, 1991; Harding *et al.*, 2003a; Harding *et al.*, 2003b; Cotteleer and Gardebroek, 2006; Habito *et al.*, 2010).

It is straightforward to obtain from [3] that when P tends to zero (infinity), θ tends to zero (infinity), as $q(\theta)$ tends to infinity (zero). Consequently, given the negative slope of [5] and the fact that price is always positive, only one long term equilibrium deriving from the intersection of the two curves exists in the model (see point A in *Figure* 1).

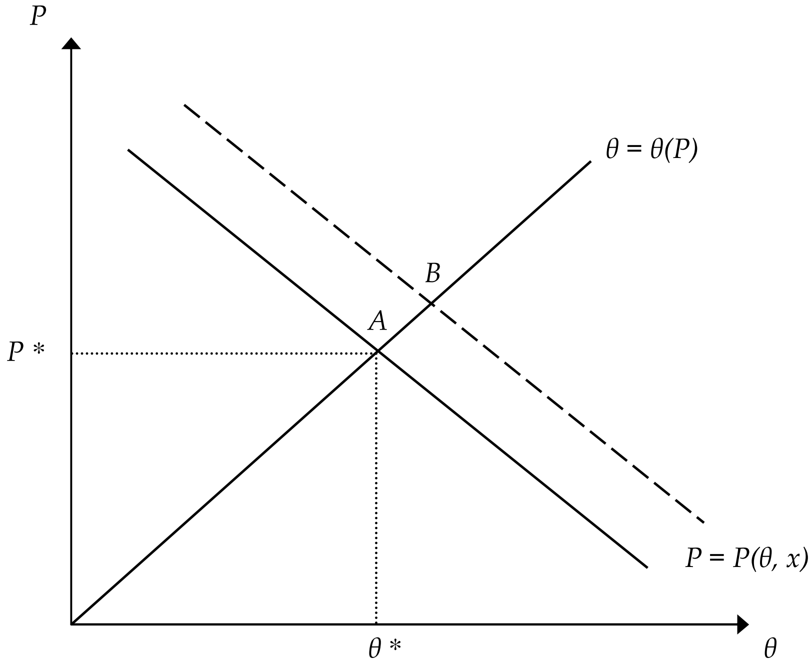

Figure 1. Equilibrium

Finally, normalizing the population in the housing market to the unit, 1 = s + b, and using the definitions of equilibrium tightness ($\theta = \theta^*$) and vacancies, it is straightforward to obtain the share of sellers,

$$s = \frac{\theta^*}{n-1+\theta^*} \qquad [6]$$

buyers,

$$b = \frac{n-1}{n-1+\theta^*} \qquad [7]$$

and the "natural" vacancy rate,

$$v = \frac{(n-1)\cdot\theta^*}{n-1+\theta^*} \qquad [8]$$

the "natural" vacancy rate is the optimal share of houses for sale on the market that prevails in long term equilibrium at which sellers make no economic profits (Arnott and Igarashi, 2000; McDonald, 2000).

The positive sign of $\partial s / \partial \theta$ makes clear the economic meaning of the two key relationships of the model: if the sale price increases, market tensions are higher because more sellers will stand in the market; whereas, if the market tensions increase, the effect of the well-know congestion externalities (see Pissarides, 2000) on the demand side will lower the price.

2.3 The hedonic price function

As in Habito *et al.* (2010), the buyer's benefit is a positive function of housing characteristics c:

$$x = x(c) \quad [9]$$

with $x'(c)>0$, since *ceteris paribus* the greater the quality/quantity of characteristics, the higher the buyer's benefit. Hence, the hedonic price function of the model is none other than equation [5]. However, unlike the standard hedonic price function, the sale price of this model depends not only on the housing characteristics but also on the market tensions and bargaining power of the parties. As a consequence, the hedonic price is equal to:

$$\frac{\partial P}{\partial c} = \frac{\gamma \cdot r \cdot x'(c)}{r + g(\theta) \cdot G(\eta, y) \cdot (1-\gamma)} > 0 \quad [10]$$

If c increases, the new equilibrium of the model is the point B shown in *Figure 1* (see the dotted line), where both the sale price and market tensions are higher. In short, the housing markets are not necessarily "thin", as suggested by Harding *et al.* (2003a) and Cotteleer and Gardebroek (2006). Nevertheless, the assumption of perfect competition fails because of the presence of search frictions.

In empirical investigations of hedonic models, an interesting issue is determining how the market price of the commodity varies as the characteristics vary (Epple, 1987). From [10] it is clear that the trend of P as c varies depends on the hypotheses underlying (the second derivative of) the function $x(c)$. It follows that the hedonic price function of this model is compatible with any empirical specification. In fact, the hedonic price theory does not give precise indications regarding the choice of the (best) functional form to be used in empirical applications (Malpezzi, 2003; Taylor, 2003).

3. Conclusions

This paper develops a theoretical model in which the matching framework *à la* Pissarides (2010) extended to the housing market is integrated with the hedonic price theory. A decentralised long-run equilibrium model is proposed, based on the costly search activity which characterises the housing market, where the formal distinction between buyer and seller becomes very subtle and the sale price depends not only on the characteristics of the house but also on the market tensions and bargaining powers of the parties. This model can be used as theoretical groundwork for any empirical specification selected to estimate hedonic (or implicit) prices, and it allows to overcome a major drawback of the hedonic pricing theory, namely the assumption of perfect competition.

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