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Consistency of the proposed additive measures of revealed comparative advantage

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Abstract

The recently proposed additive measures of revealed comparative advantage (RCA) have been argued as better alternatives to the Balassa Index. This note builds on those suggestions and provides a framework to assess their applicability by means of their consistency across the dimensions and inferences that the indices' informational content allow. A formal empirical investigation supports the presented arguments that the proposed additive constructs of RCA possess certain inconsistencies that may obscure comparative analyses based on them, whereas alternative additive measures proposed here are shown to be more consistent over space and time, and hence may serve as better instruments in related quantitative research.

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1. Introduction

The law of comparative advantage still stands as one of the core pillars of international trade theory. Yet as relative autarkic prices are not observable and the available data is likely be in post-trade equilibria, identification of the true comparative advantage is not feasible. Nevertheless the use of actual trade flows and their relative shares to construct certain indices that would reveal comparative advantage has become an empirical convention, following the seminal paper by Balassa (1965). While the vast amount of the associated literature employs the Balassa Index (BRCA) or its normalized variants[1], recent studies (Hoen & Oosterhaven, 2006; Yu et al., 2009) challenge the multiplicative nature and informational content of the index in favour of an additive construct. This note builds on those suggestions and provides a framework to investigate their applicability.

By employing Kunimoto's (1977) probabilistic framework, the recently proposed additive constructs of revealed comparative advantage (RCA) are expressed as normalized forms of the level deviations in actual trade flows from their expected levels. As RCA indices provide inference as cardinal, ordinal and dichotomous measures[2], the employed normalization should not alter the initial inferences based on those level deviations, hence provide a linear monotonic transformation[3]. Furthermore, given that those level deviations retain variation across industries, countries and time, their normalization should not alter existing variations across those dimension. On the other hand, inappropriate normalization of those deviations that distorts their distribution would result in a biased transformation[4] making the indices inconsistent with the level deviations, which would then prevent a robust comparative analysis.

Based on these arguments, in a first step, proposed additive constructs of RCA are evaluated based on their consistency with the level deviations across the three dimensions and the three inferences that their informational content allow. In a second step, the note provides alternative measures that comply with the relevant consistency arguments. A formal empirical investigation supports the presented arguments that the previously proposed additive constructs of RCA possess certain inconsistencies, which may obscure comparative analyses based on them, while proposed alternatives possess the necessary properties for a robust comparative analysis.

The rest of the note is organized as follows. Section 2 introduces the framework in which RCA is defined as deviations from the comparative neutral state (expected levels). Section 3 presents the recently proposed additive measures of RCA and discusses the relevant consistency issues. In section 4, alternative measures are presented. Section 5, then empirically

[1]e.g. Vollrath (1991), Dalum et al. (1998) and Proudman & Redding (2000)

[2]for an extensive discussion please refer to Ballance et al. (1987)

[3]while a monotonic transformation would retain the order of the RCA scores, preserving their initial inferences as ordinal and dichotomous measures, it should also be linear to retain the original shape of the distribution hence preserve their initial cardinal inference.

[4]that is either non-linear but monotonic, which retains consistency only as ordinal and dichotomous measures or non-monotonic, which retains consistency only as a dichotomous measure assuming that the transformation is sign-preserving.

investigates the consistency of the perviously and presently proposed additive measures of RCA. Section 6 concludes.

2. RCA as Deviations from the Comparative Neutral State

Kunimoto (1977) argues that within a hypothesized post-autarkic world where countries do not possess a comparative advantage, exports of a particular industry would be distributed among countries in proportion to their shares in total world exports such that, each country's expected level of exports in that particular industry would be the product of the share of that industry within world trade and the country's total exports. That is

$$E\left(X_j^i\right) = X_z^i \cdot \left(X_j^w / X_z^w\right), \tag{1a}$$

where subscript j and z, respectively, refer to the reference and the aggregate of all traded industries, while superscripts i and w, respectively, represent the examined country and the world and X stands for the export flows. Hence, in a distortion free world, the equivalence of the expected level of exports with the actual level of exports would indicate a neutral comparative advantage. That is

$$E\left(X_j^i\right) = X_z^i \cdot \left(X_j^w / X_z^w\right) = X_j^i. \tag{1b}$$

On the other hand, if the actual level of exports exceeds (falls behind) the expected level, then the country would have a comparative advantage (disadvantage) in that particular industry. Thus the deviations in the actual level of exports from their expected levels would indicate the comparative state of the country in that particular industry. If the extent of the deviation is represented as a fraction of the expected level of exports, then one would express BRCA as a ratio of actual level of exports to their expected levels. That is

$$BRCA_j^i = \frac{X_j^i}{E(X_j^i)} = \frac{\left(X_j^i / X_z^i\right)}{\left(X_j^w / X_z^w\right)}. \tag{1c}$$

Such a formulation of BRCA complies with both Kunimoto's framework and Balassa's rationale since index values greater (smaller) than *1* would indicate a comparative advantage (disadvantage), whereas at the comparative neutral state as indicated by the index value of *1*, deviations from the expected export levels are zero. Alternatively, deviations of actual level of exports from their expected levels can be expressed in levels. That is

$$\Delta E\left(X_j^i\right) = \left[X_j^i - E\left(X_j^i\right)\right]. \tag{2}$$

Once again, if the actual level of exports exceeds (falls behind) the expected level, then the country would have a comparative advantage (disadvantage) in that particular industry. However unlike BRCA, although the comparative neutral state corresponds to zero deviations from the expected level of exports, it will be prevailed as *0*. Although equation (2) may be employed in a similar fashion as BRCA, it needs to be normalized over a sensible range since the size of $\Delta E\big(X^i_j\big)$ will be in levels depending on the size of the industry, country and time at hand.

On the other hand, once appropriately normalized, one may use the transformed level deviations to reveal comparative advantage. However, given the fact that the informational content of the deviations of actual exports from their comparative neutral (or expected) levels allow a comparison across industries, countries and time, such normalization should not distort the distribution of those deviations across these dimensions. In addition, RCA measures permit interpretation as cardinal, ordinal and dichotomous measures, which should also be taken into consideration for means of consistency.

3. Existing Additive Indices

The proposition for an alternative RCA index, that challenges the multiplicative nature and informational content of BRCA[5] in favour of an additive construct, was first introduced by Hoen & Oosterhaven (2006). Instead of defining RCA as a ratio between the share of an industry within a country and that of the world, they defined RCA as the difference between them, namely the numerator and the denominator of (1c). That is

$$ARCA^i_j = \big(X^i_j/X^i_z\big) - \big(X^w_j/X^w_z\big), \tag{3a}$$

which corresponds to the level deviations of country i's exports in industry j from its comparative neutral level, scaled down by the total exports of country i. That is

$$ARCA^i_j = \frac{\Big[X^i_j - E\big(X^i_j\big)\Big]}{X^i_z}. \tag{3b}$$

Yet as the scaling factor will change from country to country, the index will be biased, which would particularly affect cross-industry and cross-temporal comparisons. Furthermore, because the magnitude of the scaling factor differs for each country, cardinal and ordinal interpretations of the index values would not match in all three dimension, least pronounced across countries[6]. Yu et al. (2009), on the other hand, propose to scale the term in brackets with the total level of world exports. That is

[5] the asymmetry caused by the inequality of the intervals of BRCA, [0,1] signifying a comparative disadvantage while [1,∞] signifies a comparative advantage, indicates that the demarcation of the index is not symmetric and the relative weight attached to specialized sectors compared to unspecialized sectors would be unrestrained. Furthermore, BRCA has an inherent bias to signify strong comparative advantage for countries and industries that comprise a small market share. Such shortcomings of BRCA are argued to be caused by its multiplicative nature. For further discussion, please refer to Yu et al. (2009).

[6] that is, normalizing $\Delta E\big(X^i_j\big)$ with X^i_z results in a non-monotonic transformation.

$$NRCA_j^i = \frac{\left[X_j^i - E\left(X_j^i\right)\right]}{X_z^w}, \tag{4}$$

where the index measures the deviations of country i's exports in industry j from its comparative neutral level, scaled down by the total world exports. Although Yu et al.'s scaling of $\Delta E\left(X_j^i\right)$ with the total world trade is consistent across time, it is biased over industries and countries. Given this bias, cross-industry and cross-country comparison of $NRCA_j^i$ will not be robust. But unlike $ARCA_j^i$, because the scaling factor will be constant within each year, the index's interpretation as a cardinal and an ordinal measure will not be biased across time[7]. Since neither of the proposed additive RCA measures allow an overall consistency nor a comprehensive comparison but rather permit limited inference, comparative analyses based on them would not be robust unless comparison across only those consistent dimensions are aimed for.

4. Appropriately Normalized Additive Indices

The appropriate normalization of $\Delta E\left(X_j^i\right)$ should not alter its distribution across industries, countries or time as well as initial inferences as cardinal, ordinal and dichotomous measures. The simplest method would be to apply linear scaling transformation. That is

$$LSRCA_j^i = \frac{E\left(X_j^i\right) - min\left[E\left(X_j^i\right)\right]}{max\left[E\left(X_j^i\right)\right] - min\left[E\left(X_j^i\right)\right]}, \tag{5a}$$

where the index will have its critical value[8] at $\frac{-min\left[E\left(X_j^i\right)\right]}{max\left[E\left(X_j^i\right)\right]-min\left[E\left(X_j^i\right)\right]}$ and deviations from this point would indicate the relative comparative state. The index has fixed upper and lower bounds at *1* and *0*, respectively. As the critical value is variable depending on the data employed and will unlikely be at *0.5*, the index will be asymmetric. Although $LSRCA_j^i$ will be consistent with $\Delta E\left(X_j^i\right)$, as it introduces no distortion in the transformed values, its interpretation is not straightforward. A possible transformation would be to calculate the ratio of the deviations of each RCA score from the critical value of the index to the critical value itself. That is

$$LSRCA^{'} = \frac{LSRCA_j^i - LSRCA^*}{LSRCA^*}, \tag{5b}$$

where $LSRCA^*$ is the critical value of $LSRCA_j^i$. Using $LSRCA^{'}$ one can consistently compare the RCA scores of countries. The sign of the index will show whether the country

[7] that is, normalizing $\Delta E\left(X_j^i\right)$ with X_z^w results in a non-monotonic transformation across countries and industries while is linear monotonic across time.

[8] indicating the comparative neutral state

has a comparative advantage or a disadvantage in that industry, while the magnitude would indicate the relative advantage or disadvantage.

Another possible normalization would be to transform the deviations of exports from their comparative neutral levels using a logistic function, which would again transform the values over the range of *[0,1]* with critical value at *0.5*. However as $\Delta E\left(X_j^i\right)$ are in levels, the transformed values will likely be out-of-range and since this type of scaling gives linear normalization over its range, it must be transformed to include those out-of range values. Theoretically, there will be an arbitrary out-of-range value encountered at either end of the range and to reduce that, one can apply a softmax scaling which uses the logistic transformation (Pyle, 1999). That is

$$SSRCA_j^i = \frac{1}{1+e^s}. \tag{6a}$$

and

$$s = \frac{\Delta E\left(X_j^i\right) - \mu_{\Delta E(X_j^i)}}{\lambda \cdot \frac{\sigma_{\Delta E(X_j^i)}}{2\pi}}. \tag{6b}$$

where s denotes the transformed value of $\Delta E\left(X_j^i\right)$ with softmax scaling, μ and σ denote the mean and the standard deviation of $\Delta E\left(X_j^i\right)$ concerning the whole sample[9], respectively, and the value of λ will determine the extent of the linear range. Although this type of normalization would be monotonic, there will be some differences in the normalized values between any two nonidentical non-transformed values (Pyle, 1999), thus cardinal consistency may not be obtained[10]. Given the difficulty in interpreting the asymmetric $LSRCA_j^i$ and the likely loss of cardinal consistency using $SSRCA_j^i$, a third option would be to normalize $\Delta E\left(X_j^i\right)$ by $\sigma_{\Delta E(X_j^i)}$[11],which would be consistent across industries, countries and time. That is

$$SNRCA_j^i = \frac{\Delta E\left(X_j^i\right)}{\sigma_{\Delta E(X_j^i)}}. \tag{7}$$

5. Consistencies of the Proposed Additive Indices

As mentioned, RCA indices permit interpretation as cardinal, ordinal and dichotomous measures across industries, countries and time; hence should be consistent vis-à-vis the level

[9] as the use of μ and σ across industries, countries or time would obscure the consistency of the transformed values across the dimensions unaccounted for.

[10] by choosing the appropriate λ one can alter this by trial and error. Here the arbitrary value of $\lambda = 8$ is used, which gives a relatively high interval for the linearized range.

[11] which is the standard deviation in $\Delta E(X_j^i)$ concerning the whole sample in order to achieve consistency across all three dimensions.

deviations from comparative neutral levels in each dimension for a robust comparison. Following Ballance et al. (1987) Pearson, Spearman and Pearson correlation coefficients will be calculated for evaluating the extent[12] of consistency as cardinal, ordinal and dichotomous measures, respectively[13]. Concerning the three dimensions, the correlation coefficients will first be calculated for the whole sample and then for pairs across industries, countries and time to assess the overall consistency of the indices and if they are inconsistent, to determine the ambiguous dimension creating the inconsistency.

Although the RCA indices were shown to be calculated by taking the whole world as the group of reference countries, such an empirical pursuit may not yield sound results, as the countries therein will be quite heterogeneous regarding the market conditions that their trading partners face. Thus instead of taking the world as a whole as the reference countries, EU15 countries will be employed since they are quite homogenous concerning the distortions that their exports face. For the export destination, EU15 countries' exports to the whole world will be considered as choosing a particular country may bias the results. For the export flows, only those of manufacturing industries will be considered. The data is obtained from OECD's STAN Bilateral Trade Database for 22 industries covering the period between 1989 and 2008, yielding 6,248 observations. Table 1 summarizes the extent of consistency of the additive RCA indices vis-à-vis the level deviations from comparative neutral levels for the whole sample.

Table 1 Consistency of the Additive RCA Indices vis-à-vis $\Delta E(X_j^i)$

Index	Cardinal Measure	Ordinal Measure	Dichotomous Measure
$ARCA_j^i$	0.572	0.887	1.000
$NRCA_j^i$	0.917	0.989	1.000
$LSRCA_j^i$	1.000	1.000	1.000
$SSRCA_j^i$	0.933	1.000	1.000
$SNRCA_j^i$	1.000	1.000	1.000

As the normalization methods do not alter the sign of the deviations, consistency of all indices as dichotomous measures can be presumed. Evidently all indices are perfectly consistent in revealing the relative comparative state of the export flows. On the other hand neither of the previously proposed indices show consistency as cardinal nor ordinal measures for which the former is more severe[14], whereas the three alternative indices proposed here obtain perfect ordinal consistency. Furthermore $LSRCA_j^i$ and $SNRCA_j^i$ are also perfectly consistent as cardinal measures. Given these results, a dimensional breakdown will be pursued only for cardinal and ordinal consistencies to verify whether the problematic dimensions are indeed the ones expressed before. Table 2 presents the dimensional breakdown of the relevant consistencies, the first 3 rows indicating average cardinal consistencies while the last 2 rows indicate average ordinal consistencies.

[12] a perfectly consistent index should yield a perfect correlation.

[13] for testing the consistency as a dichotomous measure, indices' binary transformations are employed.

[14] the likelihood of a change in the cardinal value that also alters the ordinal value would be less than the overall likelihood of a change in the cardinal value.

Table 2 Dimensional Breakdown for Inconsistent Indices

Index	Across Industries	Across Countries	Across Time
$ARCA_j^i$	0.654	0.923	0.631
$NRCA_j^i$	0.922	0.925	1.000
$SSRCA_j^i$	0.980	0.980	0.965
$ARCA_j^i$	0.773	0.978	0.891
$NRCA_j^i$	0.976	0.979	1.000

The correlations presented in table 2 correspond to the previously emphasized points. As $ARCA_j^i$ coresponds to $\Delta E(X_j^i)$ scaled by the total trade of country i at time t, its cross-country consistency as a cardinal measure is much higher than its consistency across industries and time. On the other hand scaling of $\Delta E(X_j^i)$ by the total world trade at time t makes $NRCA_j^i$ perfectly consistent across time while inconsistent across industries and countries. As for $SSRCA_j^i$, the cardinal outcomes depend on the chosen extent of linear range used in the relevant softmax scaling and, as stated, a certain degree of inconsistency was expected. Considering the dimensional breakdown of ordinal consistencies, $NRCA_j^i$ is found to be perfectly consistent only across time whereas $ARCA_j^i$ obtains its highest rank correlation across countries, while shows higher inconsistency across industries and time. These results correspond to the previously emphasized points, as well as the findings in table 1. More explicitly, cardinal inconsistency is further shown to be a much more problematic issue compared to ordinal inconsistency.

6. Conclusion

This note shows that the applicability of the recently proposed additive RCA indices as means of comparative instruments is rather questionable. Although these indices fundamentally differ from the Balassa Index due to their additive construct, they do comply with the latter within Kunimoto's framework. Nevertheless they are shown to be inconsistent with the level deviations from the comparative neutral levels as cardinal and ordinal measures, and this inconsistency is due to the inappropriate normalization of those deviations. From a statistical point, such normalization should not alter the distribution of those deviations across industries, countries and time while preserving the initial inferences. Although Yu et al.'s proposed index allows a more robust comparison than Hoen & Oosterhaven's additive index, it is still inconsistent across industries and countries, which would obscure any comparison conducted across those dimensions. Whereas cross-temporal comparisons based on their index would permit a robust comparison. On the other hand the three alternative measures proposed here are shown to be more consistent over space and time compared to the previously proposed indices and hence may serve as better instruments in related quantitative research.

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