Economics Bulletin

Volume 31, Issue 3

Semi-endogenous growth when population is decreasing

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Abstract

The paper analyzes the effect of a negative population growth rate on per capita income growth using a simple model of semi-endogenous growth. It is shown that there is a non-monotonous relationship between population growth rates and long-run per capita income growth rates. Compared to the case of positive population growth the dynamics are richer and depend on the rate of depreciation. Semi-endogenous growth becomes partly endogenous.

I would like to thank an anonymous referee for his helpful comments. Any remaining shortcomings are my own.
Citation: Thomas Christiaans, (2011) "Semi-endogenous growth when population is decreasing", *Economics Bulletin*, Vol. 31 No. 3 pp. 2667-2673.
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Submitted: March 23, 2011. **Published:** September 16, 2011.

1 Introduction

Following a long history of increasing population nowadays more and more countries experience a decline of their populations or at least have growth rates of population near zero (cf. e.g. United Nations, Department of Economic and Social Affairs, Population Division, 2007, Table A.8). The theory of economic growth is nevertheless mostly concerned with non-negative population growth rates. There are only a few theoretical papers that consider negative population growth rates. Samuelson (1975) was the first to discover that a steady state will in general not exist unless the saving rate is also negative. Ritschl (1985) gives further references and discusses the instability of Samuleson's steady state, which he ascribes to the shape of the saving function. He then shows that a stable steady state exists in case of a special saving function dependent on profits. While Ritschl (1985) uses the standard Solow (1956) model in its Cobb-Douglas form, Ferrara (2011) has recently analyzed the Rebelo (1991) AK-model of endogenous growth in case of negative population growth.

To the best of my knowledge the case of semi-endogenous growth with negative population growth has not been considered up to now. As it is well known that the long-run per capita income growth rates in semi-endogenous growth models usually are proportional to the growth rate of population (Jones, 1995), this case nevertheless seems to be the most interesting. Moreover, models of semi-endogenous growth need less knife-edge conditions than endogenous growth models and they do not involve scale effects.[1] The present paper analyzes the dynamics of a simple semi-endogenous growth model under the assumption of a constant positive saving rate. Among the main results is that there is a non-monotonous relationship between population growth rates and long-run per capita income growth rates. In particular, there is a region of negative population growth rates that lead to negative per capita income growth. For very negative rates of population growth, per capita income growth becomes positive. Compared to the case of positive population growth the dynamics are richer and depend on the rate of depreciation. In a particular sense to be specified later on growth becomes endogenous in a standard semi-endogenous growth model.

2 A simple model of semi-endogenous growth

2.1 Positive population growth

This section briefly reviews a simple model of semi-endogenous growth with positive population growth (the model is part of the two-sector model in Christiaans, 2008). Each of a large number of completely identical firms j is assumed to use labor L_j and capital K_j to produce its output Y_j according to the Cobb-Douglas production function

$$Y_j = K_j^{\alpha}(K^{\beta/(1-\alpha)}L_j)^{1-\alpha}, \quad 0<\alpha<1, \quad 0\le\beta<1, \quad \alpha+\beta<1.$$

[1] Scale effects appear in endogenous growth models such as Romer's (1990) seminal contribution, where an increase in the size of an economy permanently increases its long-run per capita income growth rate. The elimination of this scale effect led Jones (1995) to the formulation of a non-scale model in which long-run per capita growth rates do not depend on population size but on its growth rate. In the absence of particular knife-edge conditions, growth in non-scale models is semi-endogenous, that is, the long-run growth rates are independent of policy instruments (cf. Christiaans, 2004).

The aggregate quantities are given by $L = \sum_j L_j$, $K = \sum_j K_j$, and $Y = \sum_j Y_j$. The presence of the aggregate capital stock K implies labor-augmenting technical progress akin to the learning by doing respectively learning by investment formulation of Sheshinski (1967) or the endogenous growth model of Romer (1986). Notice that the exponent of K, $\beta/(1-\alpha)$, is smaller than one due to the assumption that $\alpha+\beta<1$. If $1-\alpha-\beta=0$, the model would involve scale-effects and endogenous growth (cf. Jones, 1999; Christiaans, 2004). Although the individual production functions will not be used in the sequel, they are discussed here in order to emphasize that an external effect of learning by investment is assumed. If learning was internal to firms, perfect competition would be impossible.[2]

Under perfect competition, the individual production functions j can be aggregated to yield an aggregate production function:[3]

$$Y = K^{\alpha+\beta}L^{1-\alpha}, \quad 0<\alpha<1,\ 0\le\beta<1,\ \alpha+\beta<1. \tag{1}$$

For the sake of simplicity L shall equal the population. If $\beta = 0$, the model reduces to the Solow model in Cobb-Douglas form as a special case. If $\beta > 0$ and $0 < \alpha+\beta < 1$, growth is semi-endogenous.

Dividing the production function by L^γ yields the *scale adjusted* per capita income $y = Y/L^\gamma$:[4]

$$y = k^{\alpha+\beta}, \quad \text{where} \quad k = K/L^\gamma \quad \text{and} \quad \gamma = \frac{1-\alpha}{1-\alpha-\beta}$$

Observe that $\gamma>1$ if $\beta>0$ and $\gamma=1$ if $\beta=0$. Using the scale adjusted per capita income and the scale adjusted capital intensity k is reasonable because these variables are constant in long-run equilibrium [cf. the analysis of equation (2)].

The short-run equilibrium requires that gross investment I equals gross saving sY, that is $I = sY$, where s is the constant saving rate. Net investment equals the increase in the capital stock: $\dot{K} = I - \delta K$, where δ is the rate of depreciation. It follows from $\dot{K} = sY - \delta K$ that the growth rate of capital g_K is $g_K = \dot{K}/K = sY/K - \delta$. The growth rate of population is $g_L = n$. Logarithmical differentiation of the scale adjusted capital intensity $k = K/L^\gamma$ with respect to

[2]All semi-endogenous growth models of closed economies display a similar long-run behavior of growth rates, independent of the particular engine of growth. As an example it is shown in Christiaans (2003, Appendix E) that an R&D-driven growth model yields very similar dynamics to the learning by doing approach followed here.

[3]Varying a proof of Sargent (1987, p. 10) for constant returns to scale without externalities, rewrite the individual production function as $Y_j = K^\beta K_j^\alpha L_j^{1-\alpha} = K^\beta (K_j/L_j)^\alpha L_j$. Since K^β is an external effect, the ratio of marginal productivities is just $[(1-\alpha)/\alpha](K_j/L_j)$. Under perfect competition on the factor markets this ratio must equal the ratio of factor prices, which is the same for all firms. All firms therefore choose the same capital-labor ratio which hence must equal the aggregate capital-labor ratio, K/L. Substituting this result into the individual production functions and aggregating yields

$$Y = \sum_j Y_j = \sum_j K^\beta (K/L)^\alpha L_j = K^\beta (K/L)^\alpha \sum_j L_j = K^\beta (K/L)^\alpha L = K^{\alpha+\beta}L^{1-\alpha}$$

[4]To prove this result notice that

$$\frac{Y}{L^\gamma} = K^{\alpha+\beta}\cdot\frac{L^{1-\alpha}}{L^\gamma} = \left(\frac{K}{L^\gamma}\right)^{\alpha+\beta}\cdot\frac{L^{(\alpha+\beta)\gamma}L^{1-\alpha}}{L^\gamma} = k^{\alpha+\beta}\cdot L^{(\alpha+\beta-1)\gamma+1-\alpha}$$

and $(\alpha+\beta-1)\gamma+1-\alpha=0$ according to the definition of γ.

time yields $g_k = \dot{k}/k = g_K - \gamma g_L = sY/K - \delta - \gamma n$. Multiplying by $k = K/L^\gamma$ leads to

$$\dot{k} = s\frac{Y}{K}\frac{K}{L^\gamma} - (\delta + \gamma n)\frac{K}{L^\gamma} = s\frac{Y}{L^\gamma} - (\delta + \gamma n)k,$$

or, using $Y/L^\gamma = k^{\alpha+\beta}$,

$$\dot{k} = sk^{\alpha+\beta} - (\delta + \gamma n)k. \tag{2}$$

If $\beta = 0$, this is *Solow's fundamental growth equation* in case of a Cobb-Douglas production function.

The analysis of equation (2) in case of $\beta = 0$ is well known and carries completely over to the case where $\beta > 0$ and $0 < \alpha + \beta < 1$. It is therefore not repeated here. In summary, there exists a unique long-run equilibrium $\hat{k} > 0$ that is globally asymptotically stable for all initial values $k > 0$. As is usual in a steady state or long-run equilibrium, all growth rates are constant there. For the constancy of k implies $g_k = g_K - \gamma n = 0$ and therefore $g_K = \gamma n$. Since $g_K = sY/K - \delta$, it follows that Y/K must be constant, too. This implies

$$g_Y = g_K = \gamma n. \tag{3}$$

The convergence to the steady state implies that it is reasonable to call the rates in (3) the *long-run growth rates*. Per capita growth follows from $g_{Y/L} = g_Y - n$:

$$g_{Y/L} = (\gamma - 1)n \tag{4}$$

This result is well known for semi-endogenous growth models. As $\gamma - 1 > 0$ if $\beta > 0$, it seems to imply that long-run per capita income growth rates are positive if $n > 0$, zero if $n = 0$, and negative if $n < 0$. It will be shown in the next section, however, that the final statement is only valid as long as n is not too negative.

2.2 Negative population growth and $\boldsymbol{\delta + \gamma n \leq 0}$

The long-run equilibrium condition $\dot{k} = sk^{\alpha+\beta} - (\delta + \gamma n)k = 0$ in case of $\delta + \gamma n \leq 0$ can only be met for a positive k if the saving rate is $s \leq 0$. As Ritschl (1985) has shown for the Solow model, the equilibrium is unstable in this case. As actual saving rates are positive in most developed countries even if their population declines, a positive saving rate appears to be the more interesting case, however.

Figure 1 shows the dynamics of the model for $\delta + \gamma n < 0$ and $s > 0$. As $sk^{\alpha+\beta}$ and $(\delta + \gamma n)k$ do not intersect in the upper part of the figure except for $k = 0$, a long-run equilibrium with positive scale adjusted capital intensity does not exist. According to (2), $\dot{k}$ is the difference between the two curves shown in the upper part of figure 1. Thus, the lower part of the figure shows that $\dot{k} > 0$ for all $k > 0$ and k continues to increase for ever. While there is no finite steady state, the economy can be said to approach an *asymptotic steady state* because the growth rates will be shown to approach constants as $k \to \infty$.

The growth rate of per capita income Y/L follows from logarithmic differentiation of $Y/L = K^{\alpha+\beta}L^{-\alpha}$ as

$$g_Y - g_L = (\alpha + \beta)g_K - \alpha n = (\alpha + \beta)(sY/K - \delta) - \alpha n.$$

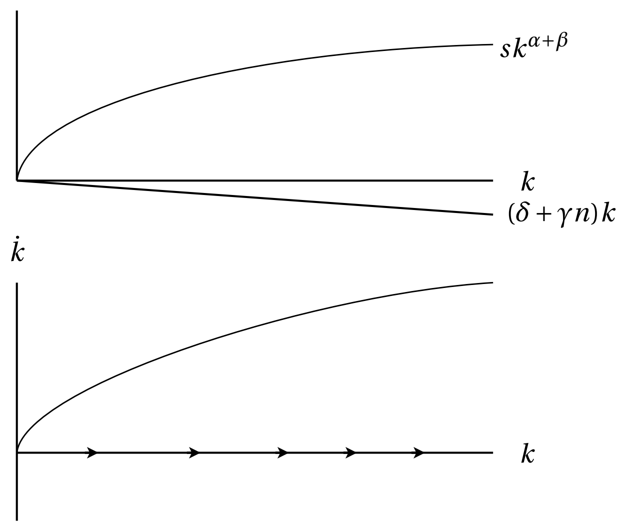

Figure 1. The neoclassical growth model for $0<\alpha+\beta<1$, $s>0$ and $\delta+\gamma n<0$

Using $sY/K = sy/k = sk^{\alpha+\beta}/k = sk^{\alpha+\beta-1}$ yields

$$g_{Y/L} = (\alpha+\beta)sk^{\alpha+\beta-1} - (\alpha+\beta)\delta - \alpha n \tag{5}$$

Since figure 1 shows that $k\to\infty$ as time approaches infinity one gets the constant per capita growth rate in the asymptotic steady state

$$g_{Y/L} = -(\alpha+\beta)\delta - \alpha n \tag{6}$$

This rate is positive if

$$n < -\frac{\alpha+\beta}{\alpha}\delta,$$

while the assumption $\delta+\gamma n<0$ implies that

$$n < -\frac{\delta}{\gamma} = -\frac{1-\alpha-\beta}{1-\alpha}\delta.$$

Observe that $-(\alpha+\beta)/\alpha < -(1-\alpha-\beta)/(1-\alpha)$. These results imply an astonishing non-monotonous dependency of the long-run per capita income growth rate on n, shown in figure 2.

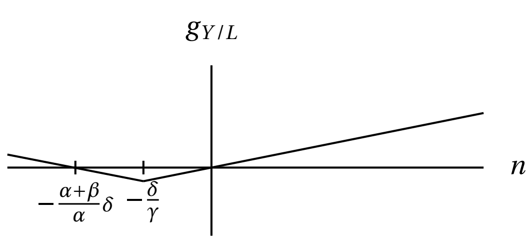

Figure 2. Dependency of long-run per capita income growth rates on n

As long as $\delta+\gamma n>0$, that is if $-\delta/\gamma<n\le 0$, the analysis in the preceding section leading to (4) is valid and $g_{Y/L}=(\gamma-1)n<0$ if $n<0$. The economic reasoning underlying this result is that a steady state with a constant value of $k = K/L^\gamma$ exists where K decreases faster than L (if $\gamma > 1$) in order to keep k constant. Thus, per capita income decreases. If, however, $\gamma n \leq -\delta$, the steady state ceases to exist and the long-run growth rate is $g_{Y/L} = -(\alpha+\beta)\delta - \alpha n$ according to (6).[5] Both K and L decrease. Ceteris paribus, the decrease in K implies a smaller income per capita while the decrease in L implies a higher income per capita. As long as $\alpha n > -(\alpha+\beta)\delta$, the detrimental effect $-(\alpha+\beta)\delta$ of a decreasing capital stock dominates the advantageous effect αn of a decreasing population and the per capita income growth rate is negative. This rate becomes positive, however, if $\alpha n < -(\alpha+\beta)\delta$, since the advantageous population effect then dominates the detrimental capital effect. The growth rate of population must be very negative for this latter result as it must overcompensate the depreciation of capital that even involves an externality measured by β. Growth continues to be semi-endogenous in any case since the long-run growth rates do not depend on the saving rate but only on exogenous production parameters and population growth.

It should be observed that it is a positive rate of depreciation that makes the dynamics interesting. In case of positive population growth δ can often be neglected as it does not change anything substantial concerning the long-run dynamics. In the present case, however, $\delta = 0$ would imply that the two numbers in figure 2 are both zero and the kink in the diagram would be at the origin. A region of negative per capita growth rates would not exist.[6]

As has been noted before, the Cobb-Douglas-Solow model is the special case where $\beta = 0$. Setting $\beta = 0$ in the computed growth rates in (4) and (6) and in the numbers in figure 2 yields figure 3. Notice that per capita income growth is always zero if $n \geq -\delta$ and positive if $n < -\delta$. The semi-endogenous case therefore yields much more plausible dynamics.

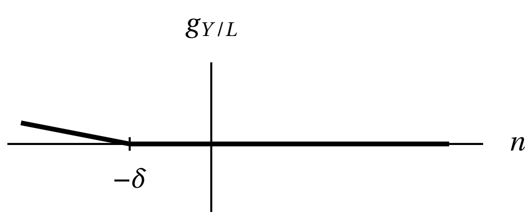

Figure 3. Dependency of long-run per capita income growth rates on n for $\beta = 0$

Figures 2 and 3 are concerned with the asymptotic steady state. It should be recalled, however, that a finite steady state does not exist and that such an asymptotic steady state will never be reached.[7] The per capita growth rate off of the steady state has been calculated in (5). For the case where $n < 0$ and $\delta + \gamma n \leq 0$ the asymptotic steady state is always infinitely far away. The growth rate provided in (5) is therefore the reasonable description of long-run growth in this case. In this sense, semi-endogenous growth becomes endogenous because this rate depends on the saving rate s. By increasing its saving rate the economy is able to increase its long-run per capita income growth.

[5]Observe that the growth rates in (4) and (6) both equal $g_{Y/L} = -\frac{\beta}{1-\alpha}\delta$ if $n = -\delta/\gamma$.

[6]The importance of the rate of depreciation in case of negative population growth is also emphasized by Ferrara (2011).

[7]Even a finite steady state at some $\hat{k}$ will not be reached in finite time if the economy starts off of the long-run equilibrium. However, it is always possible to determine the finite time in which the economy puts aside say one half of the way to the steady state. In contrast, an asymptotic steady state is always infinitely far away.

3 Conclusion

This paper analyzes negative population growth rates and adds to existing results on the Solow model and the AK-model by considering a simple but typical model of semi-endogenous growth. The results are astonishing as per capita income growth rates turn out to vary in a non-monotonous way with population growth rates. It also shows that depreciation is important as it implies that there is a region of negative per capita income growth rates that will be relevant unless the population growth rates are very negative.

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