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Complexity and bidder behavior in iterative auctions

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Abstract

We extend the finite automata approach to evaluate complexity of strategies in iterative adjustment processes such as auctions. Intuitively, a strategy's complexity is equal to the number of different contingencies in which qualitatively different behaviors are prescribed. Complexity may explain bidder choice of strategies in multi-unit iterative auctions.

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1. Motivation and approach

Open iterative adjustment processes, such as multi-object ascending auctions, are often argued to have an advantage over their sealed-bid analogs in terms of allocative efficiency, and sometimes revenue raising properties as well (Milgrom 2000). Yet, these auctions may be also prone to efficiency-reducing behaviors, such as bidder collusion (Cramton 1988), jump bidding (Isaac et al. 2007), or demand reduction (Kagel and Levin 2001). Understanding bidder choice of strategies in iterative auctions is therefore important.

One explanation for bidder choice of strategies is strategy complexity. Experimental evidence indicates that people favor less complex choices in many settings (Sonsino et al. 2002). We suggest a way to evaluate complexity of strategies in iterative auctions and show that competitive bid-according-to-value strategies are simple, which may make them focal. Other strategies, such as collusion, are more complex, but may be still adopted by bounded-complexity bidders in relatively simple settings.

There are many aspects to strategy complexity in auctions, including computational complexity (Cramton et al. 2006), communication complexity (Grigorieva et al. 2006) and complexity of learning optimal strategies. We focus on complexity of implementing strategies, and extend the finite automata approach previously applied to repeated games (Abreu and Rubinstein 1988; Kalai and Stanford 1988) to iterative auctions. Whereas most of the existing research on strategy complexity is constrained to finite action spaces and considers repeated settings,[1] we extend the finite automata approach to admit a continuum of actions (bids), and to allow bids to depend on the current auction prices. This is done by measuring strategy complexity by the number of distinct bidding rules, or bidding functions, that a bidder may use to determine bids given current auction prices. Intuitively, a strategy's complexity is equal the number of different contingencies in which qualitatively different behaviors are prescribed. We illustrate by comparing complexities of competitive and collusive strategies in a multi-object ascending-price auction. The note is largely exploratory in nature.

2. Complexity of multi-object ascending auctions

We discuss a simple case of a multi-object non-combinatorial ascending auction where bidder valuations are separable across goods. Extensions to auctions allowing for package bidding and other potential applications are discussed at the end of this note.

2.1 The model

Let $K = \{1, .., k\}$ be the set of objects for sale, and $N = \{1, .., n\}$ be the set of bidders. For each bidder $i \in N$, let $v_i = (v_{i1}, ...v_{ik})$ be the vector of his values for the goods, and assume v_i belongs to a compact subset of R^k_+. The institution is simultaneous ascending price auction (Brusco and Lopomo 2002), which proceeds in rounds. In each

[1]Sabourian (2004) and Gale and Sabourian (2005) study complexity of strategies in bargaining games where some elements of strategy spaces (prices) are continua.

round $t = 1, 2, ...,$ and for each object $j \in K$, each bidder $i \in N$ submits a bid $b_{ij} \geq 0$; to be acceptable, the bid has to exceed the current price of the object p_j, by at least a minimum bid increment $\delta > 0$. (Without loss of generality, we may interpret $b_{ij} = 0$ as "no bid.") The price of good j in round t, p_j^t, is defined recursively by $p_j^1 = 0$, and $p_j^t \equiv \max\{p_j^{t-1}, \max_{i \in N} b_{ij}^{t-1}\}$, for $t \geq 2$. Let $p^t = (p_1^t, .., p_k^t)$ denote a current price vector, and let $P \subseteq R_+^k$ be the set of all possible prices. The auction ends when no acceptable bids are submitted in the current round, and each object is then assigned to the highest bidder for this object, who pays the current price. Ties are broken randomly.

For simplicity, assume all bids are observable to all bidders.[2] Bidder i's behavioral (pure) strategy for each round t of the auction is a mapping from the previous history of bids, up to round $(t - 1)$, into the set of bids B_i, with $b_i = (b_{i1}, .., b_{ik}) \in B_i$. Assume $B_i = R_+^k$ for all $i \in N$, and let $B = \times_{i=1}^n B_i$, with $b \in B$ denoting a bid profile.

2.2 Measuring complexity

To evaluate strategy complexity in this iterative auction, we build upon the finite automata approach of the repeated games literature (Abreu and Rubinstein 1988; Kalai and Stanford 1988), with necessary modifications to accommodate for the features of iterative processes.

In the complexity literature on repeated games, the players are assumed to use finite automata (Moore machines) to implement their strategies, and the complexity of a strategy is measured as the number of states in a player's machine, with each state prescribing a distinct action. This approach builds on the precise feature of a repeated game, that is, that the same stage game with a finite number of actions is played in every period.

The repeated games approach to measuring complexity is appealing but can not be adopted without modification to iterative auctions for two reasons. First, as in the model above, iterative auctions are often modeled with actions (bids) defined on convex subsets of Euclidean spaces, implying an infinite and uncountable number of actions in each round. Second, unlike repeated games, iterative processes evolve from round to round, and the set of acceptable bids changes accordingly. Consider, for example, a single-unit ascending bid (English) auction, as described above, and take a simple strategy of bidding the minimal increment above the current price until the latter reaches the bidder value. Since the bid depends on the current price p, then, according to the repeated-game approach to measuring complexity, a separate action, and thus a separate machine state, would have to be assigned to each possible level of p, yielding an infinite complexity measure. Yet, this value-bidding strategy may be described by a simple rule: given the current price, the rule prescribes, for each iteration, either to bid a minimal increment above the outstanding bid, or not to bid at all. Effectively, the rule partitions the set of all possible action states into only two distinct parts, and we can say that this strategy has a rule complexity of two.

[2] A similar modeling framework may be applied to iterative auctions where only current auction prices and identities of highest bidders are observable. It also applies to both complete and incomplete information settings regarding bidder values.

We use the above insight to adopt the finite automaton approach to measure strategy complexity in iterative processes. Bidders use automata to implement their strategies. Even though, unlike repeated games, iterative auctions evolve from round to round, bidders may still apply the same bidding rules at many price levels. We account for this feature by introducing the notion of a *rule state.* Whenever the automaton is in a given rule state, it uses a certain bidding function to determine bids given current auction prices. Complexity of a strategy is measured by the number of rule states in a bidder's automaton.

Formally, a *rule automaton* for bidder i is a four tuple $(F_i, f_i^1, \lambda_i, \mu_i)$. F_i is a finite set of rule states for bidder i, and f_i^1 is the initial rule state. The behavioral function $\lambda_i : F_i \times P \to B_i$ specifies the bid vector b_i that the automaton submits whenever it is in rule state f_i, as a function of the current price vector p. With some abuse of notation, we identify the rule state f_i with the corresponding bidding function $f_i(p)$; to distinguish between different rule states, we require each $f_i(\cdot)$ to be continuous in price p. The function $\mu_i : F_i \times B \to F_i$ governs the transition of the automaton from one rule state to another.

Analogously to the repeated games complexity literature, many bidder strategies in iterative auctions may be represented by such finite rule automata. Therefore, we can define the *rule complexity of a strategy* as the number of rule states in the smallest rule automaton describing it.

2.3 Illustration: comparing competition and collusion

To illustrate, first consider the rule complexity of competitive bid-according-to-value (hereafter, value-bidding) strategy in simultaneous ascending-price auction. Take bidder $i \in N$ and assume she only bids the minimum increment δ above the price in an attempt to win an object. Then, for each object $j \in K$, this strategy has two rule states: the "bidding" state f_{ij}^1, which prescribes $b_{ij} = p_j + \delta$; and the "non-bidding" state f_{ij}^2, which prescribes $b_{ij} = 0$. The automaton is in the bidding state f_{ij}^1 for object j whenever bidder i is not the current highest bidder for the object, and $p_j \leq v_{ij} - \delta$ (this is also the initial state); it is in the non-bidding state f_{ij}^2 otherwise. Since there are two rule states per object and k objects, and the rule states may change independently across objects, then the rule complexity is equal to the number of distinct rule state combinations across objects, 2^k.

Figure 1 illustrates a rule automaton for the value-bidding strategy in a single-object ascending (English) auction.

Observation 1 *The rule complexity of the value-bidding strategy in simultaneous k-object auction is 2^k, and is increasing in the number of objects for sale. This strategy is the least complex among all bidding strategies that prescribe at least two qualitatively different actions per object, and allow for bids that are independent across objects.*

It is well-known that all bidders playing such competitive value-bidding strategies constitutes a subgame-perfect Nash equilibrium (or a Perfect Bayesian Equilibrium, if

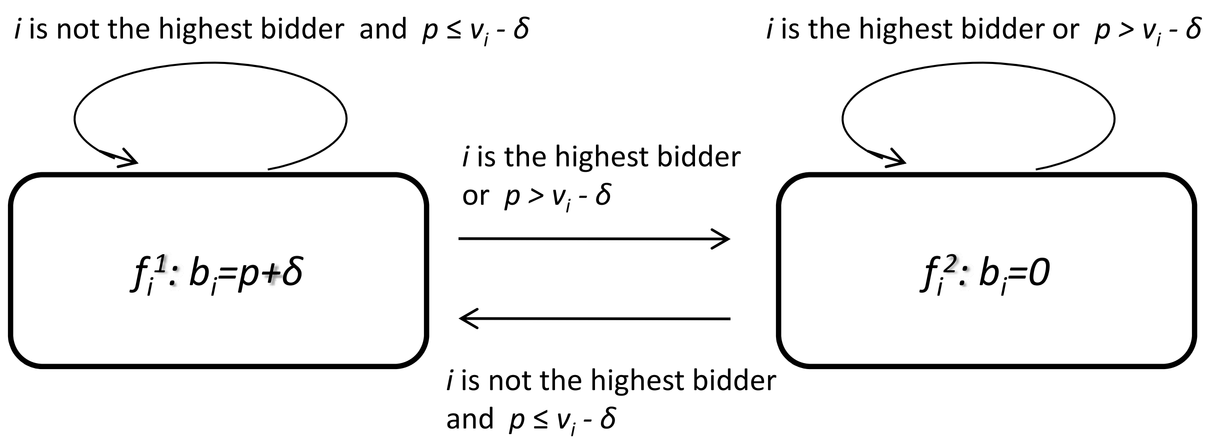

Figure 1: An automaton for value-bidding strategy in ascending English auction.

bidder values are their private information). We next compare this competitive value-bidding strategy with the collusive market splitting strategy, which constitutes an alternative, higher-payoff equilibrium for bidders in simultaneous ascending auctions (Brusco and Lopomo 2002). Under the latter, the bidders split the objects among themselves and buy at minimal (reserve) prices; deviations are deterred by the threat to revert to competitive bidding. Such collusive trigger-type strategies are more complex than competitive ones, because they include both collusive states and punishment states. Let a market-splitting strategy prescribe bidder i to submit the minimal bids for the set $K^i \subseteq K$ of her "designated" objects, and not to bid on other objects, unless a deviation occurs. Hence, there are two collusive rule states: (1) collusive bidding state f_i^1, which prescribes to bid $b_{ij} = \delta$ for $j \in K^i$, and $b_{ij} = 0$ for $j \in K \setminus K^i$ (this is the initial state); and (2) collusive non-bidding state f_i^2, which prescribes not to bid, $b_{ij} = 0$ for all $j \in K$ (this is the state in all but the initial round if no deviations are observed). In addition, there are 2^k competitive punishment states, that the automaton transitions into if a deviation occurs. Thus the rule complexity of any trigger-type collusive strategy is at least $2^k + 2$. Complexity of a collusive strategy may be much higher if collusion calls for an initial bidding phase where the set of designated objects is determined for each bidder.

Observation 2 *The rule complexity of any trigger-type collusive strategy in simultaneous k-object ascending auction is higher than the complexity of the competitive value-bidding strategy.*

3. Discussion

The above analysis indicates that the rule complexity of a strategy in the simultaneous auction increases with the number of objects for sale. Further, any collusive strategy that allows for reversion to competitive bidding is more complex than the competitive bidding strategy. That is, collusion is more complex than competition.[3]

Suppose that, due to bounded rationality, bidders can only use strategies not exceeding a certain complexity bound. Our approach may reasonably explain experimentally observed phenomena that most bidders easily follow the value-bidding strategy in single-object ascending (English) auction, but may have troubles bidding on multiple objects in simultaneous multi-object auctions.[4] Further, it explains why bidders do not always successfully collude in simultaneous ascending auctions, even when collusive equilibria exist (Kwasnica and Sherstyuk 2007).

The suggested approach to measuring complexity may be applied to evaluate complexities of other observed strategies of interest, such as jump bidding (Isaac et al. 2007), demand reduction in multi-unit auctions (Kagel and Levin 2001), or late bidding in hard-close auctions (Ariely et al. 2005). It may also be extended to combinatorial auctions (Milgrom 2007), where strategies are likely to become much more complex.

[3] In a repeated game setting, Fershtman and Kalai (1993) show that firm multi-market operations are more complex than single-market operation, and that collusion between firms is complex.

[4] Isaac and Schnier (2005) note that in simultaneous multiple good silent auctions, it may be costly for bidders to switch bidding attention from one object to another.

The approach has some obvious limitations. As in the repeated game setting, it ignores complexity issues related to computing and learning optimal strategies, and concentrates instead on the complexity of implementing them. Gell-Mann (1995) discusses several general aspects of complexity. Computational complexity of auctions is analyzed, for example, in Cramton et al. (2006, Part 3). The theories of learning in games (Fudenberg and Levine 1998) may help explain complexity of learning issues. Combining the measure of strategic rule complexity with measures of computational and learning complexity may be a promising avenue in understanding behavior of boundedly rational bidders.

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