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Extreme values dependence of risk in Latin American markets

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Abstract

This paper aims to determine which extreme value copula is best suited to the bivariate relationships between shocks of U.S market with Brazilian, Argentine and Mexican markets. We used prices of S&P500, Ibovespa, Merval and IPC from January, 3, 2009 to December, 31, 2010, totaling 483 observations. We estimated Gumbel, Galambos, Husler Reiss and Student's t (extreme-value - TEV) copulas. Results allow concluding that there is a strong dependence on the tails of the joint probability distribution of these markets. Nevertheless, the Gumbel copula for Brazil, Husler-Reiss copula for Argentina and TEV copula for Mexico had the best fit. Thus, its important use an extreme value properly diversify the risk in a portfolio that consist of assets in these countries, especially in turbulence periods.

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1. INTRODUCTION

Managing and monitoring major financial assets are routine for many individuals and organizations. Therefore careful analysis, specification, estimation and forecasting of the dynamics of returns of financial assets, construction and evaluation of portfolios are essential skills in the toolkit of any financial planner and analyst (Caporini and McAleer, 2010). Practitioners and academicians generally use Gaussian processes because of their tractable properties for computation. However, it is well known that asset returns are fat-tailed. Gaussian assumption is also the key point to understand the modern portfolio theory.

Since the introduction of the mathematical theory of portfolio selection (Markowitz, 1952) and of the Capital Asset Pricing Model (CAPM – Sharpe, 1964; Lintner, 1965; Mossin, 1966), the issue of dependence has always been of fundamental importance to financial economics. Within this context, the knowledge of the behavior of correlations and covariances between asset returns is an essential part in asset pricing, portfolio selection and risk management (Baur, 2006).

In the context of international diversification, there is the need for minimizing the risk of specific assets through optimal allocation of resources. Therefore, it is necessary to understand the multivariate relationship between different markets. Thus we need a statistical model able to measure the temporal dependence between shocks of different countries.

An inappropriate model for dependence can lead to suboptimal portfolios and inaccurate assessments of risk exposures. Traditionally, correlation is used to describe dependence between random variables, but recent studies have ascertained the superiority of copulas to model dependence, as they offer much more flexibility than the correlation approach, because a copula function can deal with non-linearity, asymmetry, serial dependence and also the well-known heavy-tails of financial assets marginal and joint probability distribution. Thus, this loss function is a very appropriate choice to a researcher is able to model extreme events, as the issues inherent to the financial risk management (Embrechts *et al*., 2003).

These two difficulties (Gaussian assumption and joint distribution modelling) can be treated as a problem of Copulas. A copula is a function that links univariate marginals to their multivariate distribution. Since it is always possible to map any vector of random variables into a vector with uniform margins, we are able to split the margins of that vector and a digest of the dependence, which is the copula. The concept of copula was introduced by Sklar (1959) and studied by many authors such as Deheuvels (1979), Genest and MacKay (1986).
An important reason to consider other families of copulas in the place of the Gaussian copula is the failure of the correlation approach to capture dependence between extreme events, as shown by Longin and Solnik (2001), Bae *et al*. (2003) and Hartmann *et al*. (2004).

Extreme value copulas arise as the possible limits of copulas of component wise maxima of independent, identically distributed samples. The use of bivariate extreme-value copulas is greatly facilitated by their representation in terms of the Pickands dependence functions (Genest and Segers, 2009). However, up to now no consensus has been reached on which copula family should be used in specific applications. For this some papers (Genest and Remillard, 2008; Genest, Remillard and Beaudoin, 2009) has emerged to explain how to test the accuracy of a specific copula.

Thus, this paper aims to determine which family of extreme value copulas is best suited to the relationship between bivariate shocks of U.S. and Latin American financial markets (Brazil, Argentina and Mexico) considering the period after the recent financial crisis of 2007/2008. These Latin American emerging markets rank among the most mature markets within the universe of emerging countries and they actually attract a particular attention from global investors thanks to their great market openness. This question is investigated testing the fitting performance of copulas for daily prices of S&P500, Ibovespa, Merval and IPC from January, 3, 2009 to December, 31, 2010, totaling 483 observations.

First the residuals were filtered through Dynamic Conditional Correlation Generalized Auto Regressive Conditional Heteroscedasticity (GARCH-DCC), in order to eliminate the serial dependence and capture the dynamic dependence between markets. After we estimated copula families of Gumbel, Galambos, Husler Reiss and Student's *t* extreme-value (TEV), in order to identify, through a selection criterion, which presents the best fit for the sample. A rank-based version of the familiar Cramér–von Mises statistic is employed to determine which family of copula has the best fit to the data studied. This choice was made in the sample because in this paper the financial/economic objective is to model the dependence of the U.S. and Latin markets, in order to verify the risk diversification. In other type of study, as the assessment of the efficiency of the Value at Risk measure, as example, one can extend this test of fit to the out of sample without problems.

The rest of the paper is structured as follows: Section 2 outlines the theoretical background concerning the modelling of Copulas. Section 3 describes the method of the study. Section 4 presents and discusses the results. Finally, section 5 gives the final considerations of the research.

2. COPULAS

This section is divided into two parts: definitions and concepts, introducing the fundamental properties of copulas; and families of extreme value copulas, showing the main classes of copulas used in finance to model financial risk.

2.1 Definitions and concepts

Dependence between random variables can be modeled by copulas. A copula returns the joint probability of events as a function of the marginal probabilities of each event. This makes copulas attractive, as the univariate marginal behavior of random variables can be modelled separately from their dependence (Kovadinovic and Yan, 2010).

The concept of copula was introduced by (Sklar 1959). However, only recently its applications has become clear. A detailed treatment of copulas as well as of their relationship to concepts of dependence is given by Joe (1997) and Nelsen (2006). A review of applications of copulas to finance can be found in Embrechts *et al.* (2003) and in Cherubini *et al.* (2004).

For ease of notation we restrict our attention to the bivariate case. The extensions to the *n*-dimentional case are straightforward. A function $C : [0,1]^2 \rightarrow [0,1]$ is a *copula* if, for $0 \leq x \leq 1$ and $x_1 \leq x_2$, $y_1 \leq y_2$, (x_1, y_1), $(x_2, y_2) \in [0,1]^2$, it fulfills the following properties:

$$C(x,1) = C(1,x) = x, \quad C(x,0) = C(0,x) = 0. \tag{1}$$

$$C(x_2, y_2) - C(x_2, y_1) - C(x_1, y_2) + C(x_1, y_1) \geq 0. \tag{2}$$

Property (1) means uniformity of the margins, while (2), the *n-increasing property* means that $P(x_1 \leq X \leq x_2, y_1 \leq Y \leq y_2) \geq 0$ for (X,Y) with distribution function *C*.

In the seminal paper of Sklar (1959), was demonstrated that a Copula is linked with a distribution function and its marginal distributions. This important theorem states that:

(i) Let *C* be a copula and F_1 and F_2 univariate distribution functions. Then (3) defines a distribution function *F* with marginals F_1 and F_2.

$$F(x,y) = C\big(F_1(x), F_2(y)\big), \ (x,y) \in R^2. \tag{3}$$

(ii) For a two-dimensional distribution function F with marginals F_1 and F_2, there exists a copula C satisfying (3). This is unique if F_1 and F_2 are continuous and then, for every $(u,v) \in [0,1]^2$:

$$C(u,v) = F\left(F_1^{-1}(u), F_2^{-1}(v)\right). \quad (4)$$

In (4), $F_1^{-1}$and F_2^{-1} denote the generalized left continuous inverses of F_1 and F_2. However, as Frees and Valdez (1997) note, it is not always obvious to identify the copula. Indeed, for many financial applications, the problem is not to use a given multivariate distribution but consists in finding a convenient distribution to describe some stylized facts, for example the relationships between different asset returns.

Melo and Mendes (2009) emphasize that in order to measure upper tail dependence one may use the coefficient λ_U defined in (5).

$$\lambda_U = \lim_{\alpha\to 0^+} \lambda_U(\alpha) = \lim_{\alpha\to 0^+} Pr\left(x > F_1^{-1}(1-\alpha) | y > F_2^{-1}(1-\alpha)\right). \quad (5)$$

Provided the limit $\lambda_U \in [0,1]$ exists, and where $F_1^{-1}$and F_2^{-1} denote the generalized left continuous inverses of F_1 and F_2. The lower tail dependence is defined in a similar way. If $\lambda_U = 0$ $(\lambda_L = 0)$, the two variables x and y are said to be asymptotically in the upper (lower) tail.

2.2 Extreme Value Copulas

The most frequently used copulas are Elliptical and Archimedean (Yan and Kojadinovic, 2010). However, it is often reasonable to assume that the dependence structure of a bivariate continuous distribution belongs to the class of extreme-value copulas, because it is more efficient to model financial risk with these copulas, due to the fact that it is precisely in the tails of the returns distribution, that lies the biggest challenge of diversifying a portfolio (Genest *et al*., 2011).

C is an extreme value copula when there exists a function A: [0,1] → [0.5,1] such that for all $(u,v) \in [0,1]^2$, there is a relation as expressed in (6).

$$C(u,v) = uv^{A[\log(v)/\log(uv)]}. \quad (6)$$

It was shown by Pickands (1981) that C is a copula if and only if A is convex and $\max(t, t-1) \leq A(t) \leq 1$. By reference to this work, the function A is often referred to as the Pickands dependence function (Genest and Segers, 2009).

Among the extreme value copulas, which are characterized by capture the tail dependence, we highlight the families: Gumbel, Galambos, Husler Reiss and TEV.

The Gumbel family has been introduced by Gumbel (1960). Since it has been discussed in Hougaard (1986), it is also known as the Gumbel-Hougaard family. Another important reference is Hutchinson and Lai (1990). The Gumbel copula is defined as (7).

$$C(u,v) = exp\left\{-[(-\ln u)^\alpha + (-\ln v)^\alpha]^{1/\alpha}\right\}. \quad (7)$$

In (7), the range for α is$[1,+\infty)$. The coefficient of tail dependence is given by $\lambda_U = 2 - 2^{1/\alpha}$.

Another family of extreme value copulas is the Galambos, which was proposed by Galambos (1987). It is represented by formulation (8).

$$C(u,v) = uvexp\left([(-\ln u)^\alpha + (-\ln v)^{-\alpha}]^{-1/\alpha}\right). \quad (8)$$

In (8), the range for α is [0,+∞). The coefficient of tail dependence is given by $\lambda_U = 2 - 2^{1/\alpha}$.

The Husler Reiss, an extreme value family of copulas, was developed by Hüsler and Reiss (1989). It is described in (9).

$$C(u,v) = exp\left(-\hat{u}\Phi\left[\frac{1}{\alpha} + \frac{1}{2}\alpha\log\left(\frac{u}{v}\right)\right] - \hat{v}\Phi\left[\frac{1}{\alpha} + \frac{1}{2}\alpha\log\left(\frac{u}{v}\right)\right]\right). \quad (9)$$

In formulation (9), $\hat{u} = -\log u$, $\hat{v} = -\log v$, Φ is the cumulative density function of a standard normal. The range of α is [0,+∞). The coefficient of tail dependence is given by $\lambda_U = 2 - 2\,\Phi(1/\alpha)$.

The Student's t family is originally as elliptical class of copula. However, as can be found in Demarta and Mcneil (2005), this copula has an adaptation to extreme values. The TEV copula with correlation ρ and υ degrees of freedom is defined as in (10).

$$C_{\rho,\upsilon}(u,v) = exp\left(\log(uv)A_{\rho,\upsilon}\frac{\log(u)}{\log(uv)}\right). \qquad (10)$$

In (10), $A_{\rho,\upsilon}$ is the Pickands dependence function, based in the bivariate Student's t probability function with correlation ρ and υ degrees of freedom.

3. EMPIRICAL METHOD AND DATA

In order to verify which family of copulas is best suited to the bivariate relationship between residuals (shocks) of United States, Brazil, Argentina and Mexico, we collected data of the daily prices of S&P500, Ibovespa, Merval and IPC, respectively, from January, 3, 2009 to December, 31, 2010, totaling 483 observations. These indices were chosen because they are commonly used in academic papers as proxies for the financial markets in these countries. All are compound by the stocks that are more representative in terms of liquidity and value.

Were initially employ the ADF test (Dickey Fuller Aumented) in both sets of prices as in their differences of logarithms (the returns), to eliminate problems of non-stationarity. The ADF test, proposed by Dickey and Fuller (1981) is represented by (11).

$$\Delta P_t = \gamma P_{t-1} + \sum_{i=1}^{n} \delta_i \Delta P_{t-i} + \varepsilon_t. \qquad (11)$$

In the formulation (11), ΔP_t is the price change at time t, γ and $\delta_i$are constant, and ε_t is a white noise series. If the null hypothesis cannot be rejected, the price series $\{P\}$ contains a unit root, with non-stationarity. The equations are estimated by Ordinary Least Squares and the parameter values are compared to critical values from tables generated by Dickey and Fuller (1981), based on Monte Carlo simulations.

Since there is serial dependence and heteroscedasticity in the series, the results may contain bias of estimation. Thus, was employed the GARCH-DCC model, proposed by Engle and Sheppard (2001) The DCC model is a two-step algorithm to estimate the parameters. In the first stage, the conditional variance is estimated by means of univariate GARCH model, respectively, for each asset. In the second step, the parameters for the conditional correlation are estimated. Finally, the DCC model includes conditions that make the variance matrix positive definite at all points in time and the variance between assets' volatility a stationary process. The DCC model is represented by the formulation (12).

$$H_t = D_t R_t D_t. \qquad (12)$$

In formulation (12), $R_t = diag\left(q_{11,t}^{-1/2} \dots q_{NN,t}^{-1/2}\right) Q_t diag\left(q_{11,t}^{-1/2} \dots q_{NN,t}^{-1/2}\right)$; $D_t = diag\left(h_{11,t}^{1/2} \dots h_{NN,t}^{1/2}\right)$, where $h_{ii,t}$ is defined similarly any univariate GARCH model;

Subsequently, by Q statistic of Ljung and Box (1978), represented for (13), which tests the null hypothesis that the data are random against the alternative of non-randomness of these, we sought to identify the presence of correlation serial on the residuals of the indices.

$$Q = n(n+2)\sum_{k=1}^{h} \frac{\hat{\rho}_k^2}{n-k}. \qquad (13)$$

In (13), n is the size of sample; $\hat{\rho}_k^2$ is the autocorrelation of sample in lag k; h is the number of lags being tested; The Ljung-Box Q statistics follows a chi-squared (χ^2) distribution.

Subsequent to this initial empirical analysis, using the residuals that were obtained through the GARCH-DCC applied to the series, we estimated the families of copulas introduced at section 2. For this the data was standardized into pseudo-observations $\boldsymbol{U}_j = \left(U_{1j}, \dots, U_{ij}\right)$ through the ranks as $U_{ij} = R_{ij}/(n+1)$. The pseudo-observations are not affected by the marginal in their transformation because the ranks are calculated based on the empirical observed data.

The next step was, to estimate the copula's parameters, it was employed the procedure of inversion of the copula based *Kendall's Tau (τ)*, that serves to measure the monotonic dependence, which is calculated as in (14).

$$\tau(x, y) = 4\int_0^1 \int_0^1 C(u, v)dC(u, v) - 1. \quad (14)$$

To determine which copula model best fits the residuals of the markets studied, we applied a rank-based version of the familiar Cramér–von Mises statistic, discussed in Genest, Rémillard and Beaudoin (2009) and extended in Genest *et. al.* (2011), which make it possible to check the validity of the dependence structure separately of the margins. These authors emphasize that it is a blanket test, i.e., a procedure whose implementation requires neither an arbitrary categorization of the data, nor any strategic choice of smoothing method, whether it be kernel- or wavelet-based, or whatever. The goodness-of-fit test employed is defined in (15), tests the null hypothesis that data is fitted by $C_{\theta n}$, a copula with vector of parameters θ.

$$S_n = \int_{[0,1]^d} \mathbb{C}_n(\boldsymbol{u})^2 \, dC_n(\boldsymbol{U}). \quad (15)$$

In (15), $C_n(\boldsymbol{U}) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(U_{i1} \le u_1;\ U_{i2} \le u_2)$ is known as the empirical copula; $\boldsymbol{U}_j = \left(U_{1j}, \dots, U_{ij}\right)$ are the pseudo-observations; $\boldsymbol{u} = (u_1, u_2) \in [0,1]^2$; $\mathbb{C}_n = \sqrt{n}(C_n - C_{\theta n})$ is the empirical process that assess the distance between the empirical copula and the estimation $C_{\theta n}$; n is the number of observations. In practice, the limiting distributions of S_n and depend on the family of copulas under the composite null hypothesis, and on the unknown parameter value θ in particular. This procedure was chosen because it can deal with non-linearity, asymmetry, serial dependence and also the well-known heavy-tails of financial assets. Further, it can verify if the copula family really fit the data, and not just indicate what family is a good model, as the usual AIC, BIC and Log-Likelihood. The pseudo-observations are not affected by the marginal in their transformation because the ranks are calculated based on the empirical observed data.

4. RESULTS

Initially, we performed the ADF test of unit root in both series in level and first difference of logarithm (daily returns). Results are shown in Table 1.

Table 1. Unit Root test of USA, Brazil, Mexico and Argentina.

Variable	ADF test	p-value
S&P500	-0.6020	0.8679
Ibovespa	-1.6507	0.4565
Merval	0.6752	0.9917
IPC	-0.3236	0.9191
Δln(S&P500)	**-15.0330**	0.0000
Δln(Ibovespa)	**-15.7937**	0.0000
Δln(Merval)	**-15.8186**	0.0000
Δln(IPC)	**-14.8784**	0.0000

*****Bold values are significant at the 1% level

As the presence of a unit root in all series was confirmed, we calculated the daily returns by the difference of logarithms of prices. Table 2 displays the descriptive statistics of these returns, whereas Figure 1 shows the temporal evolution of these series.

The results in Table 2 confirm the fact that Brazil, Argentina and Mexico being emerging countries, should have a higher standard deviation, representing greater risk and therefore requiring higher returns, as it is verified by higher values for mean and median. The U.S. market, by contrast had lower mean and median and lower standard deviation of returns, representing a more stabilized economy. It is also noticed that all sets of returns are leptokurtic, a fact quite common, being widely recognized by financial professionals.

Table 2. Descriptive statistics of daily returns of Ibovespa and S&P500.

Statistic	**Δln(USA)**	**Δln(Brazil)**	**Δln(Argentina)**	**Δln(Mexico)**
Mean	0.0007	0.0011	0.0025	0.0011
Median	0.0012	0.0014	0.0026	0.0020
Minimum	-0.0543	-0.0540	-0.0770	-0.0563
Maximum	0.0684	0.0638	0.0712	0.0618
Standard deviation	0.0151	0.0171	0.0201	0.0142
Skewness	-0.0734	0.0432	-0.1404	0.0428
Kurtosis	5.3436	4.6304	4.7111	6.0122

The results in Table 2 confirm the fact that Brazil being an emerging country, should has a higher standard deviation, representing greater risk and therefore requiring higher returns, as is verified by higher values for mean and median. The U.S. market, by contrast had lower mean and median and lower standard deviation of returns, representing a more stabilized economy. It is also noticed that both sets of returns are leptokurtic and negatively asymmetrical, a fact quite common, being widely recognized by financial professionals.

Figure 1 endorses these results. It confirms visually the greater dispersion of the daily returns of the Latin American markets compared to the U.S. It is noteworthy that there is a volatility cluster at begin of the observations, extending for about 100 trading days. It was the vestiges of the American financial crisis.

Subsequently, it was estimated a GARCH-DCC model to obtain the residuals and filter their serial dependence. The information about this model will be omitted due to lack of space. In addition, Table 3 presents the statistics Q of the residues obtained from the estimated GARCH-DCC model.

The results in Table 3 suggest that the estimated residuals from the GARCH-DCC model do not exhibit significant serial correlation. Thus, such residuals may be used for estimation of families of copulas proposed in this study.

After this initial empirical analysis of the indexes presented in this paper, the parameters of the copulas Gumbel, Galambos, Husler-Reiss and TEV were estimated, through inversion of Kendall's Tau. Then, there was statistically verified the goodness of fit of the estimated copulas by the S_n test exposed in the method of this study. The results of the estimated parameters, as well as values and significance of the S_n tests are shown in Table 4.

The results presented in Table 4 support the conclusion that, after the American financial crisis, the null hypothesis was rejected, at the 5% level of significance, of fit the bivariate relationships the copula Husler-Reiss for the relationship of U.S and Mexican markets. Nevertheless, Mexico had the largest value for the measure of monotonic dependence Kendall's Tau, obtaining the value of 0.297.

Regarding the goodness of fit of the families of copulas estimated, it had the lowest value for the S_n statistical, and consequently the least distance between the hypothetical and empirical copula, the Gumbel copula, for the relationship of U.S. market with Brazil, Husler-

Reiss copula with the Argentine market and, TEV copula for the Mexican market. In the case of the Argentine and Mexican markets, the Gumbel copula obtained very similar fit to that of the copulas with lower S_n test value. This result implies that after the period of turbulence, i.e. the beginning of recovery of these financial markets, the modeling of the joint probability distribution of its residuals showed some similarities, corroborating with the study of Righi and Ceretta (2011). It is noteworthy that although it has the lowest value for the monotonic dependence measures, the Argentine market had fitted all the copulas estimated to the relationship with the U.S. market.

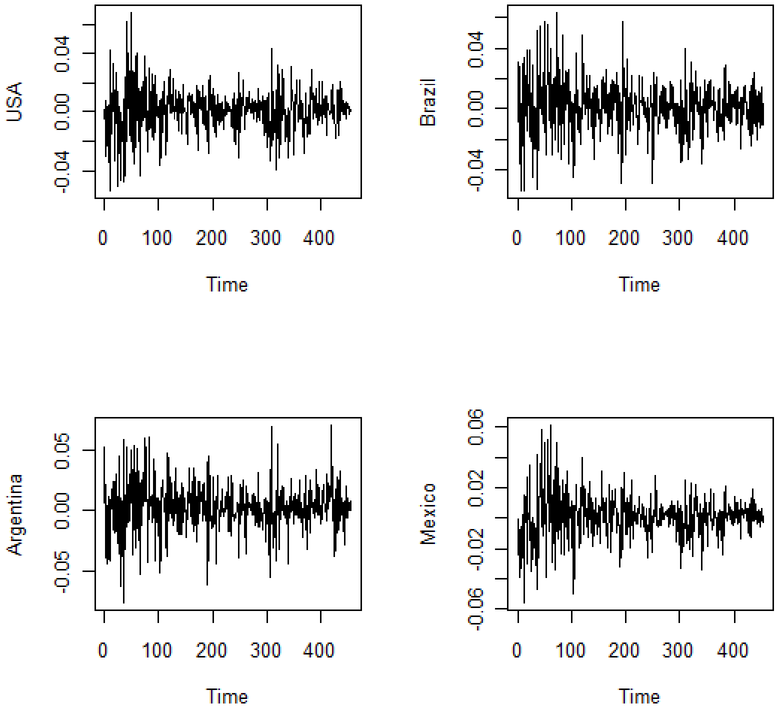

Figure 1. Time series of daily returns of S&P500 (USA), Ibovespa (Brazil), Merval (Argentina) and IPC (Mexico).

Table 3. Ljung-Box Q statistic for residuals of daily returns of S&P500 (USA), Ibovespa (Brazil), Merval (Argentina) and IPC (Mexico) after crisis period estimated by GARCH-DCC.

	USA/Brazil		**Brazil**		**USA/Argentina**		**Argentina**		**USA/Mexico**		**Mexico**	
Lag	Q-test	sig.	Q-test	sig.	Q-test	sig.	Q-test	sig.	Q-test	sig.	Q-test	sig.
1	0.038	0.845	0.260	0.610	0.686	0.407	0.932	0.334	0.046	0.830	0.830	0.362
2	0.885	0.643	0.288	0.866	0.710	0.701	1.272	0.529	0.215	0.898	1.139	0.566
3	1.871	0.600	0.336	0.953	0.993	0.803	1.899	0.594	0.358	0.949	1.728	0.631
4	2.320	0.677	0.337	0.987	1.188	0.880	1.946	0.746	0.435	0.980	1.762	0.779
5	2.491	0.778	0.630	0.987	1.279	0.937	1.961	0.855	0.444	0.994	1.787	0.878
6	2.518	0.866	0.697	0.995	1.375	0.967	2.178	0.903	0.951	0.987	1.994	0.920
7	2.568	0.922	0.914	0.996	1.435	0.984	2.178	0.949	3.384	0.847	1.997	0.960
8	3.186	0.922	0.992	0.998	2.047	0.980	2.918	0.939	4.773	0.782	2.666	0.954
9	3.246	0.954	1.930	0.993	2.086	0.990	4.130	0.903	5.023	0.832	3.862	0.920
10	3.725	0.959	2.041	0.996	2.418	0.992	4.328	0.931	6.340	0.786	4.036	0.946

* None of the values are significant at 5% level.

Table 4. Estimated parameters of the copulas, values and significance of the S_n tests in the bivariate relationships of USA, Brazil, Argentina and Mexico.

	USA-Brazil Tau=0.197			**USA-Argentina** Tau=0.035			**USA-Mexico** Tau=0.297		
Copula	Parameter	S_n test	Sig.	Parameter	S_n test	Sig.	Parameter	S_n test	Sig.
Gumbel	1.2450	0.0064*	0.9356	1.0361	0.0350	0.5459	1.4234	0.0649	0.1094
Galambos	0.5007	0.0113	0.8257	0.2249	0.0341	0.5509	0.6938	0.0757	0.0554
Husler-Reiss	0.8662	0.0148	0.7388	0.5000	0.0339	0.5439	1.1045	**0.0850**	0.0454
TEV	0.5021	0.0071	0.9256	0.0203	0.0602	0.2692	0.6783	0.0648	0.1004

Bold values are significant at 5% level. * indicates the better fit.

This result further highlights the importance of risk management in terms of international diversification. This is because such a higher concentration of probability in the tails, in particular for lower values indicate empirically that shocks of the Latin and U.S. markets have dependence above the normally expected in the extreme values of their distributions. This increased joint probability in the tails shows that it is difficult to minimize the risk of a portfolio based on asset allocation in these countries, especially in times of negative innovations, which is exactly when active managers need to protect their investments.

5. CONCLUSION

Using data of the daily prices from the proxies of Brazilian, Argentine, Mexican and American markets, we calculated their returns and residuals. We latter estimated a GARCH-DCC model in order to filter the serial dependence of the data. With the residuals, by reversing the measure of association, Kendall’s Tau, we estimated the following copulas: Gumbel, Galambos, Husler-Reiss and TEV. Based on this estimation it was found, initially, a high degree of association between shocks in both countries, as indicated by scatter plots and significant linear correlation.

Nevertheless, the main objective of this paper was verify which of the estimated copulas had the best fit to joint distribution of residuals of both markets. To that end, we used the rank-based version of the familiar Cramér–von Mises statistic. The Gumbel, for the relationship of U.S. market with Brazil, Husler-Reiss with the Argentine market, and TEV for the Mexican market. In the case of the Argentine and Mexican markets, the Gumbel copula obtained very similar fit to that of the copulas with lower S_n test value. This result can be explained by the fact that not only returns, but also residuals of financial assets possess fat tails, which are best represented by probability distributions that have more concentration in the extreme values, compared with others probability functions.

This result allows us to conclude that, such joint dependence of the residuals, especially in the tails of the distribution, make important to properly diversify the risk in a portfolio that consist of assets in these countries. This situation is aggravated by the fact that it is precisely against extreme shocks such as those raised by their tails (especially the left side) of the probability distribution that active managers and administrators need to worry in a daily basis.

Finally, it is suggested for future studies to test the goodness of fit of extreme value copulas in residuals of other emergent countries, as well calculate Value at Risk (VaR) of portfolios composed by assets of these markets with their joint distribution function represented by a copula.

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