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Tax Burden and GDP: Evidence from Frequency Doman Approach for the USA

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Abstract
We employed Breitung and Candelon’s (2006) frequency domain approach to investigate the short-and long-run Granger-causality from different tax burden to GDP in the USA for the period 1947:1 –2009:3. The frequency domain analysis shows that current receipts, personal current tax, taxes on production and imports and taxes on corporate income do not Granger-cause GDP, both at the short and high frequency level; however, current tax receipts Granger-cause GDP in the frequency range of (0.9,1.9), corresponding to the cycle of 0 to 3 months to 7 months. These results suggest that when the USA looks forward to rebalancing her GDP, by means of taxation, it is preferable to reconsider the tax structure with a focus on current tax receipts. This is so because by changing the structure of current tax receipts, the USA will be able to earn more revenue, even in the initial stage. However, if the USA decides to increase welfare, with the stability and sustainability of GDP, the policy makers are advised to re-adjust the tax burden by infusing the changes of the current receipts, personal current tax, taxes on production and imports and taxes on corporate income. 

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1. Introduction

The present empirical study discusses both the short-and long-run Granger-causal relationships between alternative taxes and GDP. Theoretically, changes in the structure of tax affect output and economic growth. Tax reforms that reduce the marginal tax rate or replace the income tax with a consumption type tax, enable a country to experience increased work effort, saving, and investment and thus faster economic growth (Engen and Skinner 1996). However, if tax cuts fail to produce the projected boost in economic growth, tax revenues could decline, putting upward pressure on the deficit, leading to lower national saving and lagging economic growth (Engen and Skinner 1996). While Neoclassical Growth models hold that taxes (particularly, income tax) have a temporary effect, the Endogenous Growth models hold that tax rates affect the long-term growth rate or steady state growth rate. This study investigates the existence of a tax burden-induced economic growth (proxied by GDP) and explores how changes in tax burden distribution can affect economic growth. This enables us to provide an indicative tax policy for the United States, which influences GDP and hence economic growth. There are numerous studies using different data sets from various countries on the role of taxes as a determinant of GDP and/or economic growth. Mention must be made of Koester and Kormendi (1989), Levine and Renelt (1992), Easterly and Rebelo (1993), Mendoza et al. (1997) and Kneller et al. (1999). These studies have found that there is either a positive or, in most cases, an insignificant correlation, between the tax rate and output. In contrast, Kormendi and Meguire (1985), King and Rebelo (1990), Barro (1991), Wright (1996) and Leibfritz et al. (1997) found a negative correlation between tax and GDP. Padovano and Galli (2002) found that though tax rate had no significant growth effects, but there was a negative impact of marginal tax rates and tax progressivity on economic growth. Mamatzakis (2005) showed that in the case of Greece, though economic growth responded negatively to an increase in tax burden, there was existence of a positive impact of the tax mix on economic growth. Karagianni et al. (2012) examined the causal relationship between different types of tax rate and GDP growth of the USA, using the nonlinear Granger-causality test developed by Hiemstra and Jones (1994) and Diks and Panchenko (2006) for the period 1948:1–2008:4 and found that tax on production and imports, and tax on corporate income Granger-caused GDP.

This paper extends the body of literature (especially Karagianni et al., 2012) in the two directions: First, we increased our sample size of analysis and second, we provided evidence of the short-and long-run Granger-causality between different types of tax and GDP growth using the frequency domain approach of Breitung and Candelon (2006), which the earlier work had ignored. The motivation for extending the work lies in a timely question on the budget problems currently being faced by the USA.

2. Data and Methodology: Causality Test in Frequency Domain

The study takes quarterly seasonally adjusted data at annual rates for the period 1947:1 to 2009:3. The data are obtained from the US Bureau of Economic Analysis Statistics and are expressed in constant 2005 prices. The variables of analysis are GDP, Current Receipts (CR), Current Tax Receipts (CTR), Personal Current Taxes (PCT), Taxes on Production and Imports (TPI), and Taxes on Corporate Income (TCI). We have taken level data instead expressing taxes.
as a ratio of GDP, because using tax variables as fraction of GDP can lead to a mechanical relationship\(^1\) between GDP and the tax rate which leads to misleading results.

It is important to mention that the Granger (1969) approach to the question of whether \(X\) cause \(Y\) is to determine how much of the current \(Y\) can be explained by past values of \(Y\), and then to see whether adding lagged values of \(X\) can improve the explanation. \(Y\) is said to Granger-cause \(X\) if, \(X\) helps in the prediction of \(Y\), or if the coefficients on the lagged \(X\)’s are statistically significant.\(^2\) Further, Granger-causality measures precedence and information content, but does not indicate causality in its conventional sense. Importantly, the extent and direction of causality differs between frequency bands (Granger and Lin, 1995). The fact that a stationary series is effectively the sum of uncorrelated components, each of which is associated with a single frequency ordinate, allows the full causal relationship to be decomposed by frequency (Lemmens et al., 2008). The traditional approach to Granger-causality tacitly ignores the possibility that the strength and/or direction of the Granger-causality, (if any) can vary over different frequencies (Lemmens et al., 2008). Granger (1969) was the first study to give the idea of further disentangling the Granger-causality relationship between two time series. Granger (1969) also suggested that a spectral-density approach would give a better-off and more complete picture than a one shot Granger-causality measure that is supposed to apply across all periodicities (for example, in the short run, over the business-cycle frequencies, and in the long run). Therefore, in the study we have taken the Breitung and Candelon’s (2006) approach to Granger-causality in the frequency domain.\(^3\) This work by Breitung and Candelon’s (2006) is based on the work of Granger (1969) and Geweke (1982), which is slightly different from the other approaches such as the Partial Directed Coherence (PDC) measure. The approach provides an elegant interpretation of the frequency-domain Granger-causality as a decomposition of the total spectral interdependence between two series (based on the bivariate spectral density matrix, and directly related to the coherence) into a sum of “instantaneous”, “feedforward” and “feedback” causality terms. The innovativeness of this measure of Granger-causality is that it can be applied across all periodicities (e.g., in the short run, over the business-cycle frequencies, and in the long run) and hence, one can get to know exactly for which periodicity one variable can Granger-cause the other, which the popular one shot measure of Granger-causality test (developed either in linear or nonlinear framework) fails to measure. Though this approach of analysis has been used in quite a few studies, its scope is limited to studies in the area of monetary policy and stock market. Mention can be made of Assenmacher-Wesche and Gerlach (2007), Assenmacher-Wesche and Gerlach (2008a,b), Assenmacher-Wesche et al. (2008), Lemmens et al. (2008), Gronwald (2009) and António (2010) in this context. To the best of our knowledge this the first study in the area of public finance that applies such an innovative approach. The Breitung and Candelon (2006) approach can be explained as follows:

\(^1\) Since, we have only a two variable case, and if one of those is expressed as a ratio of the other, the increase/decrease in the other will decrease/increase the first one. For example, if taxes are expressed as ratios of GDP, increase (decrease) in GDP will decrease (increase) the tax-GDP ratio (provided variants of tax either remain constant or does not increase proportionally higher than the GDP).

\(^2\) Putting it differently, a process \(X\) is said to Granger-cause another process \(Y\), when the variance of the error in forecasting future values of \(Y\), using an (optimal) forecast based on the observed values of both \(X\) and \(Y\), is strictly smaller than the variance of the prediction error, using an (optimal) forecast based only on the observed values of \(Y\).

\(^3\) In statistics, frequency domain describes the domain for analysis of mathematical functions or signals with respect to frequency, rather than time. A very similar definition holds for the Granger causality, as in the time domain. Speaking non-technically, a time-domain graph shows how a signal changes over time, whereas a frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies.
Let \( z_t = [x_t, y_t]' \) be a two-dimensional vector of time series observed at \( t = 1, \ldots, T \) and it has a finite-order VAR representation of the form

\[
z_t = \Theta(L)z_t
\]

where \( \Theta(L) = I - \Theta_1 L - \ldots - \Theta_p L^p \) is a \( 2 \times 2 \) lag polynomial with \( L^k z_t = z_{t-k} \). We assume that the error vector \( \varepsilon_t \) is white noise with \( E(\varepsilon_t) = 0 \) and \( E(\varepsilon_t \varepsilon_t') = \Sigma ; \) where \( \Sigma \) is positive definite. For ease of exposition we neglect any deterministic terms in (1).

Let \( G \) be the lower triangular matrix of the Cholesky decomposition \( \Sigma = G'G \) such that \( IE_{tt} = (G')' \eta_t \) and \( \varepsilon_t = \psi(L) \eta_t \). If the system is assumed to be stationary, the Moving Average (MA) representation of the system is

\[
z_t = \Phi(L)\varepsilon_t = \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \]

\[
= \psi(L)\eta_t = \begin{bmatrix} \psi_{11}(L) & \psi_{12}(L) \\ \psi_{21}(L) & \psi_{22}(L) \end{bmatrix} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix}
\]

where \( \Phi(L) = \Theta(L)^{-1} \) and \( \psi(L) = \Phi(L)G^{-1} \). Using this representation the spectral density of \( x_t \) can be expressed as

\[
f_x(\omega) = \frac{1}{2\pi} \left\{ |\psi_{11}(e^{-i\omega})|^2 + |\psi_{12}(e^{-i\omega})|^2 \right\}.
\]

The measure of causality suggested by Geweke (1982) and Hosoya (1991) is defined as

\[
M_{y \rightarrow x}(\omega) = \log \left[ \frac{2\pi f_x(\omega)}{|\psi_{11}(e^{-i\omega})|^2} \right] = \log \left( 1 + \frac{|\psi_{12}(e^{-i\omega})|}{|\psi_{11}(e^{-i\omega})|} \right).
\]

If \( |\psi_{12}(e^{-i\omega})|^2 = 0 \), then the Geweke’s measure will be zero, then \( y \) will not Granger-cause \( x \) at frequency \( \omega \).

If the elements of \( z_t \) are I(1) and cointegrated, in that case in the frequency domain the measure of causality can be defined by using the orthogonalized MA representation

\[
\Delta z_t = \tilde{\Phi}(L) \varepsilon_t = \tilde{\psi}(L) \eta_t,
\]

where \( \tilde{\psi}(L) = \tilde{\Phi}(L)G^{-1} \), \( \eta_t = G\varepsilon_t \), and \( G \) is a lower triangular matrix such that \( E(\eta_t \eta_t') = I \). Note that in a bivariate cointegrated system \( \beta' \tilde{\psi}(L) = 0 \), where \( \beta \) is a cointegration vector such that \( \beta'z_t \) is stationary (Engle and Granger, 1987). As in the stationary case the resulting causality measure is

\[
M_{y \rightarrow x}(\omega) = \log \left( 1 + \frac{|\tilde{\psi}_{12}(e^{-i\omega})|}{|\tilde{\psi}_{11}(e^{-i\omega})|} \right).
\]

To test the hypothesis that \( y \) does not cause \( x \) at frequency \( \omega \) we consider the null hypothesis
within a bivariate framework. Breitung and Candelon (2006) present this test by reformulating the relationship between \( x \) and \( y \) in Vector Autoregressive (VAR) equation:

\[
x_t = a_1 x_{t-1} + \ldots + a_p x_{t-p} + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + \varepsilon_t
\]

The null hypothesis tested by Geweke, \( M_{y \rightarrow x}(\omega) = 0 \), corresponds to the null hypothesis of \( H_0 : R(\omega)\beta = 0 \)

\[
R(\omega) = \begin{bmatrix}
\cos(\omega) & \cos(2\omega) & \ldots & \cos(p\omega) \\
\sin(\omega) & \sin(2\omega) & \ldots & \sin(p\omega)
\end{bmatrix}
\]

The ordinary \( F \) statistic for (11) is approximately distributed as \( F(2, T - 2p) \) for \( \omega \in (0, \pi) \). It is interesting to consider the frequency domain Granger-causality test within a cointegrating framework. To this end Breitung and Candelon (2006) suggested to replace \( x_t \) in regression (10) by \( \Delta y_t \), with the right-hand side of the equation remaining the same (see Breitung and Candelon, 2006, for more detailed discussion on this and for the case when one variable is \( I(1) \) and other is \( I(0) \)). Further, it is important to mention that in cointegrated systems the definition of causality at frequency zero is equivalent to the concept of “long-run causality” and in stationary framework there exists no long-run relationship between time series, a series may nevertheless explain future low frequency variation of another time series. Hence, in a stationary system, causality at low frequencies implies that the additional variable is able to forecast the low frequency component of the variable of interest one period ahead.

3. Empirical Findings

The study begins with the tests of stationarity and cointegration. We find that all the variables under analysis are integrated of order one (i.e., \( I(1) \)) and variants of tax are not cointegrated with GDP.\(^4\) Therefore, for analysis of Granger-causality, we transformed data into the first difference form which makes the variables stationary and gives efficient results. We incorporate different types of taxes to examine the Granger-causality of alternative tax burden distribution on GDP. The results are presented in Appendix 2, Figure-1 to Figure-5. These figures report the test statistics, along with their 5\% critical values (broken lines) for all frequencies \( (\omega) \) (which are expressed as fraction of \( \pi \) ) in the interval \((0, \pi)\). The frequency, \((\omega)\), on the horizontal axis can

\(^4\) A descriptive statistics and graphical plot of the variables analyzed are presented in Appendix 1. For testing of stationarity property of data series, we used ADF and PP test. Cointegration was tested using Johansen’s (1988) test. The choice of the appropriate specification was based on the Pantula principle (Johansen, 1992, 1995). On the basis of the Pantula principle, we estimate three models. These models are model 2, includes intercept in the cointegration relation; Model 3 which allows deterministic trends in level; and Model 4 which allows for trend in the cointegration space. The test procedure then was to move through from the most restrictive model to less and at each stage compare the trace test statistic to its critical value. The selection process only stopped at the first time where the null hypothesis was not rejected. In our case, following Pantula principle, our selection process was stopped in Model 3 with rank zero. Using this model of cointegration, we did not find cointegration relationship between the test variables we were interested in. The results of unit root and cointegration are not presented here but can be accessed from the author.
be translated into a cycle or periodicity of $T$ months by $T = \frac{2\pi}{\omega}$ where $T$ is the period.\(^5\) Note that since high frequencies are having short periods and vice versa, figures of Granger-causality at frequency domain stand reversed, with short-term fluctuations/cycles at the right end and long-term movements/cycles at the left.

First, we analyzed the short-and long-run Granger-causality from CR to GDP, and reported its result in Figure 1. Figure 1 shows that CR does not Granger-cause GDP, both at the short-and high-frequency level, as the null hypothesis of no predictability is not rejected at 5% level of significance for all frequencies in the interval $(0, \pi)$. This implies that CR is unable to forecast the low and high frequency component of GDP, one period ahead. Subsequently, we analyzed the short-and long-run Granger-causality from CTR to GDP and reported its result in Figure 2. It is evident from Figure 2 that the null hypothesis of no predictability is rejected at 5% level of significance in the range $\omega \in (0.9,1.9)$. This implies that CTR is able to forecast the intermediate frequency component (that is medium run fluctuations/cycles) of GDP one period ahead. In other words, CTR is able to forecast GDP at frequencies corresponding to 3 months to 7 months cycle.

In the next step, we analyzed Granger-causality from PCT to GDP and reported its result in Figure 3. It is evident from Figure 3 that the null hypothesis of no predictability is not rejected at 5% level of significance for all frequencies. This implies that PCT does not Granger-cause GDP. Additionally, we analyzed Granger-causality from TPI to GDP and reported its result in Figure 4. It is evident from Figure 4 that the null hypothesis of no predictability is not rejected at 5% level of significance for all frequencies in the interval $(0, \pi)$. This implies that TPI does not Granger-cause GDP. Finally, we analyzed Granger-causality from TCI to GDP and reported its result in Figure 5. It is evident from Figure 5 that the null hypothesis of no predictability is not rejected at 5% level of significance for all frequencies in the interval $(0, \pi)$. This implies that TCI does not Granger-cause GDP for all levels of frequencies.

4. Concluding Remarks

In this paper, we used the frequency domain approach of Breitung and Candelon (2006) to investigate the short- and long-run Granger-causality from different tax burden measures to GDP for the period 1947:1–2009:3 for the USA. The different tax burdens were Current Receipts (CR), Current Tax Receipts (CTR),\(^6\) Personal Current Taxes (PCT), Taxes on Production and Imports (TPI), and Taxes on Corporate Income (TCI).

Stationarity and cointegration property of the data series revealed that all variables were nonstationary in level form, and stationary in first difference form (i.e., $I(1)$) and the variants of tax were not cointegrated with GDP. Results of frequency domain approach showed that CR, PCT, TPI and TCI did not Granger-cause GDP, at both the short-and high-frequency level as the null hypothesis of no predictability was not rejected at 5% level of significance for all

\(^5\) The frequency, $\omega$, of a cycle is related to its period, $T$, measured in number of observations, by the formula $T = \frac{2\pi}{\omega}$; $\pi$ takes its usual value i.e., $\pi = \frac{22}{7}$. Thus a frequency of $\frac{\pi}{4}$ corresponds to a period of 8 observations, or 2 years given the quarterly observations and 8 months for monthly observation.

\(^6\) Here, current tax receipts are the tax revenues received by government from all sources. It is the sum of personal current taxes, taxes on production and imports, taxes on corporate income, and taxes from the rest of the world. Current receipts includes current tax receipts, contributions for government social insurance, income receipts on assets, interest and miscellaneous receipts and dividends, current transfer receipts and current surplus of government enterprises.
frequencies in the interval $(0, \pi)$. However, CTR was found to be a good predictor of GDP as the null hypothesis of no predictability was rejected at 5% level of significance in the range $\omega \in (0.9, 1.9)$, corresponding to the cycle length of 3 months to 7 months. The reason for these medium run business cycles could be the delayed effect that CTR might assert over GDP through the channels of public income (that is tax collection authorities). Our results are comparable to Karagianni et al. (2012). Karagianni et al. (2012) who found evidence of Granger-causality running from TPI and TCI to GDP, while the present study revealed evidence of Granger-causality from CTR to GDP.

We can explain this difference in the findings by the following three reasons. First, this study used long span data; Second, this study did not use taxes as a ratio of GDP; Third that we have analyzed Granger-causality across the frequencies whereas, Karagianni et al. (2012) used one shot measure of Granger-causality in a non-linear framework.\footnote{Our results are comparable with relevant studies mentioned in the first section because of wide difference in the form of taxation used and methodology adopted.}

These findings have important policy implications. For example, when the USA looks forward to influencing (rebalancing) its GDP, by means of taxation, it is preferable to reconsider the tax structure with a focus on CTR. This is because by changing the structure of CTR government will be able to earn more revenue even at the initial stage. Hence, for the USA we suggest to get off from the fiscal crisis to change the structure of CTR. However, if the USA government decides to increase the welfare with the stability and sustainability of GDP, then the USA policy makers should readjust the tax burden by infusing the changes of the CR, PCT, TPI, and TCI. These would be a safer option rather than, rebalancing the tax burden distribution on CTR. If, a change made by tax authorities affects CTR, there is possibility that it might affect or create unbalance in GDP. The non-evident causality from the side of CR to GDP growth in the frequency domain could be because of the simultaneous effect that CR might assert over GDP through the channels of public income and public expenditure. In addition, non-evident causality of PCT, TCI and TPI, can be attributed to factors like unemployment, shadow economy and the low tax authorities’ enforcement power that distort the evidence of the personal income taxation causality on GDP (Karagianni et al., 2012). From our study, it is evident that frequency bands are an important factor in the investigation of Granger-causal relationships between alternative tax and GDP. Future studies might extend this work in the direction of identifying and verifying reasons for differences in the Granger-causality in frequency domain. One may also look the results and compare them by using a recent approach developed in the same framework by Lemmens et al. (2008).
References


### Table 1: Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Ln(GDP)</th>
<th>Ln(CR)</th>
<th>Ln(CTR)</th>
<th>Ln(PCT)</th>
<th>Ln(TPI)</th>
<th>Ln(TCI)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>8.574673</td>
<td>7.269927</td>
<td>6.995904</td>
<td>6.206787</td>
<td>6.012324</td>
<td>5.150359</td>
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<tr>
<td><strong>Median</strong></td>
<td>8.645516</td>
<td>7.379098</td>
<td>7.061137</td>
<td>6.322187</td>
<td>6.035274</td>
<td>5.129139</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>7.477604</td>
<td>5.894196</td>
<td>5.785217</td>
<td>4.697210</td>
<td>4.868761</td>
<td>4.203822</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>0.599458</td>
<td>0.689555</td>
<td>0.588131</td>
<td>0.704524</td>
<td>0.568461</td>
<td>0.375275</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>-0.147220</td>
<td>-0.295242</td>
<td>-0.240623</td>
<td>-0.310684</td>
<td>-0.297799</td>
<td>0.224148</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>1.847343</td>
<td>1.906767</td>
<td>2.020430</td>
<td>1.980839</td>
<td>2.067562</td>
<td>3.126566</td>
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<tr>
<td><strong>Probability</strong></td>
<td>0.000611</td>
<td>0.000312</td>
<td>0.001972</td>
<td>0.000581</td>
<td>0.001659</td>
<td>0.321530</td>
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<tr>
<td><strong>Sum</strong></td>
<td>2152.243</td>
<td>1824.752</td>
<td>1755.972</td>
<td>1557.903</td>
<td>1509.093</td>
<td>1292.740</td>
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<td><strong>Sum Sq. Dev.</strong></td>
<td>89.83758</td>
<td>118.8714</td>
<td>86.47461</td>
<td>124.0886</td>
<td>80.78699</td>
<td>35.20782</td>
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<tr>
<td><strong>Observations</strong></td>
<td>251</td>
<td>251</td>
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#### Figure 1: Graphical plots of the variables analyzed
Appendix 2

Figure 1: $H_0: CR \overset{does\ not\ Granger-cause}{\rightarrow} GDP$ (lag=2)

Frequency $(\omega) = 2\pi$/cycle length $(T)$.

Figure 2: $H_0: CTR \overset{does\ not\ Granger-cause}{\rightarrow} GDP$ (lag=3)

Frequency $(\omega) = 2\pi$/cycle length $(T)$. 
Figure 3: $H_0: PCT \xrightarrow{does \ not \ Granger-cause} GDP \ (lag=4)$

Frequency $(\omega) = 2\pi/cycle \ length \ (T)$.

Figure 4: $H_0: TPI \xrightarrow{does \ not \ Granger-cause} GDP \ (lag=4)$

Frequency $(\omega) = 2\pi/cycle \ length \ (T)$.
Figure 5: $H_0: TCI \overset{does \ not \ Granger--cause}{\rightarrow} GDP \ (\text{lag}=4)$

Note: The solid line shows the Granger-causality measure from tax burden distribution to GDP. The horizontal broken line shows the 5% critical value. The horizontal axis shows the frequency ordinates as fractions of $\pi$ and vertical axis shows calculated value of $F$ statistic.