Economics Bulletin

Volume 32, Issue 1

Cointegration between carbon spot and futures prices: from linear to nonlinear modeling

Julien Chevallier
University Paris Dauphine

Abstract

This paper develops two nonlinear cointegration models - a VECM with structural shift and a threshold cointegration model - applied to carbon spot and futures prices. The results extend the previous findings by Chevallier (2010), who studied this topic with a linear VECM. First, in the VECM with structural shift, we observe that the returns of carbon spot and futures prices correct the deviations to the long-term equilibrium, with the futures price being the leader in the price discovery. Besides, we identify a breakpoint in July 2008, which may be related to the financial crisis and its effects on the carbon market. Second, we use Hansen and Seo's (2002) methodology, which points out the need to consider threshold cointegration models. We find strong error-correction effects for the carbon futures price. Asymmetry is implied in the sense that the carbon futures price governs most of the adjustment from the short-run to the long-run equilibrium of the model above or below the estimated threshold.

Julien Chevallier is Member of the Center for Geopolitics and Raw Materials (CGEMP) and the Dauphine Economics Lab (LEDa).
Address for correspondence: Place du Maréchal de Lattre de Tassigny 75775 PARIS Cedex 16 France. Email: julien.chevallier@dauphine.fr
Citation: Julien Chevallier, (2012) "Cointegration between carbon spot and futures prices: from linear to nonlinear modeling", *Economics Bulletin*, Vol. 32 No. 1 pp. 160-181.
Contact: Julien Chevallier - julien.chevallier@dauphine.fr.
Submitted: November 09, 2011. **Published:** January 13, 2012.

1 Introduction

Chevallier (2010) has proposed a linear cointegration exercise between carbon spot and futures prices, exchangeable under the European Union Emissions Trading Scheme (EU ETS)[1]. The author finds a cointegrating relationship between spot and futures CO_2 allowances, with the futures price being the leader in the long-term relationship. However, by considering a purely *linear* model, it is possible that the econometrician is either mis-specifying the model, or ignoring a valid cointegration relationship.

That is why, in this paper, we apply two *nonlinear* cointegration models to the analysis of carbon spot and futures prices. The first model consists in a straightforward extension of the cointegration methodology by Lütkepohl et al. (2004) to include an unknown structural break. In our framework, this model could potentially be useful in order to take into account the 2008 financial crisis.

The second model is based on threshold cointegration, initiated by Balke and Fomby (1997). Implicit in the definition of cointegration is the idea that every small deviations from the long-run equilibrium will lead to error correction mechanisms. Threshold cointegration extends the linear cointegration case by allowing the adjustment to occur only after the deviation exceeds some critical threshold. Furthermore, it allows capturing asymmetries in the adjustment, whereby positive and negative deviations are not corrected in the same way. To investigate this question, we apply the methodology by Hansen and Seo (2002) to the time series of carbon spot and futures prices.

The main results of the paper may be summarized as follows: *(i)* we highlight the need to resort to nonlinear cointegration techniques when investigating the relationship between carbon spot and futures prices, and *(ii)* the ECX EUA futures contract is confirmed to be the leader in the price discovery process.

The rest of the paper is structured as follows. Section 2 presents the data used. Section 3 introduces the VECM with structural shift. Section 4 contains the threshold cointegration model. Section 5 briefly concludes.

2 Data

We study the time-series of carbon spot and futures prices, along with standard unit root tests.

[1]One ton of CO_2 emitted in the atmosphere is equal to one European Union Allowance (EUA).

2.1 Carbon Spot Price

Figure 1 presents the daily time series of EUA Spot prices traded in €/ton of CO_2 on BlueNext (BNX) from February 26, 2008 to April 26, 2011 which corresponds to a sample of 819 observations. The start of the study period corresponds to the trading of CO_2 spot allowances valid during Phase II under the EU ETS[2].The EUA Spot prices are also presented in logreturn transformation in the bottom panel of Figure 1. Descriptive statistics for all raw time-series and logreturns may be found in Table 1.

2.2 Carbon Futures Price

Figure 2 presents the daily time series of EUA Futures prices traded in €/ton of CO_2 on the European Climate Exchange (ECX). The start of the study period corresponds to the trading of CO_2 spot allowances on BlueNext, in order to match the historical data available for the two samples. The EUA Futures prices are also presented in logreturn transformation in the bottom panel of Figure 2.

2.3 Unit Root Tests

Based on the Augmented Dickey-Fuller and Phillips-Perron unit root tests, we check in Table 2 that the time series of carbon spot and futures prices are non stationary in raw form. This amounts to checking that they are difference stationary and integrated of order one (i.e. *I(1)*). The fact that the time series are integrated of the same order is indeed a pre-requisite condition for cointegration. Next, we detail the cointegration models.

3 VECM with Structural Shift

In this section, we explore the possibility of wrongly accepting a cointegration relationship, when some of the underlying time series are contaminated by a structural break. We first present the procedure for estimating a VECM with a structural shift in the level of the process, as developed by Lütkepohl et al. (2004). By doing so, we draw on the notations by Pfaff (2008).

[2]Phase I spot prices are not considered here, due to their non-reliable behavior (see Alberola and Chevallier (2009) for more details on the effects of inter-period banking restrictions on the price pattern of EUA spot prices during 2005-2007).

3.1 Cointegration Test

Let $\vec{y_t}$ be a $K \times 1$ vector process generated by a constant, a linear trend, and level shift terms[3]:

$$\vec{y_t} = \vec{\mu_0} + \vec{\mu_1} t + \vec{\delta} d_{t\tau} + \vec{x_t} \tag{1}$$

with $d_{t\tau}$ a dummy variable which takes the value of one when $t \geq \tau$, and zero otherwise. The shift point τ is unknown, and is expressed as a fixed fraction of the sample size:

$$\tau = [T\lambda], \quad 0 < \underline{\lambda} \leq \lambda \leq \overline{\lambda} < 1 \tag{2}$$

where $\underline{\lambda}$ and $\overline{\lambda}$ define real numbers, and $[\cdot]$ the integer part. Therefore, the shift cannot occur at the very beginning or the very end of the sample. The estimation of the structural shift is based on the regressions:

$$\vec{y_t} = \vec{\nu_0} + \vec{\nu_1} t + \vec{\delta} d_{t\tau} + \vec{A_1} \vec{y_{t-1}} + \ldots + \vec{A_1} \vec{y_{t-p}} + \epsilon_{t\tau}, \quad t = p+1, \ldots, T \tag{3}$$

with $\vec{A_i}, \quad i = 1, \ldots, p$ the $K \times K$ coefficient matrices, and ϵ_t the spherical K-dimensional error process. The estimator for the breakpoint is defined as:

$$\hat{\tau} = \arg\min_{\tau \in \mathcal{T}} \det \left(\sum_{t=p+1}^{T} \vec{\hat{\epsilon}_{t\tau}} \vec{\hat{\epsilon}_{t\tau}^{\prime}} \right) \tag{4}$$

with $\mathcal{T} = [T\underline{\lambda}, T\overline{\lambda}]$, and $\vec{\hat{\epsilon}_{t\tau}}$ the least squares residuals of Eq. (3). Once the breakpoint $\hat{\tau}$ has been estimated, the data are adjusted as follows:

$$\vec{\hat{x}_t} = \vec{y_t} - \vec{\hat{\mu}_0} - \vec{\hat{\mu}_1} t + \vec{\hat{\delta}} d_{t\hat{\tau}} \tag{5}$$

The test statistic writes:

$$LR(r) = T \sum_{j=r+1}^{N} \ln(1 + \hat{\lambda}_j) \tag{6}$$

with corresponding critical values found in Trenkler (2003).

According to the results in Table 3, we accept the presence of at least one cointegrating relationship ($r = 1$) between carbon spot and futures prices – considered in logarithmic form – when taking explicitly into account a

[3] Note that Lütkepohl et al. (2004) develop their analysis in the context where $\vec{x_t}$ can be represented as a VAR(p), whose components are at most $I(1)$ and cointegrated with rank r.

structural shift in the level of the process. The breakdate is identified in July 2008, i.e. in a context of high uncertainties on the carbon market regarding the effects of the financial crisis on output and, ultimately, on the demand by industrials for CO_2 allowances (Chevallier (2011)).

3.2 VECM Estimates

Next, we can specify a VECM under the form (Johansen (1988, 1991), Johansen and Juselius (1990)):

$$\Delta X_t = \Pi_1 \Delta X_{t-1} + \ldots + \Pi_{p-1} \Delta X_{t-p+1} + \Pi_p X_{t-p} + \epsilon_t \quad (7)$$

where the matrices Π_i $(i = 1, \ldots, p)$ are of size $(N \times N)$. All variables are $I(0)$, except X_{t-p} which is $I(1)$. For all variables to be $I(0)$, $\Pi_p X_{t-p}$ needs to be $I(0)$ as well.

Let $\Pi_p = -\beta\alpha'$, where α' is an (r, N) matrix which contains r cointegration vectors, and β is an (N, r) matrix which contains the weights associated with each vector. If there exists r cointegration relationships, then $Rk(\Pi_p) = r$. Johansen's cointegration tests are based on this condition. We can thus rewrite Eq. (7):

$$\Delta X_t = \Pi_1 \Delta X_{t-1} + \ldots + \Pi_{p-1} \Delta X_{t-p+1} - \beta\alpha' X_{t-p} + \epsilon_t \quad (8)$$

Table 4 reveals the error correction mechanism, which leads towards the long-term stationary relationship between carbon spot and futures prices[4]. The *negative* signs of the error-correction coefficient (ECT) estimates indicate a slow adjustment of short-term deviations to the long-term relationship. By looking at the size of these coefficients (-0.028 and -0.042 for respectively the spot and futures variables), we conclude that futures prices are the leader in the long-term price discovery. We can assert that the ECX EUA futures contract is more liquid than the BNX EUA spot contract. This may tentatively explain why we identify a leading role for the futures price in the carbon market.

Besides, the VECM explains both spot and futures price series by their own lagged values. We show in this setting that, in the long-run, carbon spot and futures prices move together according to the cointegration relationship estimated by a relatively simple dynamic repercussion.

In what follows, we take our analysis one step further by investigating whether this relationship can be considered as *asymmetric*.

[4]By combining linearly the short-term variations of the two time series, the vector error-correction mechanism allows by definition to diminish the fluctuation errors in order to achieve the cointegrating relationship between both variables.

4 Threshold Cointegration

First, we present the model used by Hansen and Seo (2002). Second, we run the threshold cointegration test. Third, we comment the VECM results.

4.1 The Model

Let x_t be a p-dimensional $I(1)$ time series which is cointegrated with one $p \times 1$ cointegrating vector β, with n observations and l as the maximum lag length. Let $w_t(\beta) = \beta' x_t$ denote the $I(0)$ error-correction term. The two-regime threshold cointegration model, or nonlinear VECM of order $l+1$, takes the form (Hansen and Seo (2002)):

$$\Delta x_t = \begin{cases} A_1' X_{t-1}(\beta) + u_t, & \text{if} \quad w_{t-1}(\beta) \leq \gamma \\ A_2' X_{t-1}(\beta) + u_t, & \text{if} \quad w_{t-1}(\beta) > \gamma \end{cases} \tag{9}$$

with A_1 and A_2 the coefficient matrices governing the dynamics of the regimes, Δ the first-order difference operator, and:

$$X_{t-1}(\beta) = [1 \quad w_{t-1}(\beta) \quad \Delta x_{t-1} \quad \Delta x_{t-2} \quad , \ldots , \quad \Delta x_{t-l}]' \tag{10}$$

The nonlinear mechanism depends on deviations from the equilibrium, above or below the threshold parameter γ. The error u_t is assumed to be a vector martingale difference sequence with finite covariance matrix $\sum = E(u_t u_t')$. The notation $w_{t-1}(\beta)$ and $X_{t-1}(\beta)$ indicates that the variables are evaluated at generic values of β.

In our setting, we have a bivariate system for the carbon spot and futures prices, and one cointegrating vector. Therefore, we may set one element of β equal to unity to achieve identification.

Eq(9) allows all coefficients (except β) to switch between the two regimes. The threshold effect only has content if $0 < \mathrm{P}(w_{t-1}(\beta) \leq \gamma) < 1$, otherwise the model simplifies to linear cointegration. Therefore, we assume that:

$$\pi_0 \leq \mathrm{P}(w_{t-1}(\beta) \leq \gamma) \leq 1 - \pi_0 \tag{11}$$

with $0 < \pi_0 < 1$ a trimming parameter set to $\pi_0 = 0.05$ (see Andrews (1993), Andrews and Ploberger (1994)). The model is estimated by maximum likelihood under the assumption that the errors u_t are i.i.d Gaussian. Let the estimates be denoted by $(\tilde{\beta}, \tilde{A}_i, \tilde{\sum})$, with $\tilde{u}_t$ the residual vectors.

The threshold model in eq(9) has two regimes, defined by the value of the error-correction term in relation to some threshold γ. It is conceivable that the error-correction may occur in one regime only, or that the error-correction occurs in both regimes but at different speeds of adjustment. Thus, this approach provides richer insights than the standard linear error-correction modelling, which assumes the same error-correction mechanism throughout the whole sample period as in Chevallier (2010).

4.2 Threshold Cointegration Test

Let us test explicitly for the presence of linear *vs.* threshold cointegration. Hansen and Seo (2002) suggest to use the LM test statistic proposed by Davies (1987):

$$\text{SupLM} = \sup_{\gamma_L \leq \gamma \leq \gamma_U} \text{LM}(\tilde{\beta}, \gamma) \tag{12}$$

with $[\gamma_L, \gamma_U]$ the search region, so that γ_L is the π_0 percentile of $\tilde{w}_{t-1}$ and γ_U is the $(1 - \pi_0)$ percentile. $\tilde{\beta}$ is the null hypothesis estimate of β (linear cointegration) against the alternative of threshold cointegration. This means that there is no threshold under the null, so that the model reduces to a linear VECM. As the function $\text{LM}(\tilde{\beta}, \gamma)$ is non-differentiable in γ, to implement the maximization defined in eq(12), it is indeed necessary to perform a grid evaluation over $[\gamma_L, \gamma_U]$. The LM statistics are computed with heteroskedasticity-consistent covariance matrix estimates.

To assess the evidence of threshold cointegration, we use the SupLM test (estimated β) with 300 gridpoints, and the p-values calculated by the parametric bootstrap (see Hansen and Seo (2002)) where the true cointegrating vector is unknown for the complete bivariate specification. Figure 3 shows the resulting LM statistics computed as a function of γ.

In Table 5, all p-values have been computed with 5,000 simulation replications. The multivariate LM test points to the presence of threshold cointegration, with a test statistic equal to 67.038. This result provides a strong rejection of the null of linear cointegration at the 1% significance level.

4.3 Threshold Cointegration Estimates

Next, we estimate and test the two-regime model of threshold cointegration between the carbon spot and futures prices. To select the lag length, we find that the AIC and BIC applied to the threshold VECM pick the value of $l = 1$. Moreover, we report our results by letting $\tilde{\beta}$ be estimated.

From the grid search procedure, the model with the lowest value of $\log \left| \tilde{\sum}(\beta, \gamma) \right|$ is used to provide the $MLE(\tilde{\beta}, \tilde{\gamma})$, with the limitation of β in eq(11). Then, we use the grid-search algorithm[5] developed by Hansen

[5] See the Appendix for more details. We thank a referee for this suggestion.

and Seo (2002) to obtain the parameter estimates, with $MLE(\tilde{A}_1, \tilde{A}_2)$ being $\tilde{A}_1 = \tilde{A}_1(\tilde{\beta}, \tilde{\gamma})$ and $\tilde{A}_2 = \tilde{A}_2(\tilde{\beta}, \tilde{\gamma})$.

Table 6 reports the parameter estimates, which were obtained by minimizing the likelihood function over a 300×300 grid on the parameters γ, β.

The estimated threshold is $\tilde{\gamma} = 4.960$. The error-correction term is defined as: $w_t = LOGSPOT_t - 0.053LOGFUT_t$. The first regime occurs when $LOGSPOT_t \leq 0.053LOGFUT_t + 4.960$. 6.47% of the observations are found in this regime, which we label the 'extreme' regime. The second regime occurs when $LOGSPOT_t > 0.053LOGFUT_t + 4.960$. This regime is relevant to 93.53% of the observations, and may be viewed as the 'typical' regime. This kind of repartition of the data in usual and unusual regimes is consistent with other studies (see for instance Chevallier (2011)).

As expected, the carbon futures price governs most of the adjustment from the short-run to the long-run equilibrium of the model: its coefficients for w_{t-1} are highly significant (3.730 and -3.873 in the first and second regimes, respectively). The coefficients on the error-correction term also indicate that the magnitude of the response for carbon futures is two times greater than the coefficient of the spot price. Therefore, we are able to confirm our earlier finding that the futures price leads the nonlinear mean-reverting behavior of the carbon price.

To allow further visual interpretation of the results, the error-correction mechanism is pictured in Figure 4 by holding other variables constant. It can be seen the strong error-correction effect for the carbon futures price (and to a lesser extent for the spot price) on both sides of the estimated threshold. This finding comes up when we take account of the *nonlinearity* in the underlying processes.

5 Conclusion

This paper develops two *nonlinear* cointegration models - a VECM with structural shift and a threshold cointegration model - applied to carbon spot and futures prices. Our results are twofold.

First, in the VECM with structural shift, we observe that the returns of carbon spot and futures prices correct the deviations to the long-term equilibrium, with the futures price being the leader in the price discovery. Besides, we identify a breakpoint in July 2008, which may be related to the financial crisis and its effects on the carbon market.

Second, we use Hansen and Seo's (2002) methodology, which points out the need to consider threshold cointegration models instead of the potentially mis-specified linear VECM. We find strong error-correction effects for the carbon futures price. Asymmetry is implied in the sense that the carbon futures price governs most of the adjustment from the short-run to the long-run equilibrium of the model above or below the estimated threshold.

Overall, these results extend the previous findings by Chevallier (2010), who studied this topic with a *linear* VECM. We show that the null hypothesis of linear cointegration is rejected in favor of the alternative hypothesis of threshold cointegration. Besides, we find that the cointegrating relationship between carbon spot and futures prices is properly modeled by taking into account the occurrence of structural breaks. A trader in carbon markets could gain from the paper one central insight: carbon spot and futures prices are cointegrated, their relationship is mainly driven by the variation of the futures price, and it is sensitive to the occurrence of various thresholds and structural breaks[6].

[6]We wish to thank a referee for this comment.

References

Alberola, E., Chevallier, J. 2009. European Carbon Prices and Banking Restrictions: Evidence from Phase I (2005-2007). *The Energy Journal* 30(3), 51-80.

Andrews, D.W.K. 1993. Tests for parameter instability and structural change with unknown change point. *Econometrica* 61, 821-856.

Andrews, D.W.K., Ploberger, W. 1994. Optimal tests when a nuisance parameter is present only under the alternative. *Econometrica* 62, 1383-1414.

Balke, N.S., Fomby, T.B. 1997. Threshold Cointegration. *International Economic Review* 38(3), 627-645.

Chevallier, J. 2010. A Note on Cointegrating and Vector Autoregressive Relationships between CO_2 allowances spot and futures prices. *Economics Bulletin* 30(2), 1-21.

Chevallier, J. 2011. Evaluating the carbon-macroeconomy relationship: Evidence from threshold vector-error correction and Markov-switching VAR models. *Economic Modelling* 28(6), 2634-2656.

Davies, R.B. 1987. Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika* 74, 33-43.

Hansen, B.E., Seo, B. 2002. Testing for two-regime threshold cointegration in vector error-correction models. *Journal of Econometrics* 110, 293-318.

Lütkepohl, H., Saikkonen, P., Trenkler, C. 2004. Testing for the cointegrating rank of a VAR with level shift an unknown time. *Econometrica* 72, 647-662.

Johansen, S. 1988. Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control* 12(2-3), 231-254.

Johansen, S. 1991. Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models. *Econometrica* 59(6), 1551-1580.

Johansen, S., Juselius, K. 1990. Maximum likelihood estimation and the demand for money inference on cointegration with application. *Oxford Bulletin of Economics and Statistics* 52, 169-210.

Johansen, S. 1995. *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press, Oxford.

MacKinnon, J.G. 1996. Numerical Distribution Functions for Unit Root and Cointegration Tests. *Journal of Applied Econometrics* 11, 601-618

Pfaff, B. 2008. *Analysis of Integrated and Cointegrated Time Series with R*. Springer, New York Dordrecht Heidelberg London.

Trenkler, C. 2003. A new set of critical values for systems cointegration tests with a prior adjustment for deterministic terms. *Economics Bulletin* 3, 1-9.

	$SPOT$	$D(LOGSPOT)$	FUT	$D(LOGFUT)$
Mean	16.1576	-0.0003	18.2264	-0.0001
Median	14.6800	0.0001	17.4500	0.0001
Maximum	28.7300	0.1054	32.2500	0.1865
Minimum	7.9600	-0.1028	8.2000	-0.2882
Std. Dev.	4.3706	0.0233	4.4594	0.0265
Skewness	1.1970	-0.2065	0.4529	-0.9802
Kurtosis	3.3463	5.4294	2.4008	16.0174
Jarque-Bera	199.6887	206.9753	76.0942	11170.4900
Prob.	0.0000	0.0000	0.0000	0.0000
Obs.	819	818	819	818

Table 1: Descriptive statistics

Note: $SPOT$ refers to the BNX Spot time series in raw form, $D(LOGSPOT)$ to the BNX Spot time series in logreturn transformation, FUT refers to the EUA Futures time series in raw form, $D(LOGFUT)$ to the EUA Futures time series in logreturn transformation, Std. Dev. to standard deviation, Prob. to the probability of the Jarque-Bera test, and Obs. to the number of observations.

Table 2: Augmented Dickey-Fuller and Phillips-Perron Unit Root Tests

Null Hypothesis: $LOGSPOT$ has a unit root	t-Statistic	Critical value (1% level)	Model	Lag
Augmented Dickey-Fuller test statistic	-0.540608	-2.567799	1	2
Phillips-Perron test statistic	-0.521879	-2.567792	1	2

Null Hypothesis: $D(LOGSPOT)$ has a unit root	t-Statistic	Critical value (1% level)	Model	Lag
Augmented Dickey-Fuller test statistic	-22.10418	-2.567799	1	1
Phillips-Perron test statistic	-26.66421	-2.567796	1	1

Null Hypothesis: $LOGFUT$ has a unit root	t-Statistic	Critical value (1% level)	Model	Lag
Augmented Dickey-Fuller test statistic	-1.668031	-3.438149	2	3
Phillips-Perron test statistic	-0.492962	-2.567792	1	2

Null Hypothesis: $D(LOGFUT)$ has a unit root	t-Statistic	Critical value (1% level)	Model	Lag
Augmented Dickey-Fuller test statistic	-15.72768	-2.567802	1	2
Phillips-Perron test statistic	-26.23415	-2.567796	1	2

Note: $LOGSPOT$ and $LOGFUT$ stand for the logarithmic transformation of the carbon spot and futures price variables, respectively. $D(LOGSPOT)$ and $D(LOGFUT)$ stand for the first-difference logarithmic transformation of the carbon spot and futures price variables, respectively. For the Augmented Dickey-Fuller unit root test, the number of lags is determined by minimizing the Schwarz information criterion. For the Phillips-Perron unit root test, the number of lags is determined based on the Newey-West Bartlett kernel. For both unit root tests, Model 3 refers to the model with constant and with deterministic trend. Model 2 refers to the model with constant and without deterministic trend. Model 1 refers to the model without constant and without deterministic trend. Critical values are MacKinnon (1996) one-sided p-values.

	test	10%	5%	1%
r $\leq$ 1	3.52	5.42	6.79	10.04
r = 0	53.33	13.78	15.83	19.85

Table 3: Johansen Cointegration Test with Structural Shift

Note: The test displays the Trace Statistic, with linear trend in shift correction. Carbon spot and futures prices series are taken in *logarithmic* form.

Cointegrating Eq:		
$LOGSPOT(-1)$	1.000000	
$LOGFUT(-1)$	-0.996687***	
	(0.048320)	
Error Correction:	$D(LOG(SPOT))$	$D(LOG(FUT))$
ECT	-0.028097***	-0.042401***
	(0.004306)	(0.004454)
$D(LOGSPOT(-1))$	-0.276619***	0.198018***
	(0.035693)	(0.017965)
$D(LOGFUT(-1))$	0.364243***	-0.107441
	(0.043359)	(0.092130)
C	-0.000314***	-0.000219***
	(0.00078)	(0.00069)

Table 4: Vector Error-Correction Estimates

Note: $LOGSPOT(-1)$ and $LOGFUT(-1)$ stand for the logarithmic transformation of the carbon spot and futures price variables lagged one period, respectively. CointEq stands for Cointegrating Equation. ECT refers to the Error-Correction Term. $D(LOGSPOT(-1))$ and $D(LOGFUT(-1))$ stand for the first-difference logarithmic transformation of the carbon spot and futures price variables lagged one period, respectively. C refers to the constant. Standard errors in parentheses. ***,**,* denote respectively statistical significance at the 1%, 5% and 10% levels. The model is estimated without intercept and without trend in the Cointegrating Equation, with intercept and no trend in the data (Johansen (1995)).

Table 5: LM Tests Results for Threshold Cointegration

LM Threshold Test Statistic	67.0382
(Asymptotic) .05 Critical Value	19.5843
Bootstrap .05 Critical Value	19.7316
(Asymptotic) p-value	0.0001
Bootstrap p-value	0.0001

Note: The model estimated is the bivariate specification with the carbon spot and futures prices. The number of gridpoints for threshold and cointegrating vector is equal to 300. For p-values, the number of bootstrap replications is set to 5,000.

Table 6: Threshold VECM Estimates

Threshold Estimate	4.9607	
Cointegrating Vector Estimate	0.0539	
Negative Log-Likelihood	1652.5329	

First Regime	$D(LOGSPOT)$	$D(LOGFUT)$
μ	0.4936***	2.6346***
	(0.1407)	(0.2573)
w_{t-1}	1.6912***	3.7307***
	(0.5494)	(0.6569)
$D(LOGSPOT(-1))$	0.0940**	-1.0164**
	(0.0364)	(0.3937)
$D(LOGFUT(-1))$	0.0137***	0.0156***
	(0.0049)	(0.0048)
Percentage of Observations	0.0647	

Second Regime	$D(LOGSPOT)$	$D(LOGFUT)$
μ	0.1038*	-0.2520
	(0.0462)	(0.1549)
w_{t-1}	-1.9300***	-3.8738***
	(0.4435)	(1.2095)
$D(LOGSPOT(-1))$	0.0553	0.1249
	(0.0330)	(0.0791)
$D(LOGFUT(-1))$	0.0109***	-0.0213***
	(0.0030)	(0.0010)
Percentage of Observations	0.9353	

Note: $D(LOGSPOT)$ stands for the carbon spot price in logreturns transformation. $D(LOGFUT)$ stands for the carbon futures price in logreturns transformation. Eicker-White standard errors are provided in parentheses. ***,**,* denote respectively statistical significance at the 1%, 5% and 10% levels. The model estimated is defined in eq(9).

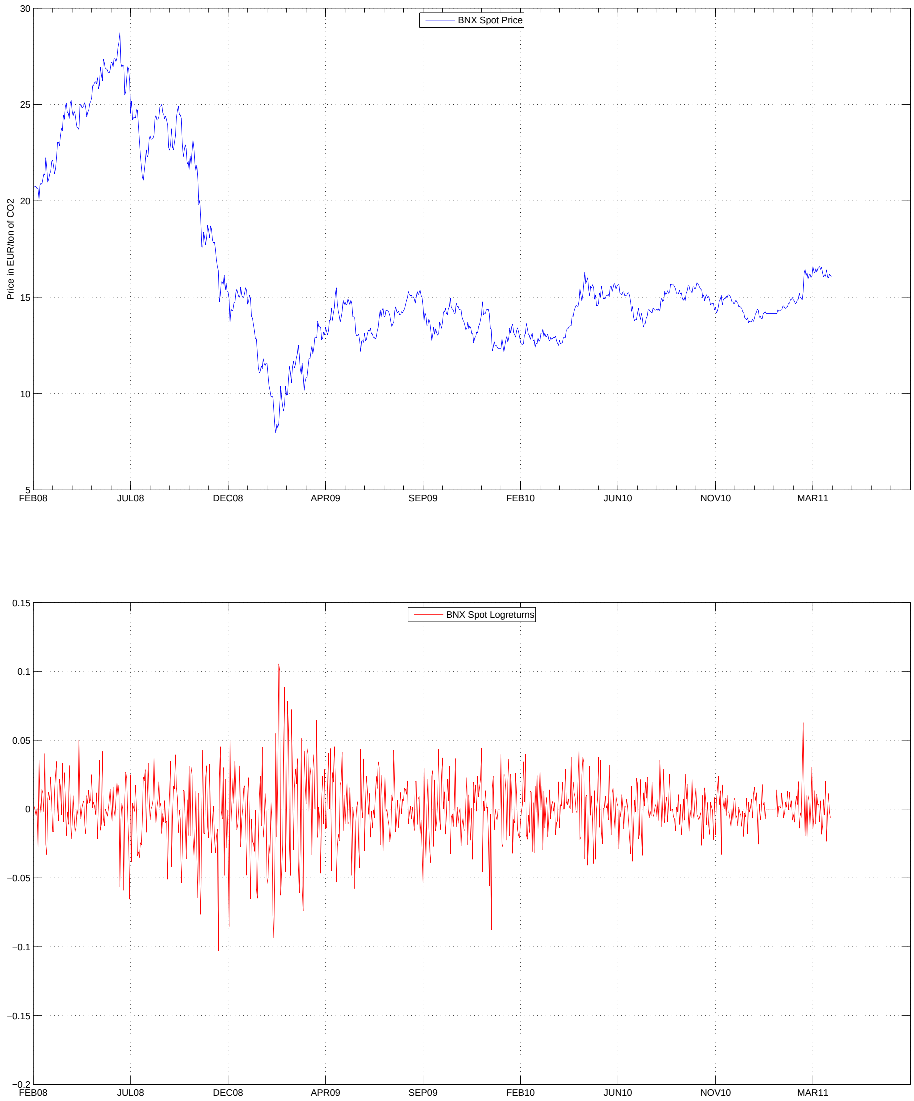


Figure 1: Time series of BNX Spot daily closing prices in raw form (top) and logreturn transformation (bottom) from February 26, 2008 to April 26, 2011
Source: BlueNext

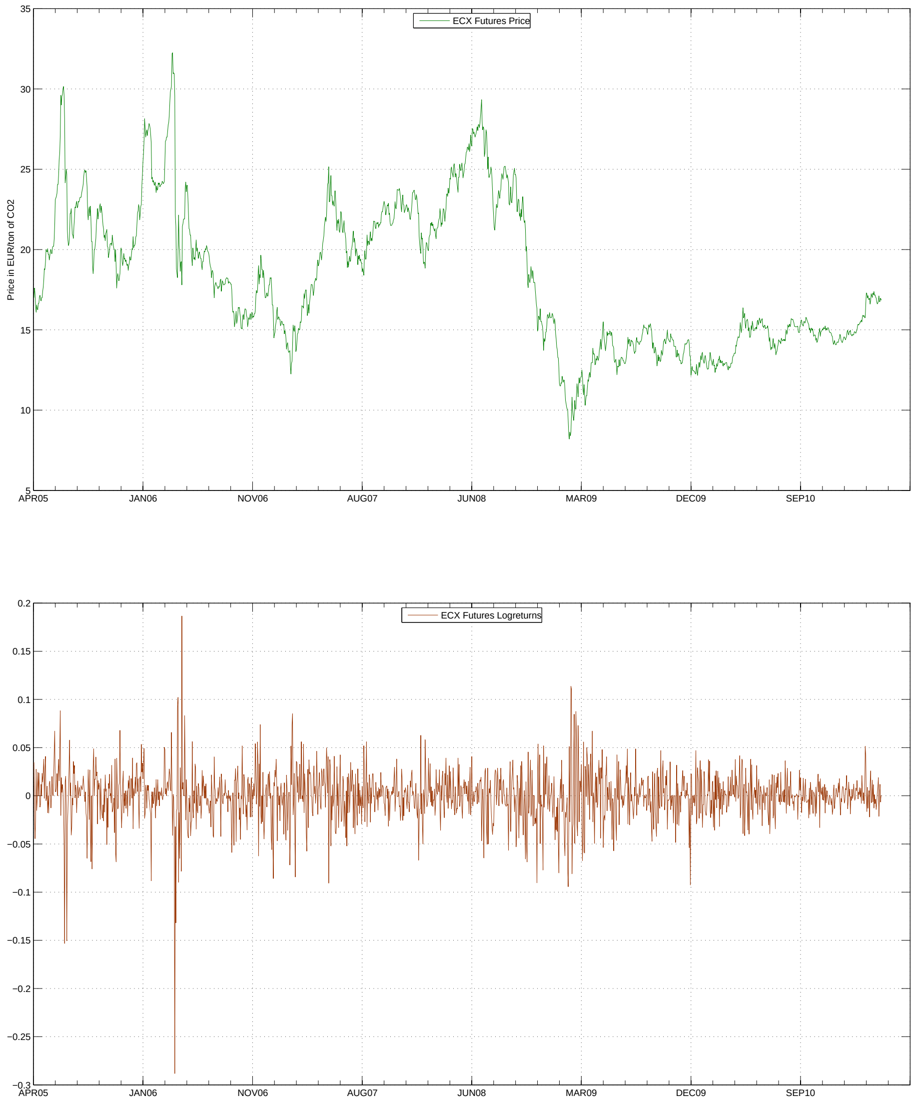


Figure 2: Time series of ECX EUA Futures daily closing prices in raw form (top) and logreturn transformation (bottom) from April 22, 2005 to April 26, 2011
<u>Source:</u> European Climate Exchange

Figure 3: LM Statistic for the Bivariate Threshold Cointegration Model

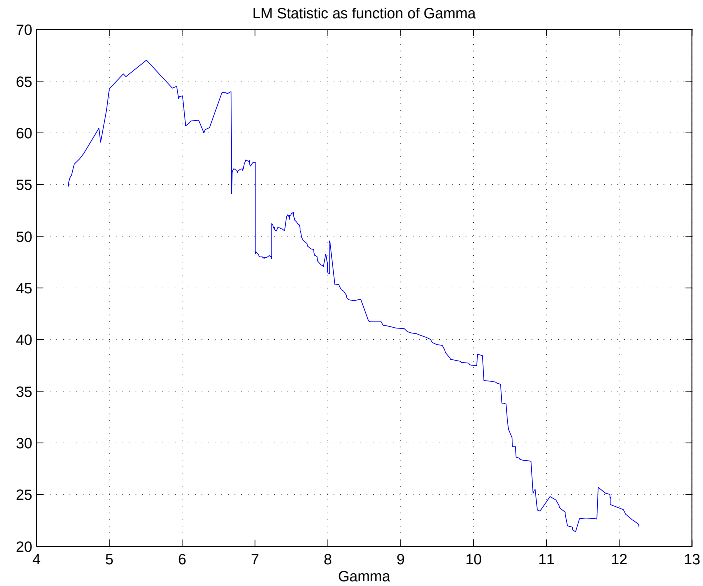
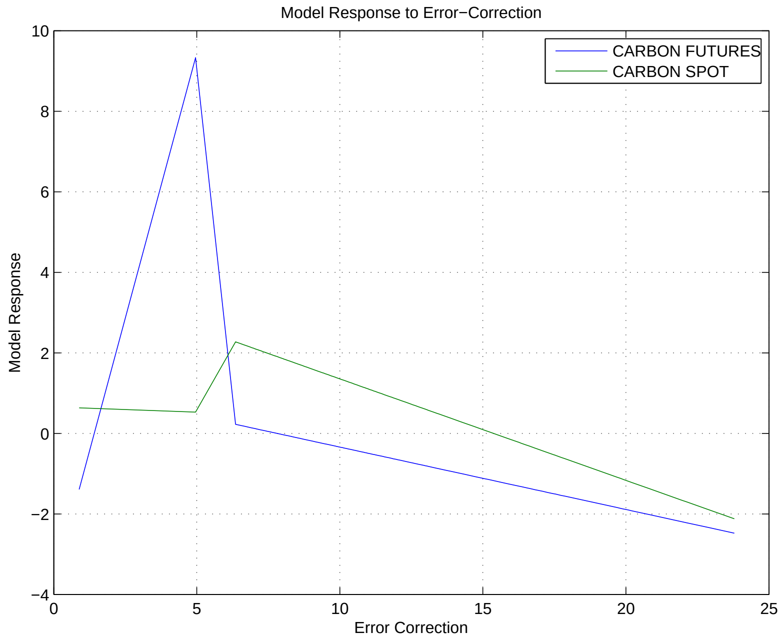


Figure 4: Threshold Cointegration Model: Response to the Error-Correction



Appendix

Hansen and Seo (2002)'s Grid-Search Algorithm

To execute a grid-search, one needs to pick up a region over which to search. Hansen and Seo (2002, p.297) suggest calibrating this region based on the consistent estimate of $\tilde{\beta}$ obtained from the linear model. Set $\tilde{w_{t-1}} = w_{t-1}(\tilde{\beta})$, let $[\gamma_L, \gamma_U]$ denote the empirical support of $\tilde{w_{t-1}}$, and construct an evenly spaced grid on $[\gamma_L, \gamma_U]$. Let $[\beta_L, \beta_U]$ denote a (large) confidence interval for β constructed from the linear estimate $\tilde{\beta}$ and construct an evenly spaced grid on $[\beta_L, \beta_U]$. The grid-search over (β, γ) then examines all pairs (γ, β) on the grids on $[\gamma_L, \gamma_U]$ and $[\beta_L, \beta_U]$, conditional on $\pi_0 \leq n^{-1} \sum_{t=1}^{n} 1(x_t'\beta \leq \gamma) \leq 1 - \pi_0$.