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A reassessment of the risk-return tradeoff at the daily horizon

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Abstract

This note makes two contributions by extending the analysis in Bali and Peng (2006) which investigates the risk-return tradeoff at the daily horizon using high-frequency data. Our first contribution is to show that the empirical relation between returns and risk is not validated for recent years. Our second contribution is to assess the importance of disentangling jumps from the continuous component using high-frequency data and recent nonparametric methods. We show that similar results are obtained using either realized variance or an alternative measure of realized variance which is robust to jumps thereby providing evidence that jumps do not improve significantly the explanatory power in the risk-return relation.

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1 Introduction

From the ICAPM Merton's (1973) model, one should observe a linear relationship between expected risk and expected return. This conclusion has only a weak support in empirical studies, beginning with French *et al.* (1987) for data from 1920 to 1984. The estimation issues are numerous when examining the risk-return tradeoff. How to form expectations? At which horizon should the relation be recovered? Which market portfolio should be used? Etc.

A recent strand of the literature focused attention on how to measure the expected risk. Ghysels *et al.* (2005) suggest to use MIDAS regression to improve forecast of risk. Adrian and Rosenberg (2008) consider two components for the market risk, one for the short term and one for the long term. Bali *et al.* (2009) and Conrad *et al.* (2009) show that a relation exists between various measures of downside risk and realized returns. Bollerslev *et al.* (2009) use the variance risk premium (the difference between implied variance and realized variance) to provide empirical evidence about the risk-return tradeoff. All these papers are interested in examining the relation between expected mean and expected risk at various horizons going from 1 month to 6 or more months.

One notable exception is Bali and Peng (2006) which emphasizes the very short-term risk-return tradeoff by examining expected risk and return at the daily horizon. The expected risk is constructed using intraday data and the so-called realized variance (or volatility). The use of realized variance provides a very reliable and much more precise estimate of the conditional variance. Interestingly, the authors show that the relation between expected risk and realized return is very significant, robust to the introduction of a number of control variables and quite stable over time.

In this note, we reinvestigate the issue of the risk-return relation at the daily horizon and extend the Bali and Peng's (2006) work in two dimensions. First, we show that their results are not valid for the most recent period. In particular, using rolling window regressions, we provide evidence that the relative risk aversion coefficient estimate is not significant anymore after mid-2006.

Our second empirical work is an investigation of the potential contribution of jumps in the risk-return tradeoff. Our interest in this question comes from recent results from the literature which has already provided evidence of the particular role of jumps for volatility forecasting (Giot and Laurent (2007), Andersen *et al.* (2007), Patton and Sheppard (2011)), in the volatility volume debate (Giot *et al.* (2011)), in modeling excess bon premia (Wright and Zhou (2009)) or the credit spread at the aggregate level (Tauchen and Zhou (2005)) or for individual firms (Zhang *et al.* (2009)).

We thus analyze the role of jumps in shaping the relation between risk and return and investigate how the distinction between jumps and the continuous component can improve the fit of the standard regression using standard realized variance.[1] It should be emphasized that our approach is nonparametric in essence and thus very different from the recent contribution by Li (2011) for which a jumps risk premium has to be specified.

The next section briefly presents the methodology to detect jumps. Section 3 provides our empirical results for a set of regressions using realized measures either robust or not to jumps to investigate the ICAPM equation. Section 4 concludes.

2 Jumps detection methodology

We first consider the so-called realized variance. For day t, the realized variance is given by the sum of the N equally spaced squared intraday returns:

$$RV_{t,N} = \sum_{i=1}^{N} r_{t,i}^2, \tag{1}$$

where the $r_{t,i}$ are intraday returns computed as $r_{t,i} = p_{t,i} - p_{t,i-1}$ for $i = 1, ..., N$. $p_{t,i}$ are intraday observations allowing to compute N continuously compounded intraday returns each day.[2] The sampling frequency is then given by $1/N$. When the frequency

[1] Identifying jumps in a stochastic process is important because it has implications for risk management, option pricing, portfolio selection and also has consequences for optimal hedging strategies.The impact of jumps in returns and volatility is studied in Andersen *et al.* (2002), Eraker *et al.* (2003), Chernov *et al.* (2003), Eraker (2004), Broadie *et al.* (2007). A risk premium (jump risk premium) can be raised in reference to jumps (Pan, 2000).

[2] We do note consider here the issue of the so-called microstructure noise when using high-frequency

of observation goes to infinity, the realized variance measure permits to treat the conditional volatility as if it were observable following the argument advanced in Merton (1980).[3] Nevertheless, the realized variance is not robust to jumps as it does include in the summation returns that are too high to be likely realization of a Wiener process.

To disentangle jumps from the continuous component, we need a measure of realized variance which is robust to jumps. Barndorff-Nielsen and Shephard (2004) propose to use the bipower variation (BPV), which is a sum of products of contiguous absolute returns or:

$$BPV_{t,N} = \xi_1 \sum_{i=1}^{N-1} |r_{t,i+1}||r_{t,i}| \tag{2}$$

where $\xi_p \equiv 2^{p/2}\Gamma(\frac{1/2(p+1)}{\Gamma(1/2)}) = E(\mid Z \mid_p)$ denotes the mean of the absolute value of standard normally distributed random variable Z. As the sampling frequency increases, the presence of jumps should have no impact because the return representing the jump is multiplied by a non-jump return which tends to zero asymptotically. This is true in case of rare jumps (one each day) when the probability of two consecutive jumps is negligible, as it is the case in the widely used Merton's (1976) model. Because the BPV is robust to jumps, the difference between the realized variance and the BPV can be used to investigate the presence of jumps. However, a small difference may be due to chance. Barndorff-Nielsen and Shephard (2004, 2006) develop a test that can be used to assess the presence of jumps when the difference between realized variance and BPV is sufficiently large. A possible test statistic using BPV as the robust-to-jump estimator is:

data. This issue is debated in, among others, Hansen and Lunde (2006) and a number of alternative estimators have been proposed in the literature to deal with this problem. Because we use S&P 500 futures data, which is a very liquid asset, and has been used previously in the rest of the literature with a 5-min sampling interval, giving satisfactory results, we adopt the same sampling frequency here. We also computed the TSRV estimator of Zhang *et al.* (2005) and found similar results in all our regressions. Results are available upon request from he authors.

[3]It can be noted that a similar idea was already in use in the investigation of the risk-return tradeoff in French *et al.* (1987) where the authors compute the monthly conditional variance using squared daily returns.

$$ZJ_{BPV}(N,d) = \sqrt{N}\frac{(RV_{d,N} - BPV_{d,N})RV_{d,N}^{-1}}{\left((\xi_1^{-4} + 2\xi_1^{-2} - 5)\max\{1, TQ_{d,N}BPV_{d,N}^{-2}\}\right)^{1/2}} \tag{3}$$

with TQ the realized tripower quarticity, which converges in probability to the integrated quarticity. This test statistic in Eq. (3) has been shown to have the best small sample properties in Huang and Tauchen (2005) and reasonable power against several empirically realistic calibrated stochastic volatility jump diffusion models (Andersen *et al.* (2007)). It has been used so far in a large number of contributions (see Giot *et al.* (2011), Giot and Laurent (2007), Tauchen and Zhou (2010), Wright and Zhou (2009) among many others).

3 Empirical analysis

From Merton's (1973) ICAPM, we know that the conditional expected excess return of the stock market index should be a linear function of the expectation of the conditional variance plus a hedging component. The theoretical relation thus has the following form:

$$\mathbb{E}_{t-1}[r_t - r_{f,t}] = \gamma\mathbb{E}_{t-1}[\sigma_t^2] \tag{4}$$

In Eq. (4), the coefficient γ is the relative risk aversion. The empirical counterpart of Eq. (4) is Eq. (5) where the realized return is used in place of the expected return can take the following form:

$$r_t - r_{f,t} = \mu + \gamma\mathbb{E}_{t-1}[RISK_t] + \pi_1 FED_{t-1} + \pi_2 DEF_{t-1} + \pi_3 TERM_{t-1} + \varepsilon_t \tag{5}$$

which is much more general than the theoretical relation. In particular, we will consider different measures of expected risk : the realized variance, the realized volatility and the implied realized variance. In addition, we explicitly model the hedging component using control variables.[4] Note that these measures of risk and hedging component are not

[4] Scruggs (1998) and Guo and Whitelaw (2006) are exemplified studies where the hedging component is of utmost interest.

original to our analysis but similar to Bali and Peng (2006) thereby ensuring comparability with their study. We thus refer the interested reader to Bali and Peng's (2006) article for the construction of the FED, DEF and $TERM$ variables and the data source employed.

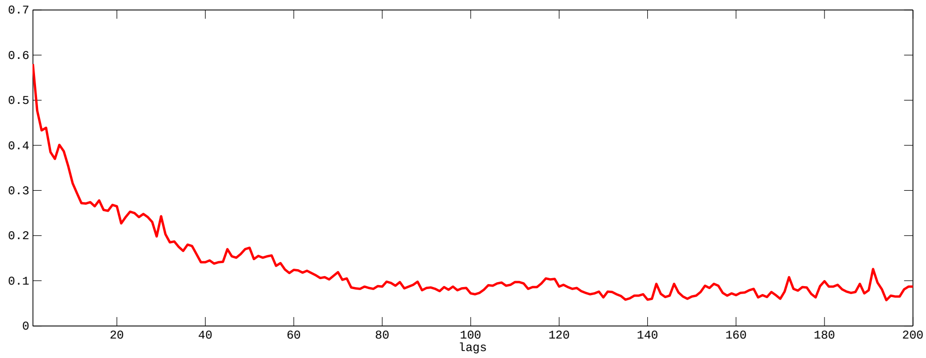

Figure 1
Autocorrelation for estimated realized variance using 5-min sampling interval

In what follows, the expected conditional market risk will be modeled using lagged valued of market risk measures. The motivation for using the lagged realized variance or volatility is the very strong persistence on the time series. This long-memory behavior for the realized variance can be observed in Figure 1 where autocorrelation coefficients estimates are significant for a large number of lags and decay hyperbolically. The long-memory behavior of the estimated conditional variance explains why more elaborated forecasts of the conditional variance (or volatility) do not help much in examining the risk-return relation. Following Andersen *et al.* (2003), Bali and Peng (2006) use an ARMA(5,5) to predict variance or volatility and demonstrate that their results are similar to the case where simple lagged values are employed. We experimented with the HAR model proposed in Corsi (2009) which mimic long-memory very accurately. As in Bali and Peng (2006), our results are worse when the time-series model forecast is used. Hence, we do not report this set of results here but these results remain naturally available upon request.

3.1 Data

The time period for our intraday (transaction) data is from January 2, 1996 to July 31, 2008. We consider a continuous time series constructed using the most active contract each day and rolling over when needed.[5] With a sampling interval of 5 minutes, we should obtain 81 intraday returns each day.[6] We do not consider days with less than 81 intraday returns, which are indicative of a shortened trading period using standard filters. In addition, we ensure that our sample only includes days with sufficient trading activity. We end with 3166 days where all these requirements are met. The average number of trades for these days is 3,090 and this variable is quite stable during the period under consideration.

In contrast with Bali and Peng (2006), we only consider the S&P 500 futures contract. This appears to be relevant in light of the consistency of empirical results when either the S&P 500 cash index or the CRSP value-weighted index are used. In addition, it can be emphasized that both the cash index and the CRSP have some notable drawbacks. In particular, these are not tradable assets and thus no available transaction data exist.

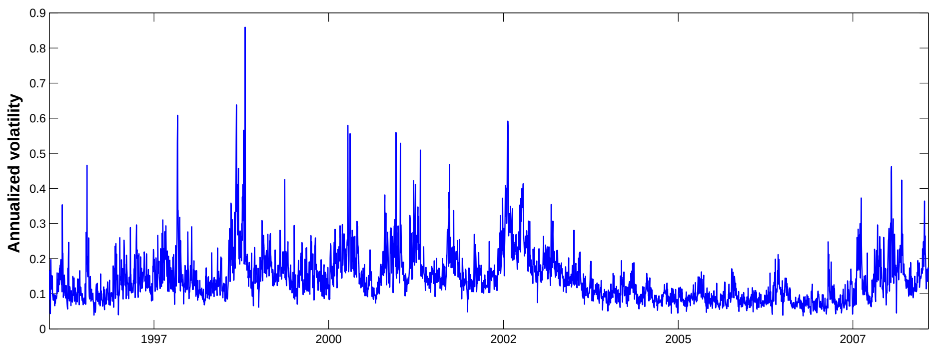

Figure 2
Annualized volatility computed from realized variance

Figures 2 and 3 represent the realized volatility (square root of realized variance) in annualized term (multiplied by $\sqrt{252}$) and the corresponding jumps extracted following

[5]To avoid calendar effects, we do not build our continuous series using a fixed number of days prior to maturity but adapt our rollover procedure to the observed trading activity.
[6]Trading of the S&P 500 futures contract occurs from 8:30 AM to 3:15 PM.

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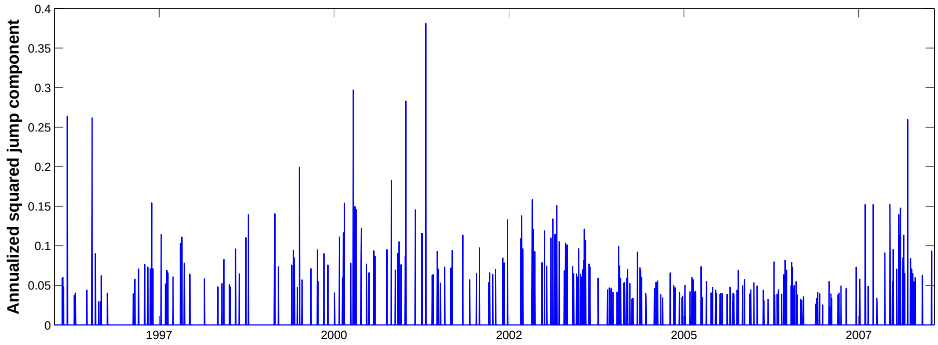

Figure 3
Annualized squared jump component computed using the Barndorff-Nielsen and Shephard (2004) test statistic

the methodology exposed in section 2, respectively. We can observe the time-varying behavior of the volatility and the clustering of associated jumps.

3.2 Empirical findings for full sample

We first consider the full sample with and without control variables as noted in Eq. (5). Regression results are reported in Table 1. These regressions provide a direct reassessment of the main result in Bali and Peng (2006). The first row of Table 1 is an estimation of the linear relation between daily excess return in the S&P 500 futures contract and the estimated realized variance for the previous day constructed using S&P 500 futures intraday data. The estimated relative risk aversion for this regression is 6.16. This estimate is shown to be significant at the 1% level in light of the reported t-statistic using Newey-West (1987) adjustment.[7] The explanatory power of the regression is rather low but in line with previous contributions using daily data, which are known to be very noisy.

Rows 2 to 4 in Table 1 complement our first finding. In row 2, the relation is investigated using the estimated realized volatility. Results are qualitatively similar in that the slope

[7]Bali and Peng (2006) note that the estimated relative risk aversion is upward biased because the overnight return is not considered when computing the realized measures of risk. This remark is valid for all our regressions in the present paper.

coefficient is significant at the 1% level and the explanatory power is pretty much the same. Row 3 provides results of the regression when the measure of risk is the lagged VIX. To obtain comparable results with the realized variance regression, we use the daily implied variance which is obtained from the VIX as follows: $[\text{VIX}/(100 \times \sqrt{252})]^2$. While the estimate of the relative risk aversion is of the same order, it is only significant at the 5% threshold. A possible explanation is that the VIX is calibrated to give an expectation of the risk associated with the index at an horizon of 22 days and not of a single day. The last regression includes control variables in addition of the realized variance. As noted above, the aim is to consider alternative investment opportunities. We confirm results in Bali and Peng (2006) in that FED, DEF and $TERM$ variables are never significant in our regressions.

Constant	RV_{t-1}	$\sqrt{RV_{t-1}}$	VIX_{t-1}	FED_{t-1}	DEF_{t-1}	$TERM_{t-1}$	Adj. R^2
-0.0004*	6.1615***						0.48%
(-1.6616)	(2.9517)						
-0.0011**		0.1523***					0.31%
(-2.3945)		(2.6315)					
-0.0007*			5.1644**				0.26%
(-1,9394)			(2.2742)				
0.0009	6.5224***			-7.4461e-06	-0.0016	5.4517e-05	0.47%
(0.4707)	(3.0335)			(-0.0301)	(-1.4082)	(0.1707)	

Table 1
Relation between daily excess market return and daily estimated risk (realized variance using 5-minute returns in this table). The dependent variable is the one-day-ahead excess return on the S&P 500 index futures and the risk free rate is the equivalent one-day rate computed from the three month Treasury bill. The rows beginning with OLS are estimated using ordinary least square. Standard deviations are computed using Newey-West (1987) HAC. Estimated coefficients are those of Eq. (5). Asterisks indicate statistical significance at the 1% (***), 5% (**) or 10% (*) level.

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Table 2 reports regression results for the BPV estimator of realized variance. Results are qualitatively similar to those in Table 1. We only note that the estimate for the relative risk aversion is slightly larger when BPV is used in place of realized variance and that the explanatory power for all three regressions is very slightly larger. However, these differences are not significant and we can unambiguously conclude that the jump component does not play any role in shaping the risk-return relation, at least when jumps are extracted following the nonparametric method we adopt in this paper.

Constant	BPV_{t-1}	$\sqrt{BPV_{t-1}}$	FED_{t-1}	DEF_{t-1}	$TERM_{t-1}$	Adj. R^2
-0,0004*	6.5752***					0.52%
(-1.7298)	(2.9767)					
-0.0011**		0.1595***				0.34%
(-2.4572)		(2.6858)				
0.0009	6.9554***		-1.2716e-05	-0.0016	5.0493e-05	0.51%
(0.4837)	(3.0551)		(-0.0512)	(-1.4317)	(0.1577)	

Table 2
Relation between daily excess market return and daily estimated risk (realized variance using 5-minute returns in this table). The dependent variable is the one-day-ahead excess return on the S&P 500 index futures and the risk free rate is the equivalent one-day rate computed from the three month Treasury bill. The rows beginning with OLS are estimated using ordinary least square. Standard deviations are computed using Newey-West (1987) HAC. Estimated coefficients are those of Eq. (5). Asterisks indicate statistical significance at the 1% (***), 5% (**) or 10% (*) level.

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3.3 Rolling window analysis

As in Bali and Peng (2006), we rely on rolling window regressions to assess the stability of the estimated risk-return tradeoff. An alternative method would be to adopt the time-varying coefficients approach but our aim in this note is to make our results comparable with previous literature. In addition, empirical assessments based on rolling windows are generally very reliable.

Plots of our rolling estimates are provided in Figures 4 and 5 for the lagged realized variance and lagged realized BPV, respectively. Following results in the previous section, the regressions do not include control variables in light of their insignificancy. In each Figure, the top panel reports the estimated value for the relative risk aversion coefficient and the bottom panel reports its associated t-statistic with the 5% threshold given by the red dashed line. Two conclusions are in order. First, the two graphical representations are undistinguishable, which reinforces our finding that realized variance and BPV yield similar results. Second, we observe that the estimated relative risk aversion coefficient is not more significant after mid-2006 and this result holds until the end of our period of investigation.

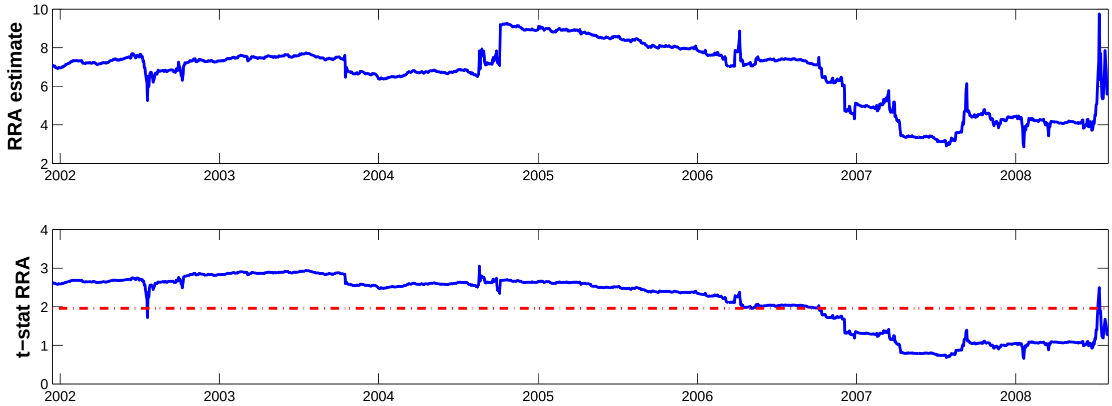

Figure 4
Rolling window estimation of the RRA coefficient using 1500 observations and RV

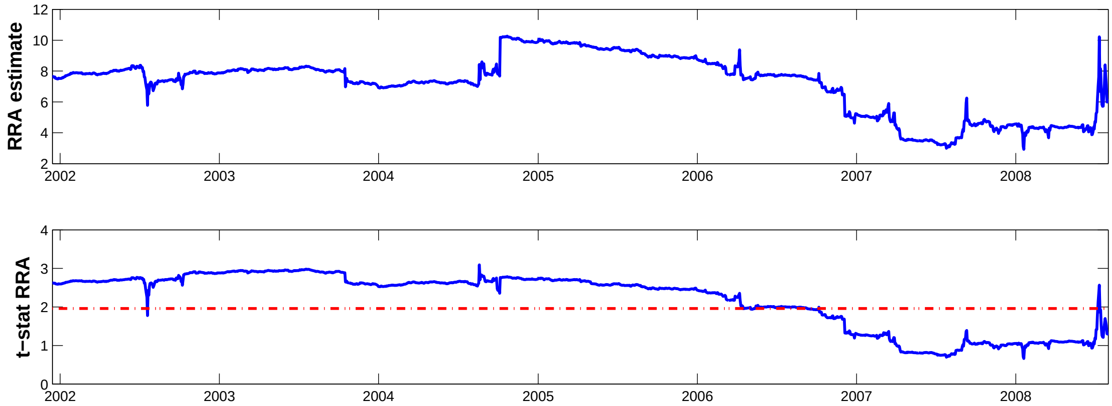

Figure 5
Rolling window estimation of the RRA coefficient using 1500 observations and the continuous component

4 Conclusion

In this note, we showed that the risk-return tradeoff at the daily horizon is questionable in recent years and that jumps do not seem to play a significant role in the relation between risk and return. Of course, our conclusion are dependent on the methodology we adopt, which is similar to the methodology presented in Bali and Peng (2006).

As a potential extension of the present work, we may investigate the explanatory power of signed jumps as defined in Patton and Sheppard (2011). Signed jumps are defined as the difference between positive and negative realized semivariance. The authors show that signed jumps significantly help to forecast volatility at horizons going from 1 to 60 days. Interestingly, signed jumps can be estimated each day and are not “rare events” as in the case of the methodology we employ here.

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