Economics Bulletin

Volume 32, Issue 1

Unanimous, reducible, anonymous social choice

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Abstract

The problem of aggregating preferences over two alternatives is considered. Three axioms are postulated: unanimity, reducibility (two divergent preferences can be replaced by their aggregation), and anonymity. It is shown that only twelve aggregation rules satisfy the three axioms: the majority rule, two myopic majority rules, three dictated rules (rules that almost always output the same outcome), and six hierarchically dictated rules (the output is determined by some priority ranking among outcomes).

Financial support from the Secretaría de Estado de Investigación of the Spanish Ministerio de Ciencia e Innovación (research project SEJ2007-67580-C02-01) is gratefully acknowledged. This paper has extraordinarily benefited from comments by reviewers of the Journal of Public Economic Theory.
Citation: Antonio Quesada, (2012) "Unanimous, reducible, anonymous social choice", *Economics Bulletin*, Vol. 32 No. 1 pp. 242-250.
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Submitted: July 06, 2011. **Published:** January 20, 2012.

1. Introduction

One of the simplest social choice problems has a group of individuals to decide between two alternatives. Each individual reports whether he or she prefers one alternative to the other or whether he or she is indifferent between them. Given the preferences revealed by the individuals, a social welfare function (SWF) determines a collective preference, which, if strict, can then be used to choose one of the alternatives. The majority rule is probably the most popular SWF. To May (1952, p. 682) is due the benchmark characterization, based on axioms of neutrality, anonymity, and monotonicity.

This paper suggests three axioms and identifies all the SWFs satisfying them. The first axiom is unanimity: if all the preferences reported by the individuals are the same, then the SWF outputs that common preference as the collective preference. The second axiom is reducibility: the aggregation of the preferences of n individuals is the result of aggregating first two different preferences into one and aggregating next this preference and the remaining $n - 2$ preferences. Postulated to characterize the majority rule, the axioms of weak path independence in Aşan and Sanver (2002, p. 411) and reducibility to subsocieties in Woeginger (2003, p. 90) also make the aggregation of a preference profile depend on the aggregation of subprofiles. The last axiom is substitutability: the collective preference of group I is not altered by replacing a member i of I with an individual j not belonging to I and assigning i's preference to j.

The result in this paper, Proposition 3.3, shows that those three axioms characterize a set of twelve SWFs, which can be divided into four types: the majority rule, a twisted majority rule in which a specific strict preference is considered an indifference and vice versa, the dictated rules (rules that are, essentially, constant), and the hierarchically dictated rules (in which the aggregation is determined by a fixed priority ranking among the three admissible collective preferences). Hence, the above three axioms solely lead to SWFs based on (i) the principle of the majority, (ii) dictated outcomes (a certain preference is almost always chosen), or (iii) priorities among outcomes.

2. Definitions and axioms

Members of the set $\mathbb{N}$ of positive integers are names for individuals. A society is a non-empty finite subset of $\mathbb{N}$. The set of alternatives or candidates is $\{\alpha, \beta\}$, with $\alpha \neq \beta$. A preference over $\{\alpha, \beta\}$ is represented by a number from the set $\{-1, 0, 1\}$. If the number is 1, α is preferred to β; if -1, β is preferred to α; if 0, α is indifferent to β. A preference profile for society I is a function $p_I : I \to \{-1, 0, 1\}$ assigning a preference over $\{\alpha, \beta\}$ to each member of I. For preference profile p_I and $i \in I$, p_i abbreviates $p_I(i)$. The set P is the set of all preference profiles p_I such that I is a society. For $n \in \mathbb{N}$, $P_n = \{p_I \in P$: I has n members$\}$. For $p_I \in P$ and $J \subset I$, the restriction p_J of p_I to society J is the member q_J of P such that, for all $i \in J$, $q_i = p_i$. For society I and $a \in \{-1, 0, 1\}$, (a^I) represents the preference profile p_I such that, for all $i \in I$, $p_i = a$. When $I = \{i\}$, a^i stands for $a^{\{i\}}$. For disjoint societies I and J, and preference profiles p_I and q_J, (p_I, q_J) designates the profile $r_{I\cup J}$ such that, for all $i \in I$, $r_i = p_i$ and, for all $i \in J$, $r_i = q_i$.

Definition 2.1. A social welfare function (SWF) is a mapping $f : P \to \{-1, 0, 1\}$.

A SWF takes as input the preferences over $\{\alpha, \beta\}$ of all the members of a given society I and outputs a preference over $\{\alpha, \beta\}$ attributed to society I. Specifically, for $p \in P$: (i) $f(p_I) = 1$ means that, according to f, society I prefers α to β; (ii) $f(p_I) = -1$, that society I prefers β to α; and (iii) $f(p_I) = 0$, that society I is indifferent between α and β. For $a \in \{-1, 0, 1\}$ and $p_I \in P$, define $n_a(p_I)$ to be the number of members of the set $\{i \in I$: $p_i = a\}$, that is, the number of individuals having preference a in preference profile p_I.

Definition 2.2. The majority rule is the SWF μ such that, for all $p_I \in P$: (i) if $n_1(p_I) > n_{-1}(p_I)$, then $\mu(p_I) = 1$; (ii) if $n_1(p_I) < n_{-1}(p_I)$, then $\mu(p_I) = -1$; and (iii) if $n_1(p_I) = n_{-1}(p_I)$, then $\mu(p_I) = 0$.

Definition 2.3. For $a \in \{-1, 1\}$, the $0/a$-myopic majority rule is the SWF $\mu^{0/a}$ such that, for all $p_I \in P$: (i) $n_{-a}(p_I) > n_0(p_I)$ implies $\mu^{0/a}(p_I) = -a$; (ii) $n_{-a}(p_I) < n_0(p_I)$ implies $\mu^{0/a}(p_I) = 0$; and (iii) $n_{-a}(p_I) = n_0(p_I)$ implies $\mu^{0/a}(p_I) = a$. A SWF f is a 0-myopic majority rule if $f \in \{\mu^{0/1}, \mu^{0/-1}\}$.

The $0/a$-myopic majority rule $\mu^{0/a}$ can be seen as the majority rule in which a means indifference and 0 means the strict preference a. For instance, $\mu^{0/1}(1^i, 0^j, 0^k) = 0$, $\mu^{0/1}(1^i, 0^j, -1^k) = 1$, and $\mu^{0/1}(1^i, 1^j, -1^k) = -1$.

Definition 2.4. A SWF f is hierarchically dictated if there is a linear ordering (a, b, c) on the set $\{-1, 0, 1\}$ such that, for all $p_I \in P$: (i) if $a \in \{p_i\}_{i\in I}$, then $f(p_I) = a$; (ii) if $a \notin \{p_i\}_{i\in I}$ and $b \in \{p_i\}_{i\in I}$, then $f(p_I) = b$; and (iii) if $a \notin \{p_i\}_{i\in I}$ and $b \notin \{p_i\}_{i\in I}$, then $f(p_I) = c$.

Definition 2.5. A SWF f is dictated if there is $a \in \{-1, 0, 1\}$ such that, for all $p_I \in P$, $f(p_I) = a$ unless, for some $b \in \{-1, 0, 1\}$, $p_I = (b^I)$, in which case $f(p_I) = b$.

In a hierarchically dictated SWF there is a ranking (a, b, c) of the three collective preferences −1, 0, and 1 such that: (i) when some individual has preference a, the SWF chooses a as the collective preference; (ii) when no individual holds preference a, but some holds b, then b is the collective preference; and (iii) otherwise, c is the preference of all the individuals, which then becomes the collective preference. A dictated SWF always outputs the same collective preference unless all the individuals have the same preference, in which case that common preference defines the collective preference.

U. *Unanimity*. For every society I and each $a \in \{-1, 0, 1\}$, $f(a^I) = a$.

U states that if all the individuals in society I have the same preference, then that preference is the preference attributed to society I.

R. *Reducibility*. For all $p_I \in P\backslash(P_1 \cup P_2)$, $i \in I$, and $j \in I\backslash\{i\}$, if $p_i \neq p_j$, then, for all $k \in \{i, j\}$, $f(p_I) = f(p_{I\backslash\{i,j\}}, f(p_{\{i,j\}})^k)$.

R is motivated by the idea of reducing the aggregation of non-unanimous preferences to the aggregation of unanimous preferences. According to R, $f(p_I)$ can be obtained as follows: take any two different preferences p_i and p_j from p_I; remove them from p_I; aggregate them into $f(p_i, p_j)$; choose a representative k for, and from, the society $\{i, j\}$; attribute $f(p_i, p_j)$ to k; aggregate preferences $p_{I\backslash\{i,j\}}$ and preference $f(p_i, p_j)$ of k. Successive application of R ensures that a preference profile in which all the preferences are identical will be reached, in which case U can be invoked.

For $p_I \in P$, $i \in I$, and $j \in \mathbb{N}\backslash\{i\}$, $p_I^{i\leftrightarrow j}$ abbreviates $(p_{I\backslash\{i,j\}}, (p_i)^j, (p_j)^i)$ if $j \in I$ and $(p_{I\backslash\{i\}}, (p_i)^j)$ if $j \notin I$. When $j \in I$, $p_I^{i\leftrightarrow j}$ is the preference profile obtained from p_I by permuting the preferences of individuals i and j. When $j \notin I$, $p_I^{i\leftrightarrow j}$ is the profile obtained from p_I by removing i from I, adding j, and ascribing to j the preference p_i held by i in p_I.

S. *Substitutability (inter-anonymity)*. For all $p_I \in P$, $i \in I$, and $j \in \mathbb{N}\backslash I$, $f(p_I) = f(p_I^{i\leftrightarrow j})$.

A. *Anonymity*. For all $p_I \in P$, $i \in I$, and $j \in \mathbb{N}\backslash\{i\}$, $f(p_I) = f(p_I^{i\leftrightarrow j})$.

A asserts that the collective preference does not depend on the identity of the members of the society: the collective preference is not altered by exchanging the preferences of two members of the society (intra-anonymity) or by replacing a member of the society with an individual not in the society (inter-anonymity). S just postulates the second possibility: for all $p_I \in P$, $i \in I$ and $j \in \mathbb{N}\backslash I$, $f(p_I) = f(p_{I\backslash\{i\}}, (p_i)^j)$.

3. Result

Remark 3.1. S implies A (with $k \in \mathbb{N}\backslash I$, $f(p_i, p_j, p_{I\backslash\{i,j\}}) = f(p_i, (p_j)^k, p_{I\backslash\{i,j\}}) = f((p_i)^j, (p_j)^k, p_{I\backslash\{i,j\}}) = f((p_i)^j, (p_j)^i, p_{I\backslash\{i,j\}}))$.

By Remark 3.1, when S holds, the SWF is anonymous. Hence, one can then dispense with the superscripts in preference profiles and, for instance, write $f(a, b)$ instead of both $f(a^i, b^j)$ and $f(b^i, a^j)$. This convention is followed in Table I (where, for example, $f(1, -1)$ is the common value assigned to $f(1^i, -1^j)$ and $f(-1^i, 1^j)$, for all $i \in \mathbb{N}$ and $j \in \mathbb{N}\backslash\{i\}$).

Lemma 3.2. If a social welfare function f satisfies U, R, and S, then only the twelve cases represented in Table I as columns 1-12 are possible.

Table I. The twelve ways of assigning value to (a, b), with $a \neq b$, given U, R, and S

	1	2	3	4	5	6	7	8	9	10	11	12
$f(1, -1)$	0	0	1	1	1	1	1	−1	−1	−1	−1	−1
$f(1, 0)$	1	0	0	−1	1	1	1	0	0	0	1	−1
$f(-1, 0)$	−1	0	0	0	0	1	−1	0	1	−1	−1	−1

Proof. By Remark 3.1, f assigns the same value to all the preference profiles abbreviated by $(1, -1)$ and $(-1, 1)$, no matter the identity of the individuals. This value is represented by the second row in Table I. The same applies to all the preference profiles $(1, 0)$ and $(0, 1)$, whose common value appears in the third row, and to all the preference profiles $(-1, 0)$ and $(0, -1)$, whose common value is in the fourth row.

• Case 1: $f(1, -1) = 0$. It will be shown that this case generates columns 1-2 in Table I.

Case 1a: $f(1, 0) = 1$.Case 1a1: $f(-1, 0) = -1$. The corresponding values constitute <u>column 1</u>. Case 1a2: $f(-1, 0) \neq -1$. By R, $f(1, 0, -1) = f(f(1, 0), -1) = f(1, -1) = 0$. This and R imply $0 = f(1, 0, -1) = f(1, f(-1, 0))$. If $f(-1, 0) = 0$, then $f(1, f(-1, 0)) = f(1, 0) = 1$: contradiction. If $f(-1, 0) = 1$, then, by U, $f(1, f(-1, 0)) = f(1, 1) = 1$: contradiction.

Case 1b: $f(1, 0) = -1$. By R and U, $f(-1, 1, 0) = f(-1, f(1, 0)) = f(-1, -1) = -1$. By R and U, $f(-1, 1, 0) = f(f(-1, 1), 0) = f(0, 0) = 0$: contradiction.

Case 1c: $f(1, 0) = 0$. By R and U, $f(-1, 1, 0) = f(f(-1, 1), 0) = f(0, 0) = 0$. By R, $0 = f(-1, 1, 0) = f(-1, f(1, 0)) = f(-1, 0)$. This set of values defines <u>column 2</u>.

• Case 2: $f(1, -1) = 1$. This case generates columns 3-7 in Table I.

Case 2a: $f(1, 0) = 0$. By R, $f(-1, 1, 0) = f(f(-1, 1), 0) = f(1, 0) = 0$. By R, $0 = f(-1, 1, 0) = f(-1, f(1, 0)) = f(-1, 0)$. This defines the case represented as column 3.

Case 2b: $f(1, 0) = -1$. By R, $f(-1, 1, 0) = f(f(-1, 1), 0) = f(1, 0) = -1$. If $f(-1, 0) = -1$, then, by R, $-1 = f(-1, 1, 0) = f(f(-1, 0), 1) = f(-1, 1) = 1$: contradiction. If $f(-1, 0) = 1$, then, by R and U, $-1 = f(-1, 1, 0) = f(f(-1, 0), 1) = f(1, 1) = 1$: contradiction. As a result, $f(-1, 0) = 0$, which defines the case displayed as column 4.

Case 2c: $f(1, 0) = 1$. When $f(-1, 0) = 0$, column 5 obtains. When $f(-1, 0) = 1$, it is column 6 that obtains. And when $f(-1, 0) = -1$, it is column 7.

• Case 3: $f(1, -1) = -1$. This case generates columns 8-12 in Table I.

Case 3a: $f(1, 0) = 0$. Having $f(-1, 0) = 0$ leads to column 8; $f(-1, 0) = 1$, to column 9; and $f(-1, 0) = -1$, to column 10.

Case 3b: $f(1, 0) = 1$. By R, $f(-1, 1, 0) = f(-1, f(1, 0)) = f(-1, 1) = -1$. By R, $-1 = f(-1, 1, 0) = f(f(-1, 1), 0) = f(-1, 0)$. These values give rise to column 11.

Case 3c: $f(1, 0) = -1$. By R and U, $f(-1, 1, 0) = f(-1, f(1, 0)) = f(-1, -1) = -1$. By R, $-1 = f(-1, 1, 0) = f(f(-1, 1), 0) = f(-1, 0)$. These values define column 12.■

Proposition 3.3. A social welfare function f satisfies U, R, and S if and only if f is the majority rule, a 0-myopic majority rule, hierarchically dictated, or dictated (a total of twelve social welfare functions).

Proof. "$\Leftarrow$" For every SWF f of the four types indicated, $f(p_I)$ does not depend on the identity of the individuals, so S holds in all cases. It is not difficult to verify that U also holds. With respect to R, it is satisfied by a dictated SWF, since any two different preferences are always collapsed into the same preference. As regards a hierarchically dictated SWF, suppose the associated linear order is (a, b, c). Consider any $p_I \in P$. If a is present in p_I, then, no matter how preferences are integrated, the aggregate preference is a. If a is not present but b is, then, no matter how preferences are integrated, the aggregate preference is b. And, finally, if neither a nor b is present, $p_I = (c, \dots, c)$, in which case R imposes no constraint.

To show that μ satisfies R, choose $p_I \in P$, $i \in I$, and $j \in I\backslash\{i\}$ such that $p_i \neq p_j$. Let $a \in \{1, -1\}$. If $\{p_i, p_j\} = \{a, -a\}$, then $\mu(p_{\{i,j\}}) = 0$. Accordingly, for all $k \in \{i, j\}$, $\mu(p_I) = \mu(p_{I\backslash\{i,j\}}, 0^k) = \mu(p_{I\backslash\{i,j\}}, \mu(p_{\{i,j\}})^k)$. If $\{p_i, p_j\} = \{0, a\}$, then $\mu(p_{\{i,j\}}) = a$. Consequently, for all $k \in \{i, j\}$, $\mu(p_I) = \mu(p_{I\backslash\{i,j\}}, a^k) = \mu(p_{I\backslash\{i,j\}}, \mu(p_{\{i,j\}})^k)$, because $(p_{I\backslash\{i,j\}}, a^k)$ is obtained from p_I by removing a component equal to zero and, possibly, renaming an individual. Lastly, 0-myopic majority rules satisfy R because each such rule is a majority rule with a reinterpretation of symbols: whereas indifference is represented by 0 in μ, it is represented by a in the myopic version $\mu^{0/a}$; and the strict preference designated by a in μ becomes represented by 0 in the myopic version.

"$\Rightarrow$" By Lemma 3.2, for $a \neq b$, $f(a, b)$ can only take values according to the columns in Table I. Part 1: column 1 implies $f = \mu$. By U, $f = \mu$ on P_1. By U and column 1, $f = \mu$ on P_2. Taking $f = \mu$ on $P_1 \cup P_2$ as the base case of an induction argument, choose $n > 2$ and suppose that $f = \mu$ on $P_1 \cup \ldots \cup P_{n-1}$. To show that $f = \mu$ on P_n, choose $p_I \in P_n$. If all the components of p_I are the same, then, by U, $f(p_I) = \mu(p_I)$. If two components p_i and p_j of p_I are different, then, by R, $f(p_I) = f(p_{I\backslash\{i,j\}}, f(p_{\{i,j\}})^i)$. By the induction hypothesis, $f(p_{I\backslash\{i,j\}}, f(p_{\{i,j\}})^i) = \mu(p_{I\backslash\{i,j\}}, \mu(p_{\{i,j\}})^i)$. Since μ satisfies R, $\mu(p_{I\backslash\{i,j\}}, \mu(p_{\{i,j\}})^i) = \mu(p_I)$.

Part 2: column 2 makes f be the dictated SWF with dictated value 0 (this is the unanimity rule, which outputs 0 when not all the preferences are the same, and, when all the preferences are the same, outputs that preference). Let $p_I \in P$. If all the components of p_I are the same, then, by U, $f(p_I)$ is that common preference. If not all the components are the same, then, being 0 the absorbing preference, the pairwise aggregation of preferences eventually generates the profile $(0, \ldots, 0)$, which, by U, is aggregated into 0.

Part 3: column 3 implies that f is the hierarchically dictated SWF f_σ with ordering $\sigma = (0, 1, -1)$. U attributes values in a way consistent with σ. Since the values of column 3 are also consistent with σ, $f = f_\sigma$ on $P_1 \cup P_2$. For the rest of profiles, notice that 0 absorbs both 1 and -1, because $f(-1, 0) = 0$ and $f(1, 0) = 0$. Therefore, if 0 is in the profile, the pairwise aggregation of preferences eventually leads to the profile $(0, \ldots, 0)$, which, by U, is aggregated into 0. If 0 is not in the profile but 1 is, then $f(-1, 1) = 1$ implies that the pairwise aggregation of preferences eventually leads to a profile of the sort $(1, \ldots, 1)$, which, by U, is aggregated into 1. And if neither 0 nor 1 appears in the profile, it must be $(-1, \ldots, -1)$, which, by U, is aggregated into -1.

Part 4: column 4 implies $f = \mu^{0/-1}$. Let $p_I \in P$. If $n_1(p_I) > n_0(p_I)$, then, thanks to R and $f(1, 0) = -1$, every 0 can be paired with some 1, so that every such pair $(1, 0)$ becomes -1.

Since $n_1(p_I) > n_0(p_I)$, some 1 remains unmatched. As $f(-1, 1) = 1$, value 1 absorbs value -1. Accordingly, the application of R will eventually generate a profile of the sort (1, … , 1), which, by U, is aggregated into 1. In sum, $f(p_I) = 1 = \mu^{0/-1}(p_I)$. If $n_1(p_I) < n_0(p_I)$, then, by an analogous reasoning, it is some 0 that survives the process of aggregating all the 1s. Given that $f(1, 0) = -1$, the removal of 1s by combining them with 0s generates -1s. Since $f(-1, 0) = 0$ and there is some 0, R leads to the profile (0, … , 0), which, by U, becomes $0 = \mu^{0/-1}(p_I)$. Finally, if $n_1(p_I) = n_0(p_I)$, every pair (1, 0) is transformed into -1, so all the 1s and all the 0s eventually cancel out. Hence, R yields a string of -1s, which, by U, is aggregated into $-1 = \mu^{0/-1}(p_I)$.

Part 5: column 5 implies that f is the hierarchically dictated SWF with ordering (1, 0, -1). The proof mimics that of part 3 (now, 1 absorbs both -1 and 0, whereas 0 absorbs -1).

Part 6: column 6 makes f be the dictated SWF with dictated value 1. This is like part 2 with 1 replacing 0.

Part 7: column 7 implies that f is the hierarchically dictated SWF with ordering (1, -1, 0). The proof mimics that of part 3 (1 absorbs both -1 and 0, whereas -1 absorbs 0).

Part 8: column 8 implies that f is the hierarchically dictated SWF with ordering (0, -1, 1). The proof mimics that of part 3 (0 absorbs both 1 and -1, whereas -1 absorbs 1).

Part 9: column 9 implies $f = \mu^{0/1}$. Let $p_I \in P$. If $n_{-1}(p_I) > n_0(p_I)$, then, thanks to R and $f(-1, 0) = 1$, every 0 can be paired with some -1, so that every such pair (-1, 0) becomes 1. Since $n_{-1}(p_I) > n_0(p_I)$, some -1 remains unmatched. As $f(-1, 1) = -1$, value -1 absorbs value 1. Accordingly, the application of R will eventually generate the profile (-1, … , -1), which, by U, is aggregated into -1. In sum, $f(p_I) = -1 = \mu^{0/1}(p_I)$. If $n_{-1}(p_I) < n_0(p_I)$, then, by an analogous reasoning, it is some 0 that survives the process of aggregating all the -1s. Given that $f(-1, 0) = 1$, the removal of -1s by combining them with 0s generates 1s. Since $f(1, 0) = 0$ and there is some 0, R leads to the profile (0, … , 0), which, by U, is transformed into $0 = \mu^{0/1}(p_I)$. Finally, if $n_{-1}(p_I) = n_0(p_I)$, every pair (-1, 0) is transformed into 1, so all the -1s and all the 0s eventually cancel out. Hence, R yields a string of 1s, which, by U, is aggregated into $1 = \mu^{0/1}(p_I)$.

Part 10: column 10 implies that f is the hierarchically dictated SWF with ordering (-1, 0, 1). The proof mimics that of part 3 (-1 absorbs both 0 and 1, whereas 0 absorbs 1).

Part 11: column 11 implies that f is the hierarchically dictated SWF with ordering (−1, 1, 0). The proof mimics that of part 3 (−1 absorbs both 1 and 0, whereas 1 absorbs 0).

Part 12: if f takes the values from column 12 in Table I, then f is the dictated SWF whose dictated value is −1. This is like part 2 with −1 replacing 0.■

Remark 3.4. No axiom in Proposition 3.3 is redundant.

The SWF f such that, for all $p_I \in P$, $f(p_I) = 0$ satisfies R and S, but not U. Let μ^* be the absolute majority rule: (i) $n_1(p_I) > n_{-1}(p_I) + n_0(p_I)$ implies $\mu^*(p_I) = 1$; (ii) $n_{-1}(p_I) > n_1(p_I) + n_0(p_I)$ implies $\mu^*(p_I) = -1$; and (iii) otherwise, $\mu^*(p_I) = 0$. Then μ^* satisfies U and S, but not R ($1 = \mu^*(1, 1, -1) \neq \mu^*(1, \mu^*(1, -1)) = \mu^*(1, 0) = 0$). Finally, let f be the SWF such that, for all $p_I \in P$, $f(p_I) = 0$, except that: (i) $f(1^1, 0^2) = 1$; (ii) for all $I \subset \mathbb{N}\setminus\{1, 2\}$, $f(1^I, 1^1, 0^2) = 1$; and (iii) for each society I and $a \in \{-1, 1\}$, $f(a^I) = a$. Then f satisfies both U and R, but not S.

Given Proposition 3.3, it should not be difficult to provide an axiomatic characterization of the majority rule by just adding some axiom or axioms inconsistent with the remaining eleven SWFs. Quesada (2011) provides one such characterization. Proposition 3.3 can also be compared with the results in Quesada (2012), where the SWFs that are characterized jointly with the majority rule are based on priorities over the set of individuals rather than priorities over the set of collective preferences.

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