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Predicting the risk of global portfolios considering the non-linear dependence structures

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Abstract

In this paper we estimated pair copula constructions (PCC) for three sets of markets: developed, Latin emerging and Asia-Pacific emerging. To that, we used daily prices from January 2003 to November 2011, totaling 1872 observations. The last 200 observations were separated for posterior validation of the estimated PCC. After, we constructed portfolios for each set of markets and we predicted their daily Value at Risk (VaR) for distinct significance levels, considering the dependence structure previously estimated, in the 200 days of the out-sample period. The results allow concluding that there were differences in the dependence structure of each set of markets. Further, the PCC were validated through backtesting of the predicted VaRs.

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1. Introduction

Since the introduction of the mathematical theory of portfolio selection and of the Capital Asset Pricing Model (CAPM), the issue of dependence has always been of fundamental importance to financial economics. In the context of international diversification, there is the need for minimizing the risk of specific assets through optimal allocation of resources. Therefore, it is necessary to understand the multivariate relationship between different markets. Thus we need a statistical model able to measure the temporal dependence between shocks of different countries.

An inappropriate model for dependence can lead to suboptimal portfolios and inaccurate assessments of risk exposures. Traditionally, correlation is used to describe dependence between random variables, but recent studies have ascertained the superiority of copulas to model dependence, as they offer much more flexibility than the correlation approach, because a copula function can deal with non-linearity, asymmetry, serial dependence and also the well-known heavy-tails of financial assets marginal and joint probability distribution.

A copula is a function that links univariate marginals to their multivariate distribution. Since it is always possible to map any vector of random variables into a vector with uniform margins, we are able to split the margins of that vector and a digest of the dependence, which is the copula. Despite the literature on copulas is consistent, the great part of the research is still limited to the bivariate case. Thus, construct higher dimensional copulas is the natural next step, even this do not being an easy task. Apart from the multivariate Gaussian and Student, the selection of higher-dimensional parametric copulas is still rather limited (Genest *et al.*, 2009).

The developments in this area tend to hierarchical, copula-based structures. It is very possible that the most promising of these is the pair-copula construction (PCC). Originally proposed by Joe (1996), it has been further discussed and explored in the literature for questions of inference and simulation (Bedford and Cooke, 2001; Bedford and Cooke, 2002; Kurowicka and Cooke, 2006; Aas *et al*, 2009). The PCC is based on a decomposition of a multivariate density into bivariate copula densities, of which some are dependency structures of unconditional bivariate distributions, and the rest are dependency structures of conditional bivariate distributions.

In this sense, this paper aims to predict the daily risk of portfolios composed by assets of distinct markets, considering the dependence structure among them. To that, we collected data from developed markets (U.S., Germany, England and Japan), Latin (Argentina, Brazil, Mexico and Chile) and Asia-Pacific (China, Hong Kong, Indonesia and Singapore) emerging markets. For each set of markets, we estimated a PCC, to compare their dependence structure. Further, in order to give robustness to these estimates, we predicted the daily Value at Risk (VaR) of portfolios composed by each set of markets.

The sequence of this paper is structured on the following way: Section 2 presents the material and methods of the study, explaining briefly about copulas and PCC, beyond of expose the data and the procedures to achieve the objective of the paper; Section 3 presents the found results and their discussion; Section 4 expose the conclusions of the paper.

2. Material and Methods

This section is subdivided on: i) Copulas, which briefly explain about definition and properties of this class of function; ii) Pair Copula Construction, which succinctly expose the concepts of this construction; iii) Empirical Method, which presents data and the applied procedures to estimate the dependence structures and predict the VaR of the portfolios.

2.1 Copulas

Dependence between random variables can be modeled by copulas. A copula returns the joint probability of events as a function of the marginal probabilities of each event. This makes copulas attractive, as the univariate marginal behavior of random variables can be modeled separately from their dependence (Kojadinovic and Yan, 2010).

The concept of copula was introduced by Sklar (1959). However, only recently its applications have become clear. A detailed treatment of copulas as well as of their relationship to concepts of dependence is given by Joe (1997) and Nelsen (2006). A review of applications of copulas to finance can be found in Embrechts *et al.* (2003) and in Cherubini *et al.* (2004).

For ease of notation we restrict our attention to the bivariate case. The extensions to the *n*-dimentional case are straightforward. A function $C : [0,1]^2 \rightarrow [0,1]$ is a *copula* if, for $0 \leq x \leq 1$ and $x_1 \leq x_2$, $y_1 \leq y_2$, (x_1, y_1), $(x_2, y_2) \in [0,1]^2$, it fulfills the following properties:

$$C(x,1) = C(1,x) = x, \quad C(x,0) = C(0,x) = 0. \tag{1}$$

$$C(x_2, y_2) - C(x_2, y_1) - C(x_1, y_2) + C(x_1, y_1) \geq 0. \tag{2}$$

Property (1) means uniformity of the margins, while (2), the *n-increasing property* means that $P(x_1 \leq X \leq x_2, y_1 \leq Y \leq y_2) \geq 0$ for (X,Y) with distribution function *C*.

In the seminal paper of Sklar (1959), it was demonstrated that a Copula is linked with a distribution function and its marginal distributions. This important theorem states that:

(i) Let *C* be a copula and F_1 and F_2 univariate distribution functions. Then (3) defines a distribution function *F* with marginals F_1 and F_2.

$$F(x,y) = C\big(F_1(x), F_2(y)\big), \ (x,y) \in R^2. \tag{3}$$

(ii) For a two-dimensional distribution function *F* with marginals F_1 and F_2, there exists a copula *C* satisfying (3). This is unique if F_1 and F_2 are continuous and then, for every $(u,v) \in [0,1]^2$:

$$C(u,v) = F\big(F_1^{-1}(u), F_2^{-1}(v)\big). \tag{4}$$

In (4), $F_1^{-1}$and F_2^{-1} denote the generalized left continuous inverses of F_1 and F_2. However, as Frees and Valdez (1998) note, it is not always obvious to identify the copula. Indeed, for many financial applications, the problem is not to use a given multivariate distribution but consists in finding a convenient distribution to describe some stylized facts, for example the relationships between different asset returns.

2.2 Pair Copula Construction

The PCC is a very flexible construction, which allows for the free specification of *n(n−1)/2* copulas. This construction was proposed by the seminal paper of Joe (1996), and it has been discussed in detail, especially, for applications in simulation and inference. Similar to the NAC, the PCC is hierarchical in nature. The modeling scheme is based on a decomposition of a multivariate density into *n(n−1)/2* bivariate copula densities, of which the first *n−1* are dependency structures of unconditional bivariate distributions, and the rest are dependency structures of conditional bivariate distributions (Aas and Berg, 2011).

The PCC is usually represented in terms of the density. The two main types of PCC that have been proposed in the literature are the C (canonical)-vines and D-vines. In the present paper we focus on the D-vine estimation, which accordingly to Aas *et al*. (2009) has the density as in formulation (5)

$$f(x_1,\cdots,x_n)=\prod_{k=1}^{n}f(x_k)\prod_{j=1}^{n-1}\prod_{i=i}^{n-j}c\begin{Bmatrix}F(x_i|x_{i+1},\cdots,x_{i+j-1}),\\F(x_{i+j}|x_{i+1},\cdots,x_{i+j-1})\end{Bmatrix}. \quad (5)$$

In (5), $x_1,\cdots,x_n$ are variables; f is the density function; $c(\cdot,\cdot)$ is a bivariate copula density and the conditional distribution functions are computed, accordingly to Joe (1996), by formulation (6).

$$F(x|\boldsymbol{v})=\frac{\partial C_{x,v_j|\boldsymbol{v}_{-j}}\{F(x|\boldsymbol{v}_{-j}),F(v_j|\boldsymbol{v}_{-j})\}}{\partial F(v_j|\boldsymbol{v}_{-j})}. \quad (6)$$

In (6) $C_{x,v_j|\boldsymbol{v}_{-j}}$ is the dependency structure of the bivariate conditional distribution of *x* and $v_j$conditioned on $\boldsymbol{v}_{-j}$, where the vector $\boldsymbol{v}_{-j}$is the vector $\boldsymbol{v}$ excluding the component v_j.

Thus, the conditional distributions involved at one level of the construction are always computed as partial derivatives of the bivariate copulas at the previous level (Aas and Berg, 2011). Since only bivariate copulas are involved, the partial derivatives may be obtained relatively easily for most parametric copula families. It is worth to note that the copulas involved in (5) do not have to belong to the same family. Hence, we should choose, for each pair of variables, the parametric copula that best fits the data.

2.3 Empirical Method

We collected daily prices from January 2003 to November 2011, totaling 1872 observations of S&P500 (U.S.), DAX (Germany), FTSE100 (England), Nikkei225 (Japan), which represents the developed markets (set 1); Merval (Argentina), Ibovespa (Brazil), IPC (Mexico), IPSA (Mexico), which are the emerging Latin markets; SSEC (China), HSI (Hong Kong), JKSE (Indonesia) and STI (Singapore), which compose the Asia-Pacific emerging markets (set 3). The last 200 observations were separated for posterior validation of the estimated PCC.

Firstly, in order to avoid non-stationarity issues we calculated the log-returns of the assets by formulation (7).

$$r_t=\ln P_t-\ln P_{t-1}. \quad (7)$$

In (7), r_t is the log-return at period *t*; P_t is the price at period *t*.

Before estimate the PCCs, for each set of assets we modeled their marginal. Initially, we used a vector autoregressive model (VAR) to obtain the estimated returns and residuals of each set. The mathematical form of the VAR(p) model used is represented by (8).

$$\boldsymbol{r}_t=\boldsymbol{\phi}_0+\boldsymbol{\Phi}_1\boldsymbol{r}_{t-1}+\cdots\boldsymbol{\Phi}_p\boldsymbol{r}_{t-p}+\boldsymbol{a}_t. \quad (8)$$

In (8), $\boldsymbol{r}_t$ is a *k*-dimensional vector of the log-returns at period *t*; $\boldsymbol{\phi}_0$ is a *k*-dimensional vector of constants; $\boldsymbol{\Phi}_i$, *i=1,...,p* are *k x k* matrixes of parameters; $\{\boldsymbol{a}_t\}$ is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix $\boldsymbol{\Sigma}$.

Subsequently, to consider the well-known conditional heteroscedastic behavior of financial assets, using the residuals of the VAR applied to each set of returns, we used estimated a copula-based GARCH model, with skew-t innovations to fit the asymmetry of the returns, as represented in (9).

$$h_{i,t}^2=c_i+b_ih_{i,t-1}^2+a_i\varepsilon_{i,t-1}^2. \quad (9)$$

Where $h_{i,t}^2$ is the conditional variance of asset *I* in period *t*; a_i, b_i and c_i are parameters; $\varepsilon_{i,t}=h_{i,t}z_{i,t}, z_{i,t}\sim skewed-t(z_i|\phi_i)$. ϕ is the asymmetry parameter. Further, the model was estimated with a student copula as multivariate distribution.

After model the marginal, we estimated a PCC for each set of returns. To that, we standardized the residuals of the VAR-GARCH approach into pseudo-observations $\boldsymbol{U}_j=(U_{1j},\ldots,U_{ij})$ through the ranks as $U_{ij}=R_{ij}/(n+1)$. Subsequently, we ordered the variables by the decreasing order of the sum of the non-linear dependence with the other variables in

the set by the Kendall's tau. Subsequently, to choose the copula that best fits each bivariate pair of variables we employed the AIC criterion.

To validate the choice of a D-vine PCC, we compared each model with their counterpart C-vine by the test proposed by Clark (2007). This test allows comparing non-nested models. For this let C_1 and C_2 be two competing vine copulas in terms of their densities and with estimated parameter sets θ_1 and θ_2. The null hypothesis of statistical indifference of the two models is:

$$H_0: P(m_i > 0) = 0.5\,,\ m_i = \log\left[\frac{C_1(u_i|\theta_1)}{C_2(u_i|\theta_2)}\right],\ \forall_i = 1, \cdots, n. \quad (10)$$

We used each fitted PCC, to determine the risk of the return distribution for equally weighted portfolios composed by each set of assets. The PCCs estimated from January 2003, to December 2010, is used to forecast one-day VaR at different significance levels for each day in the period from January 2011, to November, 2011 (200 days). The procedure, adapted from Aas and Berg (2011), is as follow. For each day *t* in the prediction period:

We computed the one-step ahead forecast of the conditional standard deviation $\sigma_{j,t}$ of each asset through the estimated GARCH models; We simulated 10,000 samples u_1, u_2, u_3, u_4 by the estimated PCCs; each set of simulation was converted to z_1, z_2, z_3, z_4 samples through the inversion of their density probability (skew-t); For each asset *j*, we determine the 10,000 simulations of the daily log return by $r_{j,t} = \sigma_{j,t} z_j$; We computed the return of the portfolio as the mean of the log-returns of each asset, being 10,000 simulations in each period; For each significance level $q = \{0.05, 0.01, 0.005\}$ we computed the one-day VaR_t^q as the *q*th quantile of the distribution of the portfolio return.

If the observed log-return of the constructed portfolio is below the predicted VaR a violation occurred. To test the significance of the difference between the realized and the expected number of violations, we use the likelihood ratio statistic by Kupiec (1995), represented by formulation (11).

$$2\ln\left[\left(\frac{x}{n}\right)^x \left(1 - \frac{x}{n}\right)^{n-x}\right] - 2\ln[\alpha^x (1-\alpha)^{n-x}]. \quad (11)$$

In (11), the null hypothesis is that the expected proportion of violations is equal to α; *x* is the number of occurred violations; *n* is the length of the sample. This statistic is asymptotically distributed as $\chi^2(1)$. We have computed p-values of the null hypothesis for each quantile *q*.

3. Results and discussion

We first calculated the log-returns of each asset for the studied period. The Figures 1, 2 and 3 exhibit the plots of these returns for each set of markets. These Figures elucidate that the developed markets has less oscillation than the emerging ones, as expected due to their economic solidity and financial liquidity. It should be noted also that there was clear vestiges of turbulence periods during the well-known financial crisis, as pointed by the volatility clusters. The most noted clusters occurred around the observations 1200 to 1400, representing the sub-prime crisis of 2007/2008.

In order to complementing this initial visual analysis, we present in Table 1 the descriptive statistics of the markets during the analyzed period. The results in Table 1 confirm that the developed markets tend to have less oscillation than the emerging ones. The mean of the log-returns in these developed markets is also slightly smaller, although no one of the calculated means was significantly different of zero. Further, all markets had leptokurtic log-returns, and, with exception of Mexico and Chile, there was a predominance of negative skewness. These results reinforce the use of a skew-t distribution to model the innovations of the log-returns.

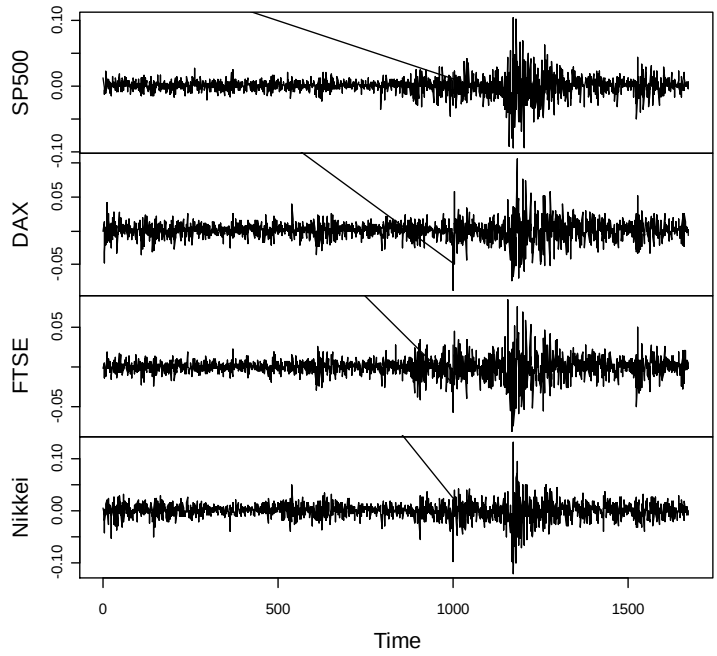

Figure 1. Daily log-returns of the developed markets (set 1) during the period from January 2003, to December 2010.

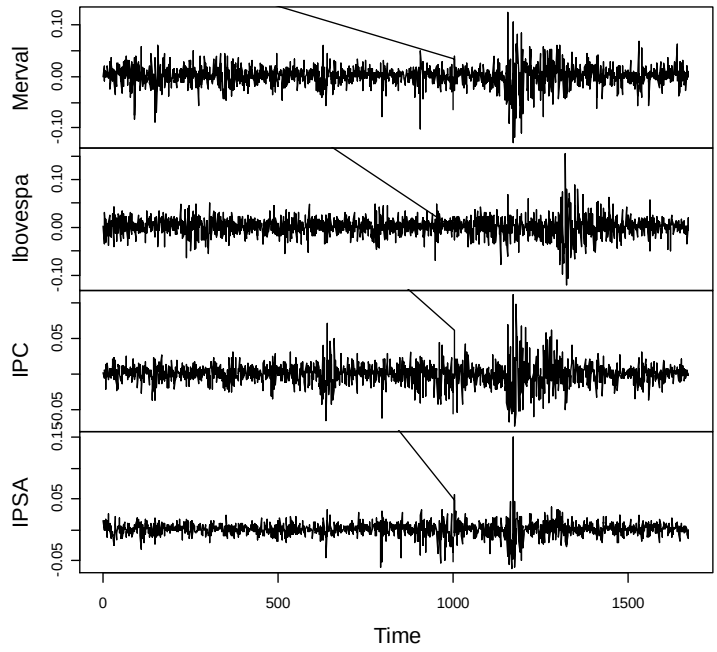

Figure 2. Daily log-returns of the Latin emerging markets (set 2) during the period from January 2003, to December 2010.

Subsequently, to choose the order of the variables in the PCC construction, we estimated the dependence matrix for each set of returns by the Kendall's tau approach. Table 2 presents the results of these dependence matrixes. The chosen criteria was order the assets conform the absolute sum of their calculated Kendall's tau with the others.

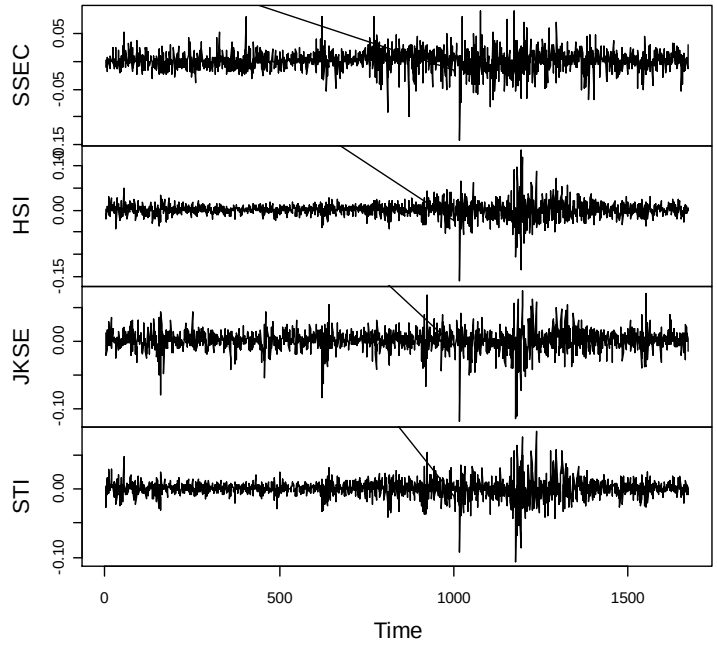

Figure 3. Daily log-returns of the Asia-Pacific emerging markets (set 3) during the period from January 2003, to December 2010.

Table 1. Descriptive statistics of the daily log-returns of the studied markets during the period from January 2003, to December 2010.

Statistic	Minimum	Maximum	Mean	St. Dev.	Skewness	Kurtosis
Set 1						
S&P500	-0.0947	0.1042	0.0001	0.0142	-0.3695	9.1437
DAX	-0.0883	0.1068	0.0003	0.0149	-0.2687	5.3675
FTSE100	-0.0818	0.0847	0.0001	0.0131	-0.1855	5.7954
Nikkei225	-0.1211	0.1323	-0.0001	0.0167	-0.6222	8.6198
Set 2						
Merval	-0.1295	0.1249	0.0009	0.0205	-0.5819	5.4675
Ibovespa	-0.1210	0.1547	0.0011	0.0200	-0.1724	4.8367
IPC	-0.0726	0.1111	0.0009	0.0152	0.1385	5.5279
IPSA	-0.0621	0.1502	0.0008	0.0117	0.5051	19.4637
Set 3						
SSEC	-0.1597	0.1341	0.0004	0.0176	-0.3919	12.4328
HSI	-0.1063	0.0835	0.0004	0.0135	-0.3978	8.1862
JKSE	-0.1147	0.0736	0.0011	0.0161	-0.8622	7.2178
STI	-0.1417	0.0903	0.0004	0.0192	-0.3907	4.5633

The results in Table 2 indicate that, in a general way, the Asia-Pacific markets are more dependent with the others, if compared to the remaining sets. The negative signal in the calculated Kendall's tau only appeared in the Latin markets, for the bivariate cases of Brazil/Argentina and Brazil/Mexico. This result corroborate with the increasing in the globalization of the financial markets, as pointed by the predominance of positive dependence among the log-returns.

With the results contained in Table 2 we decided the order of the variables in the PCCs. For set 1: DAX, FTSE100, Nikkei225 and S&P500; for set 2: IPC, Merval, IPSA and Ibovespa; for set 3: HIS, STI, JKSE and SSEC. After, we modeled the marginal of the assets through the VAR-copula based GARCH procedure explained in the subsection 2.3 of the current paper. The results of the estimation of these model were omitted due to lack of space, beyond are not the principal scope of the study. The residuals of the previous modeling were standardized and utilized to estimate the PCCs. The results of the estimation of the dependence structure of each set of markets are presented in Table 3.

Table 2. Kendall's Tau dependence matrixes of each set of daily log-returns of the studied markets during the period from January 2003, to December 2010.

Developed markets				
	S&P500	**DAX**	**FTSE100**	**Nikkei225**
S&P500	1.0000	0.0171	0.0034	0.0890
DAX	0.0171	1.0000	0.6645	0.2825
FTSE100	0.0034	0.6645	1.0000	0.2658
Nikkei225	0.0890	0.2825	0.2658	1.0000
Sum	0.1095	0.9641	0.9337	0.6373
Latin markets				
	Merval	**Ibovespa**	**IPC**	**IPSA**
Merval	1.0000	-0.0386	0.3586	0.3091
Ibovespa	-0.0386	1.0000	-0.0148	0.0043
IPC	0.3586	-0.0148	1.0000	0.3465
IPSA	0.3091	0.0043	0.3465	1.0000
Sum	0.7063	0.0577	0.7199	0.6599
Asia-Pacific markets				
	SSEC	**HSI**	**JKSE**	**STI**
SSEC	1.0000	0.2458	0.1360	0.1836
HIS	0.2458	1.0000	0.3686	0.5180
JKSE	0.1360	0.3686	1.0000	0.3792
STI	0.1836	0.5180	0.3792	1.0000
Sum	0.5654	1.1324	0.8838	1.0808

The results contained in Table 3 indicate that there is a clear predominance of the Student and BB7 copulas in the bivariate relationships among the three sets of studied markets. Gumbel and BB1 copulas also appeared as having the best fit to some data. These copulas assign, in certain degree, importance to the tails of the joint probability distribution. This fact clarify that there is more dependence among the markets in extreme events than the normally expected. This corroborate with the studies that appoint to an increase of the dependence between markets in periods of great shocks.

Regarding to the differences of the estimated PCCs, the results in Table emphasizes that in the developed markets dependence structure, the student copula was predominant, while BB7 copula obtained the best fit in the most of bivariate relationships. Again, in a general form, the Asia-Pacific markets presented the bigger dependence. Further, all the PCCs rejected the null hypothesis of the Clark test, which states that there is no distinction in the fit of the utilized D-vine approach and the C-vine construction.

The results in Table 3 fundamentally emphasize the need for a properly estimation of the dependence structure of financial assets. This procedure allied with a precise estimation of the marginal of the log-returns should lead to a trustable prediction of the dynamic risk of a portfolio. In this sense, to give robustness for the estimated PCCs, we exhibit in Figures 4,

5 and 6 the observed log-returns and the predicted one-day VaR for each portfolio, conform procedure explained in subsection 2.3, composed by the studied groups of markets for the out-sample period from January 2011, to November 2011, totalizing 200 observations .

Table 3. Pair Copula Constructions (considering the best copula for every bivariate relationship) for each set of markets during the period from January 2003, to December 2010.

Developed markets			
Pair	**Copula**	**Parameter 1**	**Parameter 2**
DAX,FTSE100	Student	0.7705	4.4751
FTSE100,Nikkei225	Student	0.2116	3.9106
Nikkei225,S&P500	BB7	1.2355	0.2081
DAX,Nikkei225|FTSE100	Gumbel	1.1038	-
FTSE100,S&P500|Nikkei225	Student	0.0606	4.2582
DAX,S&P500|FTSE100,Nikkei225	Student	0.0415	5.9611
Clark test			739 (0.0000)
Latin emerging markets			
Pair	**Copula**	**Parameter 1**	**Parameter 2**
IPC,Merval	BB7	1.4136	0.7968
Merval, IPSA	BB7	1.3288	0.6283
IPSA, Ibovespa	Student	0.0044	13.8539
IPC,IPSA|Merval	BB1	1.1918	1.1994
Merval,Ibovespa|IPSA	Gumbel (rotated 90º)	-1.0302	-
IPC,Ibovespa|Merval,IPSA	Student	-0.0080	20.1479
Clark test			907 (0.0003)
Asia-Pacific emerging markets			
Pair	**Copula**	**Parameter 1**	**Parameter 2**
HSI, STI	BB7 (rotated 180º)	2.1605	1.2245
STI, JKSE	BB7	1.5307	0.8327
JKSE,SSEC	Student	0.2076	6.2932
HSI,JKSE|STI	Student	0.2063	10.5908
STI,SSEC|JKSE	Gumbel (rotated 180º)	1.1447	-
HSI,SSEC|STI,JKSE	BB7 (rotated 180º)	1.1163	0.2604
Clark test			882 (0.0260)

As can be visually percept by the plots in Figures 4, 5 and 6, there are volatility clusters in the last fifty observations of the portfolios returns. This turbulence represents some vestiges of the European crisis, which spillover in the whole world markets. Beyond this volatility prediction, which come from the estimated GARCH models, it is notable the precision of the estimated PCCs in model the dependence structure among the analyzed markets. This because just in few days the observed log- return of the constructed portfolios was below the predict values of the one-day VaR. To statically test this robustness in the prediction of the daily risk of the constructed portfolios, we applied the back test presented in the subsection 2.3. The results of this test are present in Table 4.

Results in Table 4 explicit that only the Latin portfolio for the 5% level and the Asia-pacific portfolio for 5% and 1% levels rejected the null hypothesis of that the expected proportion of violations is equal to the significance level of the VaRs. Even for these cases, the number of violations was smaller than the expected. The obtained result confirms the robustness of the dependence structure estimated by the PCCs, once that none of the portfolios had more violations than the proportionally expected by the predict one-day VaRs. This highlights the relevance of a precise and real specification of the dependence among

markets, beyond the dynamic behavior of the conditional variance of assets in the risk management of portfolios.

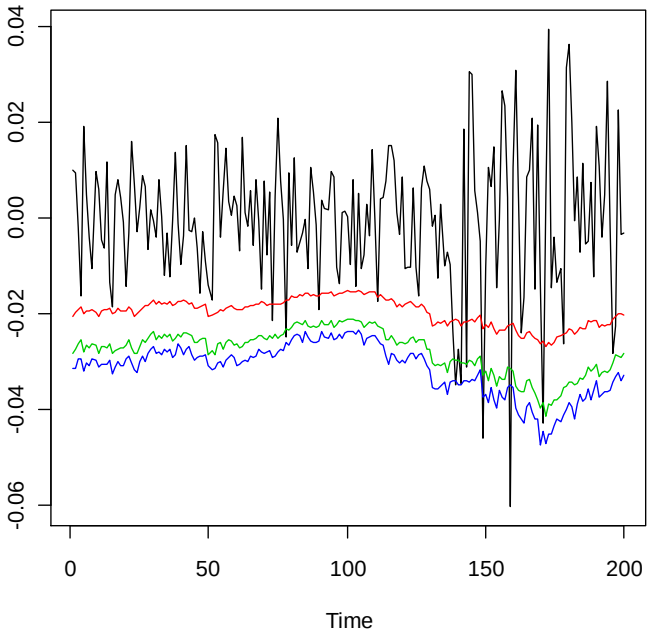

Figure 4. Observed log-returns (black) of the portfolio composed by the developed markets and the predicted one-day VaR for the 5% (red), 1% (green) and 0.5% (blue) through the Copula-GARCH-PCC construction for the period from January 2011, to November 2011.

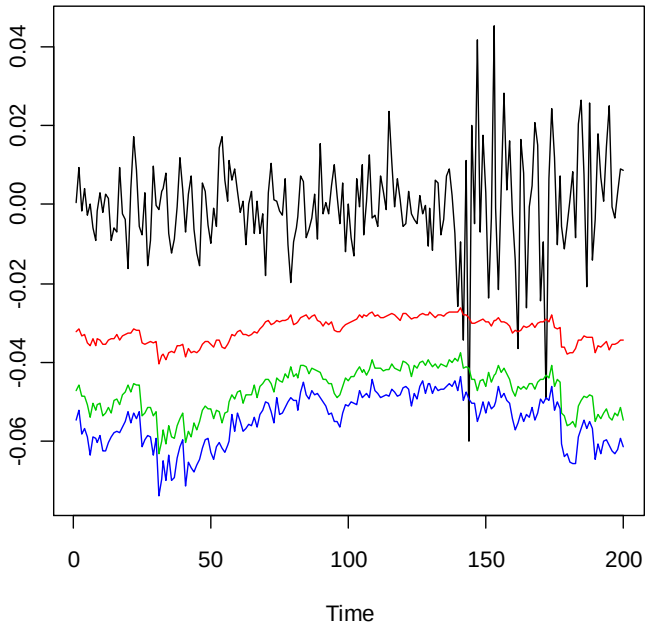

Figure 5. Observed log-returns (black) of the portfolio composed by the Latin emerging markets and the predicted one-day VaR for the 5% (red), 1% (green) and 0.5% (blue) through the Copula-GARCH-PCC construction for the period from January 2011, to November 2011.

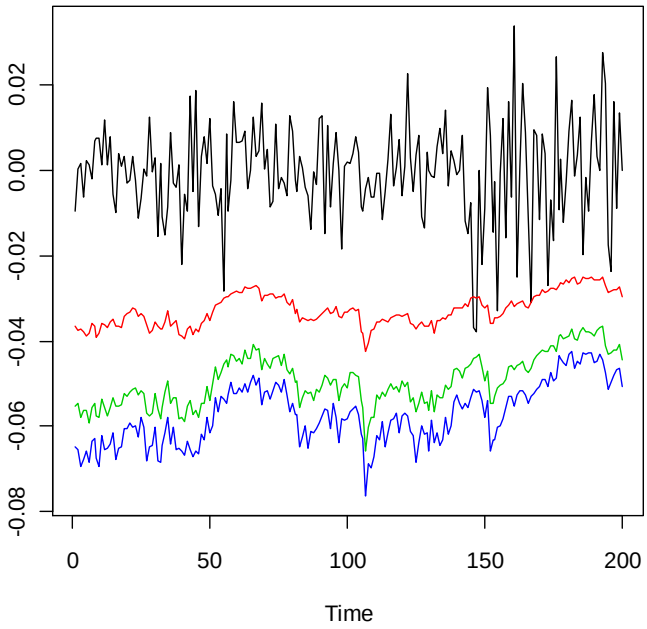

Figure 6. Observed log-returns (black) of the portfolio composed by the Asia-Pacific emerging markets and the predicted one-day VaR for the 5% (red), 1% (green) and 0.5% (blue) through the Copula-GARCH-PCC construction for the period from January 2011, to November 2011.

Table 4. Back test of the predicted one-day VaR for the log-returns of the portfolios constructed with the PCCs with each of the studied sets of markets for the out-sample period from January 2011, to December 2011.

Portfolio	Significance	Violations	Expected	Test	p-value
Developed	5%	11	10	0.1021	0.7493
	1%	5	2	3.2086	0.0732
	0.5%	3	1	19.4107	0.1061
Latin	5%	4	10	4.8572	**0.0275**
	1%	2	2	0.0000	1.0000
	0.5%	1	1	0.0000	1.0000
Asia-Pacific	5%	2	10	9.8945	**0.0016**
	1%	0	2	4.0201	**0.0450**
	0.5%	0	1	2.0050	0.1568

* Bold values are significant at 5% level.

4. Conclusions

In this paper we estimate the daily risk prediction of portfolios composed by assets of distinct markets, considering the dependence structure among them. To that, we used data from developed markets (U.S., Germany, England and Japan), Latin (Argentina, Brazil, Mexico and Chile) and Asia-Pacific (China, Hong Kong, Indonesia and Singapore) emerging markets in the period from January 2003, to December 2010.

We first estimated the marginal of the assets through a copula based multivariate GARCH model for each set of markets. Subsequently, we standardized the residuals of the

GARCH models and estimated the PCCs. The results evidenced that the Student copula predominated in the bivariate relationships of the developed markets, while the BB7 copula was the most present in the relationships of the emerging markets. Gumbel and BB1 copula also appeared in the PCCs. This fact clarify that there is more dependence among the markets in extreme events than the normally expected, once that these copulas assign, in certain degree, importance to the tails of the joint probability distribution. Thus, this dependence structure estimation reinforced the need for a properly estimation of the dependence structure of financial assets, independently of its economic stage.

After, to give robustness to the estimated PCCs, we predicted one-day VaRs for the 5%, 1% and 0.5% levels of significance for the log-returns of portfolios constructed with each set of markets for the out-sample period from January 2011, to November, 2011, totalizing 200 observations. To test the efficacy of the predictions we back test if the number of violations was different of the expected. Just few cases rejected the null hypothesis, but even for these cases, the number of violations was smaller than the expected. This confirmed the robustness of the estimated PCCs, and frizzed the relevance of real specification of the dependence among markets in the risk management of portfolios.

Finally, we suggest for future research that the PCC procedure be used for others financial applications, as the optimal allocation in a portfolio based on the non-linear dependence measure obtained through the parameters of the estimated copulas.

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