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# Retail Pricing and Clearance Sales: The Multidimensional Case

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## Abstract

In this paper I generalize Lazear's model (1986) of retail pricing to the case of a monopolist selling two goods to a potential buyer with unknown valuations. The optimal prices and profit levels are computed for different distributions of valuations using Monte-Carlo simulations. Preliminary results show that the decrease in profits of the suboptimal pure bundling strategy (where the firm has to set only 2 prices) in both periods is extremely small relative to the profits of the optimal mixed bundling strategy (where the firm has to set 6 prices).

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## 1 Introduction

In this paper I generalize Lazear's model (1986) to the case of a firm selling two goods. In Lazear original specification a firm sells a unique zero cost good to a non-strategic consumer with unknown valuation distributed uniformly on $[0, 1]$ over two periods.[1] This is the situation some firms face when launching a new product to a market with unknown initial demand. In his paper Lazear uses the story of a department store selling a one-of-a-kind designer gown to motivate the model. He shows that the optimal strategy is to charge a price of 2/3 in the first period and 1/3 in the second period generating an expected profit of 1/3.[2] Profits increase when the firm sets two prices because the first period price is used to learn about the unknown valuation for the good. If the product is not sold at price $p$, the firm must revise downwards its estimate of the value of the product. Now the firm knows that the valuation is uniformly distributed in $[0, p]$, reducing demand uncertainty and thus increasing profits.

It is interesting to investigate how the above pricing scheme is modified when the firm sells two goods. In the standard static two-good model the firm has to set three prices, two for the individual goods and one for the bundle. Adams and Yellen (1976) have shown that profits increase using this strategy (called mixed bundling) because the firm is better able to segment the market. In fact, when valuations are uniformly distributed in the unit square $[0, 1]^2$, then the optimal prices in the one-period case are given by $p_A = p_B = .667$, $p_{AB} = .8619$, generating an expected profit of $\pi = .5492$, where $p_A$, $p_B$, $p_{AB}$ are the prices of good $A$, $B$ and the bundle respectively.[3] Suppose now that the firm in Lazear original setting has two goods to sell, for example a designer gown and a bag. The firm can increase profits in the one period setting using a mixed bundling strategy. Moreover, if there is uncertainty about how much consumers value the goods, then the firm can further increase profits by using this strategy in both periods. However, what are the optimal prices in the dynamic two-period case? What is the reduction in profits of using a simple pure bundling strategy (charging only one price for the bundle) in both periods as opposed to mixed bundling? Are the results robust to different distributions of consumer valuations? In this paper I investigate these questions using Monte-Carlo methods due to the complexity of the model. I present the results of the simulations in the Appendix, interestingly it is found that the reduction in profits of using a pure bundling strategy in all periods is very small relative to the optimal multi-period mixed bundling strategy. This result is robust to different distributions of consumer valuations.

## 2 Model

The model is a mixture of Lazear and a standard multidimensional monopolist model. A monopolist has two goods to sell, denoted $A$ and $B$, with zero marginal cost to a potential consumer with unknown valuations during two periods. The monopolist has to

[1] In his model non-strategic means that a buyer who gets a positive surplus in the first period will buy immediately even if she can get a higher discounted surplus by waiting. Later on I will mention settings where this assumption is reasonable.

[2] While expected profits are 1/4 in the one-period monopolist problem.

[3] Notice that these are optimal prices *given* this particular mechanism of assigning goods. The problem of finding the optimal mechanism among *all* possible ones is a very difficult task and usually involves very exotic mechanisms where goods are assigned randomly, see Manelli and Vincent (2006). Moreover, these exotic mechanisms can be implemented as price discounts in a dynamic setting, see Runco (2010).

set three prices in each period (two for the individual goods and another for the bundle). The consumer values the goods linearly with $x_A$ and $x_B$ being the values for good A and B respectively. The values $x_i$ are distributed according to $f(x_i)$.[4] The surplus of each choice is given by $x_A - p_A,\ x_B - p_B,\ x_A + x_B - p_{AB}$ when buying only good A, good B or the bundle respectively. The consumer knows the prices the monopolist charges in each period and selects the choice that maximizes her surplus. I must note that in Lazear's and our model, the consumer does not behave strategically; in the sense that if the buyer can get a positive surplus in the first period, she will buy immediately. That is, the consumer will choose to wait until next period only if her utility-maximizing choice in the current period gives a utility of zero. The assumption is reasonable in certain environments, for example, if the price of the good is low relative to wealth then some consumers might not want to wait as long as they get a surplus today. Moreover, if the firm has a limited number of items to sell, there is a high probability that by waiting the consumer might not find the item in the next period. Suppose there are N potential buyers all with the same valuation, each consumer knows her valuation but is ignorant about the others'. For N sufficiently large, each consumer believes that if she refuses to buy during the first period the good will not be available next period. Finally, the assumption applies to the case of individual with self-control problems, see O'Donoghue and Rabin (2000).

The problem for the monopolist is to choose prices $p^t = (p^t_A, p^t_B, p^t_{AB})$ in period $t$ to maximize profits

$$\max_{(p^1,p^2)\geq 0} p^1 M^1(p^1) + p^2 M^2(p^1, p^2)$$

where the sets $M^t = (M^t_A, M^t_B, M^t_{AB}, M^t_0)$ for $t = 1, 2$ represent the mass of consumer valuations that buy good A, good B, the bundle and do not buy respectively at period t. Notice that the set of consumers who buy at $t = 1$ depends only on prices set in the first period, while the set of consumer who buy at $t = 2$ depend on first and second period prices.

The consumer problem is to choose the option that maximizes utility in each period. Let the set of possible choices at period t be $S^t = \{x_A - p^t_A, x_B - p^t_B, x_A + x_B - p^t_{AB}, 0\}$ then $M^1$ is given by (the set $M^2$ is constructed analogously)

$$\begin{aligned} M^1_A &= L(\{(x_A, x_B) : x_A - p^1_A = \max S^1\}) \\ M^1_B &= L(\{(x_A, x_B) : x_B - p^1_B = \max S^1\}) \\ M^1_{AB} &= L(\{(x_A, x_B) : x_A + x_B - p^1_{AB} = \max S^1\}) \end{aligned}$$

where $L$ is the Lebesgue measure of the set of valuations.

There is one important difference between Lazear's model and mine that makes the two-good case much more difficult to solve analytically. The one-good case problem is recursive, if the buyer does not purchase initially then the second period problem is identical to the first one (same uniform distribution but with different support). However, our problem is not recursive as shown in Figure 1. It shows the unit square and the areas $M^1_A, M^1_B$ and $M^1_{AB}$ determined by arbitrary prices $p_A, p_B, p_{AB}$. The areas $M^1_A, M^1_B$ and $M^1_{AB}$ represents the consumer valuations that choose good A, B and the bundle

[4] I assume valuations are iid, however, the simulations can easily be modified to account for different distributions and/or correlated values.

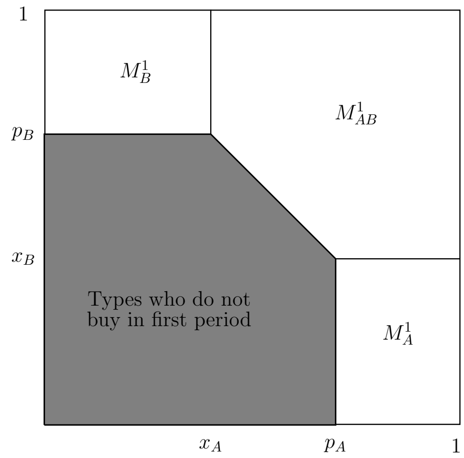


Figure 1: First period choices for different valuations using mixed bundling in first period.

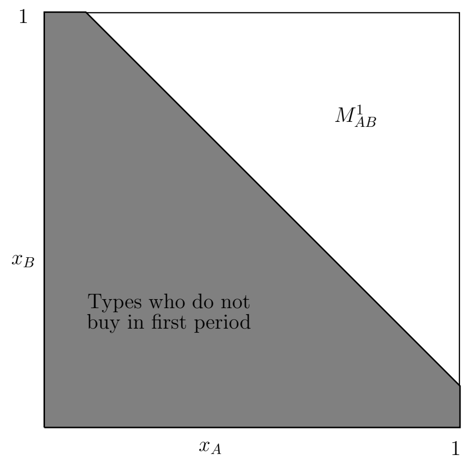


Figure 2: First period choices for different valuations using pure bundling in first period.

respectively in the first period, all other valuations do not buy initially. Therefore the gray area represent the new support of the distribution the monopolist uses to set the prices in period 2. The distribution of values on $[0,1]^2$ is different than that on the set in one important aspect. Namely, if the values $x_A, x_B$ are distributed independently on the unit square, then they will be negative correlated on the gray set.

Given the difficulty of the setting, I have solved the monopolist problem using Monte-Carlo simulations. One hundred thousand consumers are created with values for each good distributed according to $f(x_i)$ on $[0,1]$. A grid of prices is created and the monopolist's profits are computed for every combination of prices and the profit maximizing combination is selected. This procedure was repeated 100 times. In Table 1 in the appendix I present the average and standard deviation of optimal prices and profits for different strategies and distributions. For the sake of completeness I decided to also include the intermediate case of the pricing strategy based on using pure bundling in the first period and mixed bundling in the second. The column APB (Always Pure

Bundling) refers to the strategy of pure bundling in both periods. PBMB (Pure Bundle, Mixed Bundling) refers to using pure bundling in the first period and mixed bundling in the second. Finally, AMB (Always Mixed Bundling) refers to the strategy of using always mixed bundling.

It is known that mixed bundling will dominate the other two strategies in terms of profits. However, it is surprising to see that profits in the very simple pure bundling strategy are very close to the optimum. From a lower bound of 0.039% profit difference in the case of the normal distribution to an upper bound of 0.963% difference in the case of the uniform distribution. The result is interesting considering the fact that if a firm sells $N$ different goods, then in the optimal two-period mixed bundling strategy it has to set $2^{N+1} - 2$ prices, whereas in the simple pure bundling strategy it has to set only 2 prices with a minor penalty in terms of profits.

The simple two-period pure bundling strategy generates almost the same profit has the optimal two-period mixed bundling because the set of values of consumers who do not buy at $t = 1$ is negatively correlated, as shown in Figure 2. And as is well know since Adams and Yellen, the benefit of bundling increases in this circumstance. By just charging one price in the first period, the firm is able to generate a more negatively correlated set of valuations than by charging three prices as seen in Figure 1, thus increasing the benefit in the subsequent period.

## 3 Conclusion

In this paper I have analyzed a model of a monopolist selling two goods to a consumer with unknown valuation on $[0, 1]^2$ over two periods. I have computed the optimal prices and profits for different distribution of valuations and have shown that the reduction in profits of using a simple pure bundling strategy (with 2 prices) in all periods is very small relative to the optimal multi-period mixed bundling strategy (with 6 prices). This result is robust to different distribution of consumer valuations.

## 4 Appendix

| | Strategy APB | Strategy PBMB | Strategy AMB |
|---|---|---|---|
| Uniform Distribution | $p^1_{AB} = 1.0818$ (0.0128)<br>$p^2_{AB} = 0.6219$ (0.0102) | $p^1_{AB} = 1.0805$ (0.0117)<br>$p^2_{AB} = 0.6204$ (0.0094)<br>$p^2_B = p^2_A = 0.6178$ (0.0106) | $p^1_{AB} = 1.1132$ (0.0094)<br>$p^1_B = p^1_A = 0.8696$ (0.0094)<br>$p^2_{AB} = 0.6420$ (0.0065)<br>$p^2_B = p^2_A = 0.6186$ (0.0134) |
| Profits | $\pi = 0.6958$ (0.0013) | $\pi = 0.6957$ (0.0009) | $\pi = 0.7025$ (0.0010) |
| Truncated Normal Distribution $\mu = .5, \sigma = .2$ | $p^1_{AB} = .9919$ (0.0060)<br>$p^2_{AB} = .6378$ (0.0062) | $p^1_{AB} = .9924$ (0.0062)<br>$p^2_{AB} = .6388$ (0.0073)<br>$p^2_B = p^2_A = .5425$ (0.0133) | $p^1_{AB} = .9930$ (0.0055)<br>$p^1_B = p^1_A = .8462$ (0.0133)<br>$p^2_{AB} = .6720$ (0.0049)<br>$p^2_B = p^2_A = .5556$ (0.0137) |
| Profits | $\pi = .7602$ (0.0008) | $\pi = .7604$ (0.0008) | $\pi = .7605$ (0.0007) |
| Truncated Log-Normal Distribution $\mu = 0, \sigma = 1$ | $p^1_{AB} = 1.0659$ (0.008)<br>$p^2_{AB} = 0.6250$ (0.0083) | $p^1_{AB} = 1.0674$ (0.0073)<br>$p^2_{AB} = 0.6269$ (0.0074)<br>$p^2_B = p^2_A = 0.6151$ (0.0126) | $p^1_{AB} = 1.0774$ (0.0091)<br>$p^1_B = p^1_A = 0.9090$ (0.0083)<br>$p^2_{AB} = 0.6534$ (0.0060)<br>$p^2_B = p^2_A = 0.6385$ (0.0108) |
| Profits | $\pi = 0.7548$ (0.0011) | $\pi = 0.7549$ (0.0009) | $\pi = 0.7555$ (0.009) |
| Truncated Exponential Distribution $\lambda = 1$ | $p^1_{AB} = 0.9338$ (0.0082)<br>$p^2_{AB} = 0.4971$ (0.0085) | $p^1_{AB} = 0.9332$ (0.0082)<br>$p^2_{AB} = 0.4975$ (0.0082)<br>$p^2_B = p^2_A = 0.4977$ (0.0088) | $p^1_{AB} = 0.9643$ (0.0097)<br>$p^1_B = p^1_A = 0.8204$ (0.0060)<br>$p^2_{AB} = 0.5245$ (0.0059)<br>$p^2_B = p^2_A = 0.5256$ (0.0092) |
| Profits | $\pi = 0.5607$ (0.0011) | $\pi = 0.5605$ (0.0009) | $\pi = 0.5624$ (0.0010) |

Table 1: Average optimal prices and profits for different priors. Standard deviations in parenthesis.

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