Supermajority Rules and the Swing Voter's Curse

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Abstract

The Swing Voter's Curse is extended to incorporate a class of supermajority rules.

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1. Introduction

The Swing Voter's Curse (Feddersen and Pesendorfer, 1996) (FP) is an important contribution to the understanding of strategic voting. An independent voter is interested in casting her vote for the option that is "best/correct", but lacks the knowledge of which is it. FP show that less informed, indifferent voters strictly prefer to abstain rather than vote for either option even when voting is costless. She may be better off leaving the decision to informed voters because if her choice goes against theirs she could cause the wrong outcome to arise. Thus, a substantial number will abstain even though all abstainers strictly prefer one candidate over the other.

The result has been used to explain voter turnout (Lassen, 2005) and experimental evidence of such behavior has been provided (Battaglini, Morton, and Palfrey, 2006). The environment considered is one where (nonpartisan) agents share a common objective. Information constrains the members of the group from knowing with certainty which option to support. Examples of such environments include juries convicting or acquitting a defendant, academic committees making curriculum decisions, and administrative committees setting common standards (e.g. accounting standards set by FASB).

What is common among these examples is that supermajority rules are often used. That is to adopt an alternative policy to the status quo a proportion of the votes greater than one-half is required. Supermajority rules give preferential treatment to one policy, the status quo, over all others. It is unanswered whether the Swing Voter's Curse holds with such voting rules.

Thus, the objective is to extend the result of FP. It is shown that the Swing Voter's Curse holds for a class of supermajority rules. This class includes the commonly used thresholds of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.

2. Extension of the Swing Voter's Curse

The notation used is the same as in FP and Fey and Kim (2002). There are two states, $Z = \{0,1\}$, and two options, $X = \{0,1\}$. Without loss in generality refer to option 0 as the status quo and option 1 as the alternative. There are three types of agents, $T = \{0,1,i\}$. Type 0 and 1 are partisans and have a dominant strategy to support option 0 and 1 respectively. Type i agents are independents. Utility of an independent is

$$U(x,z) = \begin{cases} -1 & \text{if } x \neq z \\ 0 & \text{if } x = z \end{cases} . \tag{1}$$

First, nature chooses state 0 with probability α. Nature also selects a set of agents by taking $N+1$ independent draws. In each draw nature selects an agent with probability $1-p_\phi$. If an agent is selected, then she is type t with probability $\frac{p_t}{1-p_\phi}$. After the state and set of agents is selected, each learns her type and receives a message $\eta \in M = \{0, \alpha, 1\}$. The message is private information. Hence, if $\eta \in \{0,1\}$ she learns the state. Thus, all informed agents receive the same message. The probability an agent is informed is q. A type i agent with $\eta = \alpha$ believes $z = 0$ with probability α and is referred to as an uninformed independent agent, or rather, UIA. Assume $p_0, p_1, p_i, p_\phi, \alpha, q \in (0,1)$.

Every agent selects an action $s \in \{\phi, 0, 1\}$ where ϕ indicates abstention and 0 and 1 represents voting for the option. A mixed strategy is denoted by $\tau : T \text{ x } M \rightarrow [0,1]^3$, where τ_s is the probability of taking action s.

Suppose that for every a votes 0 receives b votes must be received for 1 for it to win. This is a $\frac{b}{a+b}$-majority rule. Thus, $a = b$ is simple majority and $a < b$ is a supermajority rule. Attention is restricted to integers, $a, b \in \mathbb{Z}_{++}$. Furthermore, assume $N = (a+b)\, m$, $m \in \mathbb{Z}_{++}$.[1]

Define $\sigma_{z,x}(\tau)$ as the probability a random draw by nature results in a vote for x if the state is z. Hence,

$$\sigma_{z,x}(\tau) = \begin{cases} p_x + p_i (1-q)\, \tau_x & \text{if } z \neq x \\ p_x + p_i (1-q)\, \tau_x + p_i q & \text{if } z = x \end{cases}. \tag{2}$$

Define $\sigma_{z,\phi}(\tau)$ as the probability a random draw by nature does not result in a vote for either option. Thus,

$$\sigma_\phi(\tau) = p_i (1-q)\, \tau_\phi + p_\phi. \tag{3}$$

To determine an UIA's optimal voting behavior one must identify the probability her vote influences the outcome. There are three ways this can happen. First, an UIA may break a tie. Define $\pi_t(z,\tau)$ as the probability the voting of the other agents has resulted in a tie. Thus,

$$\pi_t(z,\tau) = \sum_{j=0}^{m} \frac{N!\, \sigma_\phi(\tau)^{N-(b+1)j}}{j!\,(bj)!\,(N-(b+1)\,j)!} \left[\sigma_{z,0}(\tau)\, \sigma_{z,1}(\tau)^b\right]^j. \tag{4}$$

[1] FP consider only simple majority voting and assume an odd number of voters (N is even) so that $m = \frac{N}{2}$. This setup replicates Fey and Kim (2002). It is straightforward to generalize the setup by assuming $N = (a+b)\, m + r$, but the assumption used simplifies the analysis. Furthermore, a and b are reduced so that they are not a multiple of the same integer (other than one).

Second, an UIA may create a tie. Define $\pi_0(z,\tau)$ and $\pi_1(z,\tau)$ as the probability a vote for 0 and 1 respectively creates a tie. Thus,

$$\pi_0(z,\tau) = \sum_{j=1}^{m} \frac{N!\sigma_\phi(\tau)^{N-(b+1)j+1}\sigma_{z,0}(\tau)^{-1}}{(j-1)!(bj)!(N-(b+1)j+1)!}\left[\sigma_{z,0}(\tau)\sigma_{z,1}(\tau)^b\right]^j. \quad (5)$$

and

$$\pi_1(z,\tau) = \sum_{j=1}^{m} \frac{N!\sigma_\phi(\tau)^{N-(b+1)j+1}\sigma_{z,1}(\tau)^{-1}}{j!(bj-1)!(N-(b+1)j+1)!}\left[\sigma_{z,0}(\tau)\sigma_{z,1}(\tau)^b\right]^j. \quad (6)$$

Finally, the voting of the others may result in 1 winning, but an UIA's vote for 0 may switch the outcome. Define $\pi_s(z,\tau)$ as this probability. This occurs if the other N agents cast l more votes for 1 than is needed for a tie and $l < b$.[2] Thus,

$$\pi_s^l(z,\tau) = \sum_{j=0}^{m-1} \frac{N!\sigma_\phi(\tau)^{N-(b+1)j-l}\sigma_{z,1}(\tau)^{l}}{k!(bj+l)!(N-(b+1)j-l)!}\left[\sigma_{z,0}(\tau)\sigma_{z,1}(\tau)^b\right]^j \quad (7)$$

so that

$$\pi_s(z,\tau) = \sum_{l=1}^{b-1} \pi_s^k(z,\tau). \quad (8)$$

If $1 = a = b$, then $\pi_s(z,\tau) = 0$ since switching outcomes with one vote is not possible.

3. Swing Voter's Curse

FP's main result is extended to a class of supermajority voting rules.

Proposition 1 *Suppose $a = 1$. For any symmetric strategy profile τ in which no agent plays a strictly dominated strategy, $Eu(1,\tau) = Eu(0,\tau)$ implies $Eu(1,\tau) < Eu(\phi,\tau)$.*

[2] With $a = 1$ the corresponding probability of the $N+1^{\text{th}}$ agent switching the outcome from 0 to 1 is zero.

Proof. $Eu(1,\tau) - Eu(0,\tau) = 0$ implies

$$\begin{aligned}
& (1-\alpha)\left(\frac{1}{2}\right)\left[\pi_0(1,\tau) + \pi_1(1,\tau) + 2\pi_t(1,\tau) + 2\pi_s(1,\tau)\right] \\
= \; & \alpha\left(\frac{1}{2}\right)\left[\pi_0(0,\tau) + \pi_1(0,\tau) + 2\pi_t(0,\tau) + 2\pi_s(0,\tau)\right].
\end{aligned}$$

Define D as the sum of the expressions within the two brackets of this equation. Solving for α

$$\widetilde{\alpha} = \frac{\pi_0(1,\tau) + \pi_1(1,\tau) + 2\pi_t(1,\tau) + 2\pi_s(1,\tau)}{D}. \tag{9}$$

Using $\widetilde{\alpha}$ from (8) it follows that $[Eu(1,\tau) - Eu(\phi,\tau)]\, 2D =$

$$\begin{aligned}
& \left[\pi_0(0,\tau) + \pi_1(0,\tau) + 2\pi_t(0,\tau) + 2\pi_s(0,\tau)\right]\left[\pi_1(1,\tau) + \pi_t(1,\tau)\right] \\
& - \left[\pi_0(1,\tau) + \pi_1(1,\tau) + 2\pi_t(1,\tau) + 2\pi_s(1,\tau)\right]\left[\pi_1(0,\tau) + \pi_t(0,\tau)\right].
\end{aligned}$$

Define the following three terms,

$$\begin{aligned}
A(\tau) &\equiv \pi_t(1,\tau)\left[\pi_0(0,\tau) - \pi_1(0,\tau)\right] + \pi_t(0,\tau)\left[\pi_1(1,\tau) - \pi_0(1,\tau)\right] \\
B(\tau) &\equiv \pi_0(0,\tau)\,\pi_1(1,\tau) - \pi_1(0,\tau)\,\pi_0(1,\tau) \\
C(\tau) &\equiv \pi_s(0,\tau)\left[\pi_1(1,\tau) + \pi_t(1,\tau)\right] - \pi_s(1,\tau)\left[\pi_1(0,\tau) + \pi_t(0,\tau)\right].
\end{aligned}$$

Hence, $[Eu(1,\tau) - Eu(\phi,\tau)]\, 2D = A(\tau) + B(\tau) + \frac{C(\tau)}{2}$. Therefore, it is sufficient to show $A(\tau) < 0$, $B(\tau) < 0$, and $C(\tau) < 0$.

Consider, $A(\tau)$. Define $\pi(z,\tau,j)$ such that $\pi(z,\tau) = \sum \pi(z,\tau,j)$. Thus,

$$\begin{aligned}
A_1(\tau) &\equiv \pi_t(1,\tau)\left[\pi_1(0,\tau) - \pi_0(0,\tau)\right] \\
&= \left(\sum_{j=0}^{m} \pi_t(1,\tau,j)\right)\left(\sum_{k=1}^{m} \pi_1(0,\tau,k) - \sum_{k=1}^{m} \pi_0(0,\tau,k)\right) \\
&= \left(\sum_{j=0}^{m} \pi_t(1,\tau,j)\right)\left(\sum_{k=1}^{m} \left[\pi_1(0,\tau,k) - \pi_0(0,\tau,k)\right]\right) \\
&= \sum_{j=0}^{m}\sum_{k=1}^{m} \pi_t(1,\tau,j)\left[\pi_1(0,\tau,k) - \pi_0(0,\tau,k)\right] \\
&= \sum_{j=0}^{m}\sum_{k=1}^{m} \frac{N!N!\sigma_\phi(\tau)^{N-(b+1)j}\,\sigma_\phi(\tau)^{N-(b+1)k+1}}{j!\,(k-1)!\,(bj)!\,(bk-1)!\,(N-(b+1)j)!\,(N-(b+1)k+1)!} \\
& \quad \text{x} \left[\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)^b\right]^j \left[\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)^b\right]^k \left(\frac{\sigma_{0,1}(\tau) - b\sigma_{0,0}(\tau)}{bk\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)}\right).
\end{aligned}$$

Additionally,

$$\begin{aligned}
A_2(\tau) &\equiv \pi_t(0,\tau)\left[\pi_1(1,\tau)-\pi_0(1,\tau)\right] \\
&= \left(\sum_{j=0}^{m}\pi_t(0,\tau,j)\right)\left(\sum_{k=1}^{m}\pi_1(1,\tau,k)-\sum_{k=1}^{m}\pi_0(1,\tau,k)\right) \\
&= \left(\sum_{j=0}^{m}\pi_t(0,\tau,j)\right)\left(\sum_{k=1}^{m}\left[\pi_1(1,\tau,k)-\pi_0(1,\tau,k)\right]\right) \\
&= \sum_{j=0}^{m}\sum_{k=1}^{m}\pi_t(0,\tau,j)\left[\pi_1(1,\tau,k)-\pi_0(1,\tau,k)\right] \\
&= \sum_{j=0}^{m}\sum_{k=1}^{m}\frac{N!N!\sigma_\phi(\tau)^{N-(b+1)j}\,\sigma_\phi(\tau)^{N-(b+1)k+1}}{j!\,(k-1)!\,(bj)!\,(bk-1)!\,(N-(b+1)j)!\,(N-(b+1)k+1)!} \\
&\quad \text{x}\ \left[\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)^b\right]^j\left[\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)^b\right]^k\left(\frac{b\sigma_{1,0}(\tau)-\sigma_{1,1}(\tau)}{bk\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)}\right).
\end{aligned}$$

Since $A(\tau)=A_1(\tau)+A_2(\tau)$ and $\left[\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)^b\right]^j=\left[\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)^b\right]^j=1$ when $j=0$, $A(\tau)<0$ if

$$\frac{\sigma_{0,1}(\tau)-b\sigma_{0,0}(\tau)}{bk\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)}+\frac{b\sigma_{1,0}(\tau)-\sigma_{1,1}(\tau)}{bk\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)}<0$$

for all k. Since $\sigma_{1,1}(\tau)=\sigma_{0,1}(\tau)+p_iq$ and $\sigma_{0,0}(\tau)=\sigma_{1,0}(\tau)+p_iq$, it follows that this expression reduces to

$$\frac{-\sigma_{0,1}(\tau)\,\sigma_{1,1}(\tau)\,p_iq-b\sigma_{1,0}(\tau)\,\sigma_{0,0}(\tau)\,p_iq}{bk\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)\,\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)}<0,$$

which holds for all k since $p_i,q>0$. Consequently, $A(\tau)<0\ \forall\tau$.

Next, consider $B(\tau)$. $\pi_0(0,\tau)\,\pi_1(1,\tau)=$

$$\begin{aligned}
&\left(\sum_{k=1}^{m}\pi_0(0,\tau,k)\right)\left(\sum_{j=1}^{m}\pi_1(1,\tau,j)\right) \\
=&\ \sum_{j=1}^{m}\sum_{k=1}^{m}\pi_0(0,\tau,k)\,\pi_1(1,\tau,j) \\
=&\ \sum_{j=1}^{m}\sum_{k=1}^{m}\left[\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)^b\right]^j\left[\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)^b\right]^k\frac{1}{jbk\sigma_{0,0}(\tau)\,\sigma_{1,1}(\tau)}
\end{aligned}$$

$$\text{x}\ \frac{N!N!\sigma_{\phi}(\tau)^{N-(b+1)j+1}\,\sigma_{\phi}(\tau)^{N-(b+1)k+1}}{(j-1)!\,(k-1)!\,(bj-1)!\,(bk-1)!\,(N-(b+1)\,j+1)!\,(N-(b+1)\,k+1)!}$$

and $\pi_1(0,\tau)\,\pi_0(1,\tau) =$

$$\left(\sum_{j=1}^{m}\pi_0(0,\tau,j)\right)\left(\sum_{k=1}^{m}\pi_1(1,\tau,k)\right)$$

$$= \sum_{j=1}^{m}\sum_{k=1}^{m}\pi_0(0,\tau,j)\,\pi_1(1,\tau,k)$$

$$= \sum_{j=1}^{m}\sum_{k=1}^{m}\left[\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)^{b}\right]^{j}\left[\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)^{b}\right]^{k}\frac{1}{jbk\sigma_{0,1}(\tau)\,\sigma_{1,0}(\tau)}$$

$$\text{x}\ \frac{N!N!\sigma_{\phi}(\tau)^{N-(a+b)j+1}\,\sigma_{\phi}(\tau)^{N-(a+b)k+1}}{(j-1)!\,(k-1)!\,(bj-1)!\,(bk-1)!\,(N-(a+b)\,j+1)!\,(N-(a+b)\,k+1)!}.$$

Since $B(\tau)=\pi_0(0,\tau)\,\pi_1(1,\tau)-\pi_1(0,\tau)\,\pi_0(1,\tau)$, $B(\tau)<0$ if $\sigma_{0,1}(\tau)\,\sigma_{1,0}(\tau)<\sigma_{0,0}(\tau)\,\sigma_{1,1}(\tau)$. This holds since $\sigma_{1,1}(\tau)=\sigma_{0,1}(\tau)+p_iq$ and $\sigma_{0,0}(\tau)=\sigma_{1,0}(\tau)+p_iq$ and $p_i,q>0$. Consequently, $B(\tau)<0\ \forall\tau$.

Finally, consider $C(\tau)$. First, $\pi_s(0,\tau)\,[\pi_1(1,\tau)+\pi_t(1,\tau)]$

$$= \sum_{l=1}^{b-1}\sum_{i=0}^{m-1}\sum_{j=1}^{m}\pi_s^{l}(0,\tau,i)\,[\pi_1(1,\tau,j)+\pi_t(1,\tau,j)]$$

$$= \sum_{l=1}^{b-1}\sum_{i=0}^{m-1}\sum_{j=1}^{m}\frac{N!N!\sigma_{\phi}(\tau)^{N-(b+1)j}\,\sigma_{\phi}(\tau)^{N-(b+1)i-l}}{j!i!\,(bj)!\,(bi+l)!\,(N-(b+1)\,j)!\,(N-(b+1)\,i-l)!}$$

$$\text{x}\ \left[\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)^{b}\right]^{j}\left[\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)^{b}\right]^{i}$$

$$\text{x}\ \left(\frac{bj\sigma_{\phi}(\tau)}{\sigma_{1,1}(\tau)\,(N-(b+1)\,j+1)}+1\right)\sigma_{0,1}(\tau)^{l}.$$

Second, $\pi_s(1,\tau)\,[\pi_1(0,\tau)+\pi_t(0,\tau)]$

$$= \sum_{l=1}^{b-1}\sum_{i=0}^{m-1}\sum_{j=1}^{m}\pi_s^{l}(1,\tau,i)\,[\pi_1(0,\tau,j)+\pi_t(0,\tau,j)]$$

$$= \sum_{l=1}^{b-1}\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{N!N!\sigma_{\phi}(\tau)^{N-(b+1)j}\,\sigma_{\phi}(\tau)^{N-(b+1)i-l}}{j!i!\,(bj)!\,(bi+l)!\,(N-(b+1)\,j)!\,(N-(b+1)\,i-l)!}$$

$$\text{x} \left[\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)^{b}\right]^{i}\left[\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)^{b}\right]^{j}$$
$$\text{x} \left(\frac{bj\sigma_{\phi}(\tau)}{\sigma_{0,1}(\tau)\,(N-(b+1)\,j+1)}+1\right)\sigma_{1,1}(\tau)^{l}.$$

Therefore, to show $C(\tau) < 0$ it is sufficient to show that this last expression is greater than the former. Notice that $\sum_{i=0}^{m-1}\sum_{j=1}^{m}\left[\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)^{b}\right]^{j}$ x $\left[\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)^{b}\right]^{i} = \sum_{i=0}^{m-1}\sum_{j=1}^{m}\left[\sigma_{1,0}(\tau)\,\sigma_{1,1}(\tau)^{b}\right]^{i}$ x $\left[\sigma_{0,0}(\tau)\,\sigma_{0,1}(\tau)^{b}\right]^{j}$. Thus, this reduces to showing that

$$\frac{bj\sigma_{\phi}(\tau)\,\sigma_{1,1}(\tau)^{l}}{\sigma_{0,1}(\tau)\,(N-(b+1)\,j+1)} > \frac{bj\sigma_{\phi}(\tau)\,\sigma_{0,1}(\tau)^{l}}{\sigma_{1,1}(\tau)\,(N-(b+1)\,j+1)}.$$

Since $\sigma_{1,1}(\tau) > \sigma_{0,1}(\tau)$ this holds. As a result, $[Eu(1,\tau) - Eu(\phi,\tau)]\,2D < 0$. Hence, for any symmetric strategy profile where no agent plays a strictly dominated strategy if an agent is indifferent between 0 and 1, then ϕ generates a strictly greater utility. ■

The environment is restricted to those with $a = 1$. This includes the common thresholds $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. Future work should relax this restriction.

4. References

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