A stackelberg duopoly with binary choices of objectives

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Abstract
This paper analyzes a Stackelberg model where firms choose their objective functions (profit or revenue) in order to achieve maximum payoff. The objective of the present work is to demonstrate that depending on the unit cost of production, firms make either asymmetric choices of objectives (low values of the unit cost) or symmetric choices (high values of the unit cost). As a result of these choices, the payoff advantage alternates between the first and the second mover in the market.
1 Introduction

The Stackelberg model of competition is one of the benchmark models of modern industrial economics. In this model, firms select their quantities (or prices) sequentially. A main pillar of the analysis focuses on the issue of which firm, the first or the second-mover, obtains a higher market profit. Gal-Or (1985) showed that if the players’ reaction functions are downwards-sloping then the first-mover achieves a higher payoff than his opponent. On the other hand, in the case of upwards-sloping reaction functions the advantage is with the second-mover. Further studies showed that this result is not robust to variations of the model. Gal-Or (1987) studied a Stackelberg model where firms compete under private information about market demand. In this model the first-mover might earn a lower profit than his opponent, as he produces a relatively low quantity so as to send a signal for low demand. Liu (2005) analyzed a model where only the first-mover has incomplete information about the demand and showed that for some cases the first-mover loses the advantage. Wardy (2004) analyzed a sequential game where observing the first-mover’s choice is costly. It is shown that being the leader has no value, no matter how small the observation cost is.

Recently, an integration of the Stackelberg model with the theory of endogenous objectives of oligopolistic firms has taken place. The latter theory was launched with the works of Fershtman and Judd (1985), Vickers (1985) and Sklivas (1987). These works endogenized the objective functions of firms in a context of management/ownership separation by postulating that owners instruct their managers to maximize a convex combination of revenue and profit or quantity and profit. This framework was applied by Kopel and Loffler (2008) to a Stackelberg duopoly with homogeneous commodities (which give rise to downwards sloping reaction functions). The authors analyzed the effect of delegation on the issue of first or second mover advantage. They showed that only the follower has incentive to delegate the production decision to a manager. As a result, the follower produces a higher quantity and achieves higher profit than the leader, irrespective of the data of the model.

However, empirical observations verify that the structure of the profit advantage in sequential oligopolies fluctuates from industry to industry or within the same industry. First-mover’s advantages have been recorded by Robinson (1988), Robinson et.al (1994), Gorecki (1986), etc. Second-mover’s advantages have been recorded by Tellis and Golder (1996), Lilien and Yoon (1990), Shankar et.al (1998), etc. In particular, researchers have emphasized the role of factors such as technology, market demand, etc. as determinants of the profit advantage in actual markets (see Liu 2005).

The current paper is motivated by the above empirical facts. Its aim
is to develop a model that will allow for the endogenous determination of the role of technology and demand on the issue of payoff advantage. We present a Stackelberg market with binary choices of objective functions: the firms’ potential objectives are the profit or the revenue functions. Our three-stage game has the following structure. In stage 0, the two firms select simultaneously between profit and revenue maximization. In the next two stages, play becomes sequential: in stage 1, the leader (firm 1) selects its quantity via the maximization of the objective chosen in stage 0; and in stage 2, the follower (firm 2) responds by choosing its own quantity, again via the maximization of the objective chosen in stage 0. We denote this game by $H$.

We focus on a duopoly market with differentiated products. The two firms face linear demand functions and compete in quantities. Products are either substitutes (corresponding to the case of downwards-sloping reaction functions) or complements (upwards-sloping reaction functions). Production is characterized by constant returns to scale for both competitors. We solve the game for its subgame perfect Nash equilibrium outcome. The main results of the paper are summarized as follows. If the unit cost of production is low, then in stage 0 firm 1 chooses to maximize its profit and firm 2 chooses to maximize its revenue function. On the other hand, if the unit cost is high then both firms choose to maximize their profit functions.

Given the above equilibrium behavior, we show that for both substitute and complement goods, the profit advantage alternates between the leader and the follower. For the case of substitute goods, and for intermediate values of the unit cost of production, the payoff advantage is with the follower; and for low or high unit cost values, the advantage is with the leader. For complement goods, the opposite takes place: for intermediate values of the unit cost, the payoff advantage is with the leader; and for low or high unit cost it is with the follower.

The rest of the paper is organized as follows. Section 2 describes the model, section 3 presents the results and section 4 summarizes the conclusions.

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2We also discuss briefly the case of price competition.
2 The framework
Consider a duopoly market with differentiated goods. Firms 1 and 2 face the inverse demand functions $p_1(q_1, q_2) = a - q_1 - \gamma q_2$ and $p_2(q_1, q_2) = a - q_2 - \gamma q_1$ respectively, where $p_i, q_i$ are the price and quantity of firm $i$, $i = 1, 2$, $a > 0$, $\gamma \in (-1, 1)$. When $\gamma \in (-1, 0)$ the products are complements and when $\gamma \in (0, 1)$ they are substitutes. The production technology is represented by a (common for both firms) linear cost function with zero fixed cost. The marginal cost is given by $c$ where $c < a$.

We construct a game with perfect information which we call $H$. $H$ has three stages. In stage 0, the two firms select their objective functions simultaneously. These choices are then made publicly known. The play then becomes sequential. In stage 1, firm 1 selects its quantity by maximizing the objective function it has chosen in stage 0; in stage 2 firm 2 responds, by selecting its own quantity, again via the maximization of the objective it has chosen in stage 0.

The two candidate objective functions are the profit function and the revenue function. Therefore, a distinction is made between the objective function and the evaluation (or payoff) function of a firm. For example, if firm $i$ chooses its quantity by maximizing its revenue, its objective function is $u_i(q_i, q_j) = p_i(q_i, q_j)q_i$. Let $q_i^* = q_i^*(q_j)$ denote its choice. The evaluation of $q_i^*$ is then given by the evaluation function $\hat{u}_i(q_i^*, q_j) = p_i(q_i^*, q_j)q_i^* - cq_i^*$. On the other hand, if a firm chooses its quantity by maximizing its profit function then its objective and evaluation functions are the same.

In the following section we identify the sub-game perfect Nash equilibrium (SPNE) outcome of $H$.

3 Endogenous objectives and payoff advantage
Firm $i$, $i = 1, 2$, has two candidate objective functions in stage 0 of $H$: its profit and its revenue objective. Let $\Pi_i$ stand for the choice of profit and $R_i$ for the choice of the revenue objective function, $i = 1, 2$. The choices of the two firms in stage 0 induce four different subgames. Let $H_{k_1, k_2}$ denote the subgame that arises if firms 1 and 2 choose the objectives $k_1$ and $k_2$ respectively, $k_i \in \{\Pi_i, R_i\}$, $i = 1, 2$. Let $\hat{u}_i(k_1, k_2)$ and $q_i(k_1, k_2)$ denote respectively the evaluation function and quantity of firm $i$, $i = 1, 2$ in $H_{k_1, k_2}$. Working backwards we first describe the two last stages of the game, namely stages 1 and 2.

As an illustration, consider the case $(k_1, k_2) = (\Pi_1, R_2)$, which gives rise
to the subgame $H_{\Pi_1,R_2}$. Under this case, firm 1 chooses, in stage 1, its quantity by maximizing the profit function $u_1(q_1, q_2) = (a - q_1 - \gamma q_2 - c)q_1$; in stage 2, firm 2 chooses its quantity by maximizing its revenue $u_2(q_1, q_2) = (a - q_2 - \gamma q_1)q_2$. Evaluations of the market quantities are given by

\[ \hat{u}_1(\Pi_1, R_2) = (a - q_1(\Pi_1, R_2) - \gamma q_2(\Pi_1, R_2) - c)q_1(\Pi_1, R_2) \]

\[ \hat{u}_2(\Pi_1, R_2) = (a - q_2(\Pi_1, R_2) - \gamma q_1(\Pi_1, R_2) - c)q_2(\Pi_1, R_2) \]

Solving for the market outcomes of the four subgames (see Proof of Proposition 1 in the Appendix) gives us the following payoff matrix in stage 0

<table>
<thead>
<tr>
<th></th>
<th>$\Pi_2$</th>
<th>$R_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi_1$</td>
<td>$\hat{u}_1(\Pi_1, \Pi_2), \hat{u}_2(\Pi_1, \Pi_2)$</td>
<td>$\hat{u}_1(\Pi_1, R_2), \hat{u}_2(\Pi_1, R_2)$</td>
</tr>
<tr>
<td>$R_1$</td>
<td>$\hat{u}_1(R_1, \Pi_2), \hat{u}_2(R_1, \Pi_2)$</td>
<td>$\hat{u}_1(R_1, R_2), \hat{u}_2(R_1, R_2)$</td>
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Given the above, we can now present our first result, namely, the SPNE outcome of $H$.

**Proposition 1.** Consider the game $H$. There exists a function $\bar{c}(\gamma)$ such that the following hold.

(i) If $0 < c < \bar{c}(\gamma)$ the SPNE outcome of $H$ is given by $(\Pi_1, R_2)$.

(ii) If $\bar{c}(\gamma) < c < a$ the SPNE outcome of $H$ is given by $(\Pi_1, \Pi_2)$.

**Proof.** Appears in the Appendix.

The threshold is given by $\bar{c}(\gamma) = 2a\gamma^2(4 - \gamma^2 - 2\gamma)/y(\gamma)$, where $y(\gamma) = 16 + \gamma^4 - 4\gamma^3 - 8\gamma^2$.

Making the equilibrium calculations in the above matrix gives two basic findings: firstly, $\Pi_1$ dominates $R_1$ for the leader, i.e., $\hat{u}_1(\Pi_1, \Pi_2) > \hat{u}_1(R_1, \Pi_2)$ and $\hat{u}_1(\Pi_1, R_2) > \hat{u}_1(R_1, R_2)$; secondly, given that $R_1$ is never played, the follower prefers $R_2$ over $\Pi_2$ if and only if $c$ is below a threshold, i.e., $\hat{u}_2(\Pi_1, R_2) > \hat{u}_2(\Pi_1, \Pi_2)$ if and only if $c < \bar{c}(\gamma)$. Hence, the SPNE outcome of $H$ is $(\Pi_1, R_2)$ when $c$ is low and $(\Pi_1, \Pi_2)$ when $c$ is high.

Let us now examine the issue of payoff advantage in $H$. Consider first the substitute goods case ($\gamma \in (0,1)$). Recall that in the classical Stackelberg model the leader always has the payoff advantage in this setting (i.e.,

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3With a slight abuse of notation an SPNE outcome of $H$ will be denoted by $(k_1, k_2)$, where $k_i \in \{\Pi_i, R_i\}$, $i = 1, 2$. 

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under the case of decreasing best replies). The classical Stackelberg model is equivalent to the outcome \((\Pi_1, \Pi_2)\) in our extended model. Thus, the leader has the advantage under this outcome (i.e., when \(c > \bar{c}(\gamma)\)). However, under the outcome \((\Pi_1, R_2)\) the advantage can be with the follower as Corollary 1 shows.

**Corollary 1.** Consider the game \(H\) where \(\gamma \in (0, 1)\). There exist functions \(\xi(\gamma), \bar{c}(\gamma)\) such that the following hold.

(i) The follower achieves higher payoff than the leader for \(\xi(\gamma) < c < \bar{c}(\gamma)\).

(ii) The leader achieves higher payoff for \(0 < c < \xi(\gamma)\) or for \(\bar{c}(\gamma) < c < a\).

**Proof.** Follows by standard calculations.

The threshold is \(\xi(\gamma) = a\gamma^2/(2(\gamma + 2))\). More compactly:

<table>
<thead>
<tr>
<th>(0 &lt; c &lt; \xi(\gamma))</th>
<th>(\xi(\gamma) &lt; c &lt; \bar{c}(\gamma))</th>
<th>(\bar{c}(\gamma) &lt; c &lt; a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leader advantage</td>
<td>Follower advantage</td>
<td>Leader advantage</td>
</tr>
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</table>

The follower obtains the payoff advantage under the SPNE outcome \((\Pi_1, R_2)\). Under this outcome and provided that \(\xi(\gamma) < c < \bar{c}(\gamma)\), the follower produces a higher quantity than the leader, i.e., \(q_2(\Pi_1, R_2) > q_1(\Pi_1, R_2)\). This happens because in \((\Pi_1, R_2)\), the follower chooses his quantity by maximizing his revenue while the leader by maximizing his profit. For the above range of values of \(c\), the higher market share of the follower results to a higher equilibrium profit for him. Note that when \(c < \xi(\gamma)\), \((\Pi_1, R_2)\) is still the subgame perfect equilibrium outcome of \(H\); however, for this range of values of \(c\), the leader has the profit advantage: for \(c < \xi(\gamma)\), the leader has a higher market share, i.e., \(q_1(\Pi_1, R_2) > q_2(\Pi_1, R_2)\).

Consider now the case of complementary goods \((\gamma \in (-1, 0))\).

**Corollary 2.** Consider the game \(H\) where \(\gamma \in (-1, 0)\). There exist functions \(\xi(\gamma)\) and \(\bar{c}(\gamma)\) such that the following hold.

(i) The leader achieves higher payoff than the follower for \(\xi(\gamma) < c < \bar{c}(\gamma)\).

(ii) The follower achieves higher payoff than the leader for \(0 < c < \xi(\gamma)\) or \(\bar{c}(\gamma) < c < a\).

\(^4\)We present the results in terms of \(c\) (the production technology); we could equivalently present them in terms of \(a\) (demand condition).
Proof. Follows by standard calculations.5

| Table 2: Structure of profit advantage in $H$ for $\gamma \in (-1,0)$ |
|-----------------|-----------------|-----------------|-----------------|
| $0 < c < c(\gamma)$ | $c(\gamma) < c < c(\gamma)$ | $c(\gamma) < c < a$ |
| Follower advantage | Leader advantage | Follower advantage |

By Proposition 1 and Corollary 2 we observe that when $\gamma \in (-1,0)$ the profit advantage is with the leader under the SPNE outcome $(\Pi_1, R_2)$. Hence for both complementary and substitute goods, the profit advantage alternates between the two firms as a function of the unit cost of production or equivalently as a function of the demand parameter $a$ (see footnote 5).

It is interesting to note that Liu (2005) derived a similar result in a different context. He analyzed a Stackelberg duopoly with one-sided incomplete information (on behalf of the first-mover) about market demand. He showed that the structure of profit advantage depends on the magnitude of the realized demand.

Finally, we point out that our analysis is based on the three-stage game $H$ where the decisions of firms regarding their objective functions are made simultaneously. Our results on the structure of the payoff advantage (Corollaries 1 and 2) will not change qualitatively if we analyze a game where the decisions of firms regarding their objective functions are made sequentially: in the first stage of the game, firm 1 selects its objective function and firm 2 responds in the next stage by choosing its own objective. Finally in the last two stages the two firms choose quantities.

### 3.1 Price competition

In this section we discuss the case where firms compete in prices. Firms 1 and 2 face the demand functions

$$q_1 = \frac{a(1-\gamma)}{1-\gamma^2} + \frac{\gamma p_2}{1-\gamma^2} - \frac{p_1}{1-\gamma^2}, \quad q_2 = \frac{a(1-\gamma)}{1-\gamma^2} + \frac{\gamma p_1}{1-\gamma^2} - \frac{p_2}{1-\gamma^2}$$

The structure of the resulting model is as in $H$ with the only exception that in stage 1 firm 1 selects $p_1$ and in stage 2 firm 2 selects $p_2$. Denote the corresponding three-stage game by $G$. The following result follows easily.

**Proposition 2.** Irrespective of $c$ the SPNE outcome of $G$ is given by $(\Pi_1, \Pi_2)$.

**Proof.** Follows by standard calculations.

5The thresholds in Corollary 2 and Corollary 1 are the same.
Notice the difference between the games $G$ and $H$. In $H$, the equilibrium dictates that whenever $c$ is low, the follower chooses the revenue objective function. On the other hand, this does not occur in $G$; both the follower and the leader always choose the profit objective function: price competition under profit maximization is already fierce. Thus if firms switched to revenue maximization, prices would be even lower.

An immediate corollary of Proposition 2 is that the result of Gal-Or (1985) on the structure of the profit advantage goes through in $G$, as both firms choose the profit objective. Hence the payoff advantage in $G$ is with the leader when $\gamma \in (-1, 0)$ and it is with the follower when $\gamma \in (0, 1)$.

4 Conclusions

In this paper is provided one explanation for the fluctuation of the profit advantage between the first and second movers observed in markets with sequential moves. Of course this is not the only possible explanation; nonetheless our approach delivers a testable implication, namely the association of cost or demand conditions with the payoff advantage in a sequential duopoly.

A few extensions of the current work can be suggested. Analyzing a model with a more general structure (in terms of demand and cost functions) is a natural extension. Finally, the analysis of a market with $n > 2$ firms is of interest.

Appendix

Proof of Proposition 1. (i), (ii). Consider the $H_{11,12}$ subgame. Firm 1 selects the quantity $q_1(\Pi_1, R_2) = \max\{(2a - \gamma a - 2c)/(4 - 2\gamma^2), 0\}$. Notice that if $\gamma \in (-1,0)$ then $q_1(\Pi_1, R_2) > 0$ whereas if $\gamma \in (0,1)$ then $q_1(\Pi_1, R_2) > 0$ if $c < c_0(\gamma) \equiv a(2 - \gamma)/2$. Further $q_2(\Pi_1, R_2) = A_2/[4(2 - \gamma^2)]$, where $A_2 = 4a - a\gamma^2 - 2\gamma(a-c)$. Notice that $q_2(\Pi_1, R_2) > 0$ for all $\gamma \in (-1,1)$. Evaluations are $u_1(\Pi_1, R_2) = (2a - \gamma a - 2c)/[8(2 - \gamma^2)]$ and $u_2(\Pi_1, R_2) = A_2(2a - 8c + 4\gamma^2c)/(16(2 - \gamma^2)^2)$ if $c < \tilde{c}(\gamma) \equiv a(4 - \gamma^2 - 2\gamma)/(8 - 2\gamma - 4\gamma^2)$ and $u_2(\Pi_1, R_2) = 0$ if $c \geq \tilde{c}(\gamma)$. Notice that $c_0(\gamma) > \tilde{c}(\gamma)$. Hence, if $c > \tilde{c}(\gamma)$, firm 1 becomes a monopolist with profit $u_1(\Pi_1, R_2) = (a-c)^2/4$.

Consider the $H_{11,22}$ subgame. The leader selects the quantity $q_1(\Pi_1, \Pi_2) = (2 - \gamma)\theta/(4 - 2\gamma^2) > 0$ where $\theta \equiv a - c$. Moreover, $q_2(\Pi_1, \Pi_2) = (4 - \gamma^2 - 2\gamma)\theta/(8 - 4\gamma^2) > 0$. Evaluations are $u_1(\Pi_1, \Pi_2) = (2 - \gamma)\theta^2/(16 - 8\gamma^2)$ and $u_2(\Pi_1, \Pi_2) = (4 - \gamma^2 - 2\gamma)^2\theta^2/(16(2 - \gamma)^2)$.

Consider now the $H_{R1,12}$ subgame. Quantities are $q_1(R_1, \Pi_2) = (2a - \gamma\theta)/(4 - 2\gamma^2) > 0$ and $q_2(R_1, \Pi_2) = \max\{((4-\gamma^2)\theta - 2a\gamma)/(8 - 4\gamma^2), 0\}$. Notice
that if \( \gamma \in (-1, 0) \) then \( q_2(R_1, \Pi_2) > 0 \) and if \( \gamma \in (0, 1) \) then \( q_2(R_1, \Pi_2) > 0 \) iff \( c < a(4 - \gamma^2 - 2\gamma)/(4 - \gamma^2) \equiv \hat{c}(\gamma) \). Evaluations are \( u_1(R_1, \Pi_2) = \max \{(2a - \theta\gamma)(2a - \theta\gamma - 4\gamma)/(16 - 8\gamma^2), 0\} \) and \( u_2(R_1, \Pi_2) = [q_2(R_1, \Pi_2)]^2 \). We note that \( p_1(R_1, \Pi_2) > c \) (and hence \( u_1(R_1, \Pi_2) > 0 \)) if \( c < a(2 - \gamma)/(4 - \gamma) \equiv c_q(\gamma) \).

Consider lastly the \( H_{R_1,R_2} \) subgame. Quantities are \( q_1(R_1, R_2) = a(2 - \gamma)/(4 - 2\gamma^2) \) \( > 0 \) and \( q_2(R_1, R_2) = a(4 - \gamma^2 - 2\gamma)/(8 - 4\gamma^2) \) \( > 0 \), provided that \( p_1(R_1, R_2) > c \) and \( p_2(R_1, R_2) > c \). Notice that \( p_1(R_1, R_2) > c \) iff \( c < a(2 - \gamma)/4 \equiv c_{**} \) and \( p_2(R_1, R_2) > c \) iff \( c < a(4 - \gamma^2 - 2\gamma)/(8 - 4\gamma^2) \equiv c_m \), where \( c_m > c_{**} \). Hence, if \( c < c_{**} \) both firms are active. When \( c > c_{**} \) firm 1 has negative profit and hence it produces 0; firm 2 becomes a monopolist producing \( q_2 = a/2; \) then \( u_2(R_1, R_2) = (a/2 - c)a/2 > 0 \) if \( c < a/2 \equiv c_e \).

Notice that \( c_e > c_{**} \) iff \( \gamma > 0 \). Therefore if \( \gamma \in (-1, 0) \) then for \( c > c_{**} \) no firm has a positive evaluation. If \( \gamma \in (0, 1) \), then for \( c_{**} < c < c_e \) firm 2 is a monopolist with positive evaluation.

We next determine the SPNE outcome.

Case A: \( \gamma \in (-1, 0) \): There are two subcases. If \( \gamma < -0.78 \) then \( c_q(\gamma) < c_{**}(\gamma) < \bar{c}(\gamma) \). Let first \( c < c_q(\gamma) \). Then \( u_1(\Pi_1, \Pi_2) > u_1(R_1, \Pi_2) \) and \( u_1(\Pi_1, R_2) > u_1(R_1, R_2) \) and \( R_1 \) is dominated by \( \Pi_1 \) for player 1. Moreover, \( u_2(\Pi_1, R_2) > u_2(\Pi_1, \Pi_2) \) iff \( c < 2a\gamma^2(4 - \gamma^2 - 2\gamma)/(16 + 4\gamma^4 - 4\gamma^3 - 8\gamma^2) \equiv \bar{c}(\gamma) \).

Note that if \( -1 < \gamma < -0.88 \) then \( \bar{c}(\gamma) > c_q(\gamma) \) and if \( -0.88 < \gamma < -0.78 \) then \( \bar{c}(\gamma) < c_q(\gamma) \). If \( c_q(\gamma) < c < c_{**}(\gamma) \) and \( c_{**}(\gamma) < c < \bar{c}(\gamma) \) then firm 1 receives 0 in the \( G_{R_1,\Pi_2} \) and \( G_{R_1,R_2} \) respectively. Finally if \( c > \bar{c}(\gamma) \), firm 1 receives 0 in \( G_{\Pi_1,R_2} \) where \( \bar{c}(\gamma) > \bar{c}(\gamma) \). The above imply that the SPNE outcome is \( (\Pi_1, R_2) \) for \( c < \bar{c}(\gamma) \) and it is \( (\Pi_1, \Pi_2) \) for \( c > \bar{c}(\gamma) \). Finally if \( \gamma > -0.78 \) then \( c_q(\gamma) < \bar{c}(\gamma) < c_{**}(\gamma) \). Then again \( \Pi_1 \) dominates \( R_1 \) and the SPNE outcome is as in the case \( \gamma < -0.78 \).

Case B: \( \gamma \in (0, 1) \): Then \( c_{**}(\gamma) < c_q(\gamma) < c_e < \bar{c}(\gamma) \). Using these inequalities one can easily see that the SPNE outcome is \( (\Pi_1, R_2) \) if \( 0 < c < \bar{c}(\gamma) \) and it is \( (\Pi_1, \Pi_2) \) otherwise.

References


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