Labour decreasing returns, industry-wide union and Cournot-Bertrand profit ranking. A note

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Abstract

While the received literature on unionized duopolies emphasized the key role of inter-union competition in reversing the standard Cournot-Bertrand profit ranking, in this paper such issue is studied in a framework with labour decreasing returns and a centralized (industry-wide) union, hence in a context where inter-union competition is clearly absent. Nevertheless, it is shown that the "reversal result" can apply provided that union is sufficiently wage oriented. Furthermore, it is pointed out that, in the special case of total wage bill maximization, it applies for a larger range of the degree of product substitutability under a central union rather than under firm-specific unions.
1. Introduction

Since Singh and Vives’s (1984) seminal paper, a tenet in duopoly theory is that, when goods are substitutes, firms’ profits are higher under competition à la Cournot than à la Bertrand. However, the robustness of such result has been challenged more recently by introducing, in the same framework of Singh and Vives (1984), a two-stage game where, in the second stage, firms compete in the product market and, in the first stage, firms’ production costs (wages) are determined in the presence of decentralized labour unions.

Particularly, Correa-López and Naylor (2004) consider a decentralized wage-bargaining game played between each firm and a firm-specific labour union assuming a (standard) constant returns to labour technology, finding that, if unions are sufficiently powerful and are distinctly wage-oriented, the standard result (i.e. firms’ profits are higher under Cournot competition) may be reversed when goods are sufficiently imperfect substitutes. Afterwards, Fanti and Meccheri (2011) extend the Correa-López and Naylor’s (2004) work by introducing labour decreasing returns into the analysis, showing that the “reversal result” may also apply in the presence of “total wage bill-maximizing” unions, that is, even if unions attach equal weight to wages and employment.

The reason for the reversal result is that wages are indeed higher under Cournot than under Bertrand competition due to the fact that unions influence wages more aggressively in the latter case. In turn, as highlighted by Correa-López and Naylor (2004), this relies on the fact that a percentage increase in the wage rate will induce a higher percentage reduction in employment under Bertrand than under Cournot competition. The union perceives this difference as a higher proportional marginal cost for a given wage increase when bargaining with a Bertrand-type firm, hence each union has a stronger incentive to settle for a lower bargained wage rate when facing a Bertrand-type competitor. Furthermore, Fanti and Meccheri (2011) show that such mechanism proves to be reinforced by the presence of diminishing returns to labour since when wages increase, ceteris paribus, the employment reduction is more severe under decreasing returns. As a consequence, strategic effects implying that Cournot equilibrium profits decrease more steeply in wages than do Bertrand equilibrium profits are also magnified by the presence of diminishing returns, leading to the outcome that the reversal result realizes notwithstanding that unions are not distinctly wage-oriented.

Given the findings above summarized, it is easy to infer the crucial role for obtaining the reversal result played by wage competition between firm-specific unions. This would seem to suggest that if firm-specific unions merge and, as a consequence, a single central union is concerned, the deriving absence of competition in the labour market should prevent the reversal result to realize. Furthermore, this belief could be reinforced by considering also that, under standard assumptions,
Dhillon and Petrakis (2002) show that a “wage rigidity result” applies in the presence of a central union, that is, the wage rate should be the same under both Cournot and Bertrand competition.\(^3\)

In this note we carry on with Fanti and Meccheri’s (2011) analysis by considering the presence and the role of labour decreasing returns in a unionized duopoly. In contrast with the reasoning above outlined, we point out that the reversal result at issue can apply also in the presence of a single industry-wide union.\(^4\) In particular, first, we highlight that in our framework a “wage rigidity result” no longer applies since the wage rate is always lower when firms compete à la Bertrand instead of à la Cournot. Secondly, we show that the wage differential between alternative competition regimes is increasing with the “wage-aggressiveness” by the union and the degree of product substitutability. Clearly, this also leaves room for the profit-ranking reversal result to apply and, indeed, we show that profits can be higher under Bertrand competition also with a centralized (industry-wide) union. Moreover, although the received literature stressed the prominent role of wage competition between unions in determining the reversal result, we provide a case in which it is more likely to occur under centralized unionization.

The rest of this work is organized as follows. In Section 2, we present the basic model where two firms compete in the product market with (imperfect) substitute goods under centralized unionization. We derive equilibrium values for the key variables of interest under both Cournot and Bertrand competition. In Section 3, we compare equilibrium profits under alternative competition regimes and also discuss our findings in relation to the received literature. Finally Section 4 concludes.

2. Model

Following Singh and Vives (1984), we consider a model of differentiated product market duopoly, in which each firm sets its output, given pre-determined wages, to maximize profits. The product market demand for the representative firm \(i\) is linear and given by:

\[
p_i(q_i, q_j) = \alpha - q_i - \gamma q_j
\]

where \(q_i\) and \(q_j\) are outputs by firm \(i\) and \(j\) (with \(i, j = 1, 2\) and \(i \neq j\)), \(\alpha > 0\) and \(\gamma \in (0, 1)\) denotes the extent of product differentiation, with goods assumed to be imperfect substitutes.

Let assume that only labour input is used for production. As already discussed in the Introduction, another literature’s standard assumption is that labour exhibits constant returns, which implies firms face with constant marginal costs. Following Fanti and Meccheri (2011), we modify such hypothesis by introducing diminishing returns to labour into the analysis. In particular, we assume the following production technology:

\(^3\) More generally, Dhillon and Petrakis (2002) show that, as long as negotiations are centralized at the industry level, the wage rate turns out to be the same independently of the type of competition in the product market, the number of firms, the degree of product substitutability, the intensity of market competition or the bargaining institution (Monopoly Union, Right-to-Manage or Efficient Bargains).

\(^4\) While firm-specific unions and decentralized wage setting are largely predominant in UK, North America and Japan, centralized unions representing all workers in an industry are widespread in Continental Europe, (e.g. Flanagan 1999).
where \( l_i \) represents the number of workers employed by firm \( i \) to produce \( q_i \) output units of variety \( i \), while \( B > 0 \) relates to the exogenously given productivity of labour. The choice of such specific technology, described by (2), allows for analytical results and also implies that firms have quadratic costs, which is a typical example of increasing costs in the literature. Hence, firm \( i \)'s profit can be written as:

\[
\pi_i = p_i q_i - w_i l_i = p_i q_i - w_i A q_i^2
\]

(3)

where \( w_i \) is the per-worker wage paid by firm \( i \), with \( w_i < \alpha \) and \( A \equiv 1/B^2 \), which is an inverse measure of labour productivity.

Following the established literature on unionized oligopolies (see, e.g., the seminal works: Horn and Wolinsky 1988, Dowrick 1989), production costs (i.e. wages) are no longer assumed to be as exogenously given for firms, but they are the outcome of a strategic game previously (with respect to product market competition) played between each firm and a labour union. We consider here the case in which firms’ wages are fixed by a monopoly central union (hence the wage is the same for both firms, i.e. \( w_i = w_j = w \)), which maximizes a standard Stone-Geary objective function (see, e.g., Pencavel 1984, 1985, Dowrick and Spencer 1994, Petrakis and Vlassis 2000):

\[
V = (w - w^0)(l_i + l_j) = A(w - w^0)(q_i^2 + q_j^2)
\]

(4)

where \( w^0 \geq 0 \) is the reservation or competitive wage and \( \theta > 0 \) captures the importance of wages with respect to employment (whose weight is normalized to one) to the unions. For instance, a value of \( \theta = 1 \) gives the rent-maximizing case (i.e., the union seeks to maximize the total rent), while values of \( \theta \) smaller (higher) than 1 imply that the union is less (more) concerned about wages and more (less) concerned about employment (see, e.g., Mezzetti and Dinopoulos 1991, Fanti and Gori 2011). Moreover, the union aims to maximize the wage bill when \( \theta = 1 \) and \( w^0 = 0 \). \(^5\)

### 2.1 Cournot equilibrium

Taking (1) and (3) into account, profit-maximization under Cournot competition leads to the following firm \( i \)'s best-reply function:

\[
q_i(q_j) = \frac{\alpha - \gamma q_j}{2(1 + Aw)}.
\]

(5)

As \( \gamma > 0 \), the best-reply functions are downward-sloping, that is, under the Cournot assumption, the product market game is played in strategic substitutes. From (5), and its equivalent for firm \( j \), we can obtain, for given \( w \), firm \( i \)'s output as:

\(^5\)Alternatively, \( 0 < \theta < 1 \) can also be thought of as the representative worker’s relative rate of risk aversion (Booth 1995, Petrakis and Vlassis 2000).
\[ q_i(w) = \frac{\alpha}{2(1 + Aw) + \gamma} \]  

and, by substituting (6) in (3), firm i’s profit as:

\[ \pi_i(w) = \frac{\alpha^2(1 + Aw)}{[2(1 + Aw) + \gamma]^2}. \]  

Before product market competition, central union optimally chooses the wage to maximize (4). By substituting (6) and its counterpart for firm j in (4) and maximizing with respect to \( w \), we get that the equilibrium wage is given by:

\[ w^C = \frac{4Aw^o + \theta(2 + \gamma)}{2A(2 - \theta)} \]  

where the apex \( C \) denotes Cournot competition in the product market. We note that for ensuring a positive wage and making our results meaningful, the technical assumption that \( \theta < 2 \) will be followed from now onwards. After substitution of (8) in (7), it follows that firm i’s Cournot-Nash equilibrium profit is:

\[ \pi^C = \frac{\alpha^2(2 - \theta)(4 + 4Aw^o + \gamma\theta)}{8(2 + 2Aw^o + \gamma)^2}. \]  

### 2.2 Bertrand equilibrium

We consider now the case in which the product market game is characterized by price-setting behaviour by firms, i.e. competition occurs \( \text{à la} \) Bertrand. From (1) and its counterpart for firm \( j \), we can rewrite firm i’s product demand as:

\[ q_i(p_i, p_j) = \frac{\alpha(1 - \gamma) - p_i + \gamma p_j}{1 - \gamma^2} \]  

hence, by using (3), its profit is given by:

\[ \pi_i(p_i, p_j) = p_i \left[ \frac{\alpha(1 - \gamma) - p_i + \gamma p_j}{1 - \gamma^2} \right] - Aw_j \left[ \frac{\alpha(1 - \gamma) - p_i + \gamma p_j}{1 - \gamma^2} \right]^2. \]  

From (11) (and taking \( w_i = w_j = w \) into account), the first-order condition for profit-maximization gives the price choice, as a function of the price chosen by firm \( j \), as:
\[ p_i(p_j) = \frac{(\alpha(1-\gamma) + \gamma p_j) \left[ 1 + 2Aw - \gamma^2 \right]}{2 \left( 1 + Aw - \gamma^2 \right)} \] (12)

thus, for \( \gamma > 0 \), the Bertrand product market game is played in strategic complements. By substituting in (12) the corresponding equation for firm \( j \) and solving for \( p_i \), we get the Bertrand equilibrium price for a given wage:

\[ p_i(w) = \frac{\alpha \left[ 2(1 + Aw) - \gamma(1 + \gamma) \right] \left[ 1 + 2Aw - \gamma^2 \right]}{4(1 + Aw)^2 - \gamma^2 \left( 5 + 4Aw - \gamma^2 \right)} \] (13)

Hence, by substituting in (10), we get the sub-game perfect output as a function of the wage, which is fixed by the union at the previous stage of the game, as:

\[ q_i(w) = \frac{\alpha \left[ 2(1 + Aw) - \gamma(1 + \gamma) \right]}{4(1 + Aw)^2 - \gamma^2 \left( 5 + 4Aw - \gamma^2 \right)} \] (14)

and, by using (13) and (11), firm \( i \)'s profit as:

\[ \pi_i(w) = \frac{\alpha^2 (1 + Aw - \gamma^2) \left[ 2(1 + Aw) - \gamma(1 + \gamma) \right]^2}{\left[ 4(1 + Aw)^2 - \gamma^2 \left( 5 + 4Aw - \gamma^2 \right) \right]^2} \] (15)

Also in the Bertrand competition case, the union's utility function is given by (4). Hence, by substituting (14) and its counterpart for \( j \) in (4), and maximizing with respect to \( w \), we get the following expression for the equilibrium wage:

\[ w^B = \frac{4Aw^o + \theta(2 + \gamma - \gamma^2)}{2A(2 - \theta)} \] (16)

where the apex \( B \) denotes Bertrand competition in the product market.

Finally, after substitution of (16) in (15), it follows that firm \( i \)'s Bertrand-Nash equilibrium profit is given by:

\[ \pi^B = \frac{\alpha^2 (2 - \theta) \left[ 4 + 4Aw^o + \gamma^2(\theta - 4) + \gamma \theta \right]}{8 \left( 2 + 2Aw^o - \gamma^2 + \gamma \right)^2} \] (17)
3. Cournot-Bertrand profit ranking

In this section, we investigate if the conventional wisdom that Bertrand competition yields, in equilibrium, lower profits with respect to Cournot competition still holds true under centralized unionization. Let’s begin by analyzing the wage differential between the two modes of competition.

Lemma. A centralized union sets higher wages under Cournot than under Bertrand product market competition. Furthermore, the wage differential is increasing with the degrees of union’s “wage-orientation” and product substitutability, and with labour productivity.

Proof. From (8) and (16), it straightforwardly derives that

\[ \Delta w = w^C - w^B = \frac{\theta \gamma^2}{2A(2-\theta)} > 0, \]

from which it clearly appears that \( \frac{\partial \Delta w}{\partial \theta} > 0, \frac{\partial \Delta w}{\partial \gamma} > 0 \) and \( \frac{\partial \Delta w}{\partial A} < 0 \) (recall that \( A \) is inversely related to labour productivity).

Q.E.D.

Moreover, the Cournot-Bertrand profit differential (based on the comparison between (9) and (17)) is given by:

\[
\Delta \pi = \pi^C - \pi^B = \frac{(\alpha \gamma)^2 (2-\theta) \left[ 4A \gamma \left( 2(\gamma - \theta - \gamma \theta) + \gamma^2 - A w^\circ \theta \right) + \gamma^3 \theta + \gamma^2 (8 - 3\theta) - 8 \gamma (\theta - 1) - 4 \theta \right]}{\left[ 2 + 2A w^\circ - \gamma^2 + \gamma \right]^2 (2 + 2A w^\circ + \gamma)^2}. \tag{18}
\]

Result 1. Provided that the central union is sufficiently wage oriented, profits are greater under Bertrand than under Cournot competition.

Proof. From (18) we get:

\[ \Delta \pi < 0 \iff \theta > \hat{\theta} = \frac{4 \gamma \left[ A w^\circ (\gamma + 2) + 2(1+\gamma) \right]}{4 \left[ (A w^\circ)^2 + (1+\gamma) \right] \left[ 4 + 8A w^\circ - \gamma^2 + 4\gamma \right]} . \tag{19} \]

Q.E.D.

However, notice from (19) that the threshold for \( \theta \), over which the reversal result applies, generally depends on the values of the other parameters of interest. Furthermore, given the non-linearity of the expression for the profit differential (Eq. 18) with respect to those other parameters, we are not able to provide a formal statement, such that related to \( \theta \), for the role of \( \gamma, A \) and \( w^\circ \).

At the same time, however, Figure 1 below can give clear-cut indications on the role of those parameters in determining the possibility for the reversal result to apply. In particular, in Figure 1, all the \( \gamma-\theta \) combinations that lie above (below) each curve inside the box (each curve being related to...
a chosen value of \( Aw^o \); see the figure’s caption for details) are those for which profits are higher under Bertrand (Cournot) competition.

Figure 1: Plot of the “threshold curves” in \( \{\gamma-\theta\}\)-space for different \( Aw^o \) values.\(^a\)

\(^a\) Each curve is drawn for a given \( Aw^o \) value (\( Aw^o = 0 \): black solid; \( Aw^o = 2 \): blue dash; \( Aw^o = 5 \): red dash dot). For all \( \gamma-\theta \) combinations along each curve, \( \Delta \pi = 0 \) (given \( Aw^o \)) holds true. For all \( \gamma-\theta \) combinations above (below) each curve, profits are higher under Bertrand (Cournot) competition, that is (given \( Aw^o \)), \( \Delta \pi < (>) 0 \).

The following points, arising from Figure 1, can be remarked:

1. when the central union is strongly employment oriented (that is, \( \theta \) is sufficiently close to zero), the reversal result never (i.e. for any \( \gamma, A \) and \( w^o \)) applies;\(^6\)

2. by contrast, when the central union is sufficiently wage oriented (that is, \( \theta \) is sufficiently large), the reversal result applies for any \( \gamma, A \) and \( w^o \);

3. ceteris paribus, the lower the degree of product substitutability \( \gamma \), the lower the threshold for \( \theta \) for which the reversal result applies;

\(^6\) In particular, notice that when \( \theta = 0 \), that is the union only cares about employment, the equilibrium wage is clearly \( w = w^o \) in both competition regimes (see also (8) and (16)). As a consequence, equilibrium results about output and profits would replicate those of standard cases with profit-maximizing firms (without unions), for which profits under Cournot are always greater than under Bertrand competition, even if labour exhibits decreasing returns (see Fanti and Meccheri 2011, appendix).
4. since an increase of the reservation wage $w^0$, ceteris paribus, shifts downwards the “threshold curve”, it makes the reversal result more likely to occur. Conversely, an increase of labour productivity (which corresponds to a decrease of $A$), by shifting upwards ceteris paribus the threshold curve, generally makes the reversal result less likely to occur. Notice, however, that labour productivity affects the possibility of the reversal result only if the reservation wage is not null, i.e. $w^0 > 0$ (see (18)).

Furthermore, since in Fanti and Meccheri (2011), where the case with firm-specific unions is considered, we concentrate on total wage bill maximizing unions (that is, the special case with $w^0 = 0$ and $\theta = 1$) and set $B = A = 1$, it could be interesting to provide a clear-cut comparison for the crucial parameter $\gamma$ in this particular case.

**Result 2.** Under a centralized total wage bill maximizing (monopoly) union, profits are greater under Bertrand than under Cournot competition if, and only if, the degree of product substitutability is lower than 0.828 (i.e. products are not close substitutes).

**Proof.** By substituting for $w^0 = 0$, $\theta = 1$ (and $A = 1$) in (18) and solving for $\gamma$ we get:

$$
\Delta \pi < \frac{\Delta}{\theta} = 0 \Leftrightarrow \gamma^2(\gamma + 5) - 4 < 0 \Leftrightarrow \gamma < \frac{\sqrt{26}}{2} = 0.828.
$$

(20)

Q.E.D.

Moreover, also the following corollary, which derives from Result 2, applies.

**Corollary.** In the presence of total wage bill maximizing (monopoly) unions and labour decreasing returns, the reversal result is more likely to occur with a centralized union structure, that is, it applies for a larger range of the degree of product substitutability under a central union rather than under firm-specific unions.

**Proof.** The proof of the Corollary simply derives by considering that in Fanti and Meccheri (2011, Result 1, p. 7) it has been established that, with labour decreasing returns and decentralized (firm-specific) unions, the profit ranking reversal result holds true for $\gamma < 0.732$. Hence, the proof immediately follows by noting that, as showed above (Result 2), under a centralized union the reversal result applies for a larger range of $\gamma$’s values.

Q.E.D.
4. Conclusion

The received literature analyzing the Cournot-Bertrand profit ranking under unionized duopolies emphasized the key role of the strategic effects induced by firms-specific unions competition in determining the “reversal result” (i.e. higher profits under Bertrand competition). In this paper, instead, we have studied this issue in a framework with labour decreasing returns and a centralized (industry-wide) union, hence in a context where inter-union competition is clearly absent.

It has been shown that the reversal result can apply provided that the union is sufficiently wage oriented. Furthermore, ceteris paribus, the lower the degree of product substitutability and labour productivity and the higher the reservation wage, the lower the threshold for union’s wage orientation for which the reversal result applies. Finally, also referring to previous work of ours (Fanti and Meccheri 2011), it has been pointed out that, in the special case of total wage bill maximization, the reversal result is more likely to occur under a central union rather than under firm-specific unions.

By concluding, future research directed to extend and investigate the robustness of our results undoubtedly deserves to be carried out. At the same time, it is important to point out that a preliminary intuition in relation to extending our model in a particular direction, namely union-firms bargaining, can be directly derived from this work. In particular, notice that the monopoly union model, that we have adopted to maintain our analysis as simple as possible, can be viewed as a limiting case of the wage-bargaining union, where the union has all the bargaining power. At the other extreme, with firms with all the bargaining power, we would get the competitive result where the wage equals the exogenous reservation level, hence, as showed in Fanti and Meccheri (2011), the reversal result would never apply. This logically suggests that our reversal result would survive also in a more general Right-to-Manage model, provided that unions’ bargaining power vis-à-vis firms is sufficiently large.

References


