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# Tax Luxury or Necessity

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## Abstract

This paper studies the optimal taxation between luxury and necessity goods. We set up a three-production-sector neoclassical growth model with inelastic labor supply, and analyze the tax incidence. We find that the two consumption taxes are neutral to economic growth and that the welfare maximization optimal tax mix involves levying the same rate on those two goods. In reality, the tax rate levied on the luxury good is usually higher, so that the government should reduce the tax rate on the luxury good and raise that on the necessity good to the same level in order to enhance the household's lifetime welfare.

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## 1. Introduction

Public economists have devoted considerable effort to measuring the welfare cost of alternative ways of financing government spending. The public expense could be paid by a consumption tax, an income tax or a factor income tax. The pivotal works by Judd (1985) and Chamley (1986) have argued that the government should tax only labor income and not capital income. Several studies have, however, revisited the issue while few studies focus on the welfare cost of the tax rate on different consumption goods. In this paper, we analyze the optimal taxation between the luxury and necessity goods.

According to Bagwell and Bernheim (1996, p.350), luxury goods have Veblen effects, defined as a willingness to pay a higher price for a functionally equivalent good, that arise from the desire to signal wealth. Ng (1987) thought that a change in the price of a luxury good (Ng used the term "a diamond good") leaves its value and the amounts of all other goods consumed, and hence it is optimal to tax a luxury good at a very high tax rate. Ireland (1994) found there arises a welfare loss from consuming the conspicuous good for status-seeking purposes, and the tax on that good may result in a welfare improvement.

Taxes on luxury goods offer an important advantage over sumptuary laws. For example, the U.S. government levied a 10 percent tax on all expenditures on automobiles, boats, aircraft, furs and jewelry above certain thresholds in 1991.[1] While a luxury tax has a function of discouraging conspicuous consumption, we wonder whether imposing a higher tax rate on luxury goods really does improve the household's welfare. In this paper, we set up a simple model with two consumption goods, the luxury good and the necessity good, and analyze the optimal taxation between those two goods.

A roadmap for this paper is as follows. We set up a three-production-sector neoclassical growth model with inelastic labor supply in Section 2. Section 3 studies the equilibrium and the optimal consumption tax. We then use numerical simulation to discuss the tax incidence. Finally, concluding remarks are offered in Section 4.

## 2 The Model

### 2.1 Environment

A representative household that supplies labor normalized to one inelastically owns the shares of firms and decides its consumption and savings at each point of time. The lifetime welfare of the representative household is

$$U = \int_0^\infty u(c_1, c_2)\ e^{-\rho t} dt, \tag{1}$$

where $c_1$ is the necessity good, $c_2$ is the luxury good and $\rho>0$ is the time preference rate. The utility function $u(c_1, c_2)$ has standard properties and is increasing and concave in $c_1$ and $c_2$: $u_i(c_1, c_2)>0>u_{ii}(c_1, c_2)$, $i$=1, 2. Moreover, we assume that both consumption goods are Pareto complements: $u_{ij}(c_1, c_2)>0$ and $u_{ji}(c_1, c_2)>0$, $i, j$=1, 2, $i\neq j$.

The point-in-time utility function takes the following form:

$$u(c_1, c_2) = c_1^{\sigma}(c_2+\eta)^{1-\sigma},\ \ \sigma \in (0,1),$$

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[1] According to Frank (1999, p.204), the thresholds are as follows: automobiles \$30,000; boats, \$100,000; aircraft, \$250,000; and fur and jewelry, \$10,000.

where $\eta>0$ captures the luxury good, $c_2$, relative to the necessity good, $c_1$.

There are three production sectors which produce the consumption good, $c_1$ and $c_2$, and the investment good, respectively. Furthermore, the production functions are $y_1=(uk)^{\alpha}$, $y_2=(sk)^{\beta}$ and $y_3=[(1-u-s)k]^{\gamma}$, where $k$ is capital per capita, $0<\alpha$, $\beta$, and $\gamma<1$ are the capital shares in the three production sectors, a fraction of capital $u$ is devoted to producing the necessity good, a fraction of capital $s$ is to producing the luxury good and the remaining fraction ($1$-$u$-$s$) is to investment, $I$. The capital accumulation equation is

$$\dot{k}=I-\delta k=[(1-u-s)k]^{\gamma}-\delta k,\ k_0 \text{ given,} \tag{2}$$

where $\delta$ is the rate of depreciation of capital.

The government taxes the necessity and the luxury goods, and transfers that tax revenue to households. So apart from (2), the household faces two other constraints as follows

$$y_1+T_1=(1+\tau_1)c_1, \tag{3}$$

$$y_2+T_2=(1+\tau_2)c_2, \tag{4}$$

where $\tau_1$ and $\tau_2$ are the consumption taxes on the necessity and the luxury goods, respectively, and $T_1$ and $T_2$ are the lump-sum transfers in the necessity good and luxury good sectors, respectively.

### 2.1 Optimization Conditions

The representative household's problem is to maximize (1), subject to (2)-(4), taking as given all the tax rates, all the lump-sum transfers, and initial capital. Let $\lambda>0$, $\lambda_1>0$ and $\lambda_2>0$ be the co-state variables associated with capital, the necessity and the luxury goods, respectively. The necessary conditions are

$$\sigma c_1^{\sigma-1}(c_2+\eta)^{1-\sigma}=\lambda_1(1+\tau_1), \tag{5a}$$

$$(1-\sigma)c_1^{\sigma}(c_2+\eta)^{-\sigma}=\lambda_2(1+\tau_2), \tag{5b}$$

$$\lambda_1\alpha u^{\alpha-1}k^{\alpha}=\lambda\gamma(1-u-s)^{\gamma-1}k^{\gamma}, \tag{5c}$$

$$\lambda_2\beta s^{\beta-1}k^{\beta}=\lambda\gamma(1-u-s)^{\gamma-1}k^{\gamma}, \tag{5d}$$

$$\dot{\lambda}=\lambda[\rho+\delta-\gamma(1-u-s)^{\gamma}k^{\gamma-1}]-\lambda_1\alpha u^{\alpha}k^{\alpha-1}-\lambda_2\beta s^{\beta}k^{\beta-1}, \tag{5e}$$

and the transversality condition is $\lim_{t\to\infty} e^{-\rho t}\lambda_t k_t=0$.

Under these conditions, (5a) and (5b) equalize the marginal utility of consumption of the necessity and the luxury goods with the marginal cost of consumption of those two goods, respectively. Equations (5c) and (5d) allocate factors optimally among the three sectors. By combining (5c) and (5d), we obtain the optimal fraction of capital in order to have the same marginal products among these three sectors. Finally, (5e) is the Euler equation, and the transversality condition is the usual transversality or "no Ponzi game" condition in relation to capital.

### 2.3 Equilibrium Conditions

In equilibrium, all the goods markets must clear. Under the assumption that the real transfer is just the tax revenues from the respective sectors, $T_i=\tau_i\ c_i$, $i$=1, 2. Therefore, $c_i=y_i$, $i$=1, 2. The perfect-foresight equilibrium is a time path $\{c_1, c_2, u, s, k_t, \lambda, \lambda_1, \lambda_2\}$ that satisfies the household's optimization, (5a)-(5e), and the clearance of all the goods markets, including $c_i=y_i$, $i$=1, 2, and (2).

Combining (5a) and (5b) yields the marginal substitution between two consumption goods which equals the relative price between those two goods as follows

$$\tfrac{\sigma}{1-\sigma}\tfrac{c_2+\eta}{c_1}=\tfrac{\lambda_1}{\lambda_2}\tfrac{1+\tau_1}{1+\tau_2}. \tag{6a}$$

Furthermore, by combining (5c)-(5e) we obtain

$$\dot{\lambda}/\lambda=\rho+\delta-\gamma(1-u-s)^{\gamma-1}k^{\gamma-1}. \tag{6b}$$

Including the two market clearance conditions of consumption into (6a), along with (5c)-(5d), yields

$$s^{\beta}k^{\beta}[1-\tfrac{1-\sigma}{\sigma}\tfrac{1+\tau_1}{1+\tau_2}\tfrac{\beta}{\alpha}\tfrac{u}{s}]+\eta=0. \tag{6c}$$

In addition, combining the two market clearance conditions of consumption, (5a) and (5c), yields

$$\lambda=\tfrac{\sigma(u^{\alpha}k^{\alpha})^{\sigma-1}(s^{\beta}k^{\beta}+\eta)^{1-\sigma}}{(1+\tau_1)}\tfrac{\alpha u^{\alpha-1}k^{\alpha}}{\gamma(1-u-s)^{\gamma-1}k^{\gamma}}\equiv\lambda(k,u,s). \tag{6d}$$

Finally, (2), (6b) and (6c), along with (6d), make up the dynamical system and determine the equilibrium paths of $u$, $s$ and $k$. The equilibrium paths of $c_1$, $c_2$, $\lambda$, $\lambda_1$ and $\lambda_2$ are in turn determined by two market clearance conditions of consumption, namely, (6d), (5a) and (5b).

In analyzing the equilibrium characterized by (2), (6b) and (6c), we note that $\dot{k}=\dot{\lambda}=0$ in a steady state. Combining (2) and (6b) determines the steady-steady level of capital and the fraction of capital allocated to the production sector of investment is as follows

$$k^{*}=(\tfrac{\gamma}{\rho+\delta})^{\frac{\gamma}{1-\gamma}}\tfrac{1}{\delta}, \tag{7a}$$

$$1-u^{*}-s^{*}=\tfrac{\gamma\delta}{\rho+\delta}. \tag{7b}$$

Thus, the changes in the two consumption taxes do not affect capital accumulation or the fraction of capital allocated to the production sector of investment. Consumption taxes are neutral to the economic growth, and only affect the two consumption goods and the household's lifetime welfare.

Putting (7a)-(7b) into (6c) yields

$$LHS(s)\equiv(\tfrac{\gamma}{\rho+\delta})^{\frac{\gamma\beta}{1-\gamma}}(\tfrac{1}{\delta})^{\beta}[(1+\tfrac{1-\sigma}{\sigma}\tfrac{1+\tau_1}{1+\tau_2}\tfrac{\beta}{\alpha})s-\tfrac{1-\sigma}{\sigma}\tfrac{1+\tau_1}{1+\tau_2}\tfrac{\beta}{\alpha}(1-\tfrac{\gamma\delta}{\rho+\delta})]=-\eta s^{1-\beta}\equiv RHS(s). \tag{7c}$$

Thus, for the given consumption tax rates, equation (7c) determines $s^*$ in a steady state. The values of $u^*$, $c_1{}^*$ and $c_2{}^*$ can then be obtained by substituting $s^*$ and $k^*$ into (7b), $c_1^*=(u^*k^*)^{\alpha}$ and $c_2^*=(s^*k^*)^{\beta}$. The left-hand side of (7c), for simplicity, is referred to as $LHS(s)$ and the right-hand side as $RHS(s)$.

From examining the left-hand side of (7c), it is clearly seen that as $s$ increases from 0 to 1, $LHS(s)$ is monotonically increasing from $LHS(s=0)=-\frac{(\frac{\gamma}{\rho+\delta})^{\frac{\gamma\beta}{1-\gamma}}}{\delta^{\beta}}\frac{1-\sigma}{\sigma}\frac{1+\tau_1}{1+\tau_2}\frac{\beta}{\alpha}(1-\frac{\gamma\delta}{\rho+\delta})<0$

to $LHS(s=1) \equiv \frac{(\frac{\gamma}{\rho+\delta})^{\frac{\gamma\beta}{1-\gamma}}}{\delta^{\beta}}[1+\frac{1-\sigma}{\sigma}\frac{1+\tau_1}{1+\tau_2}\frac{\beta}{\alpha}\frac{\gamma\delta}{\rho+\delta})]>0$. For the right-hand side of (7c), the *RHS*($s$) locus is monotonically decreasing in $s$ from *RHS*(s=0)=0 to *RHS*(s=1)=-η<0. See the locus *LHS*($s$) and *RHS*($s$) in Figure 1. Thus (7c) determines a unique $s^*$ of which the value is between 0 and 1. So, there exists a unique steady state in this model.

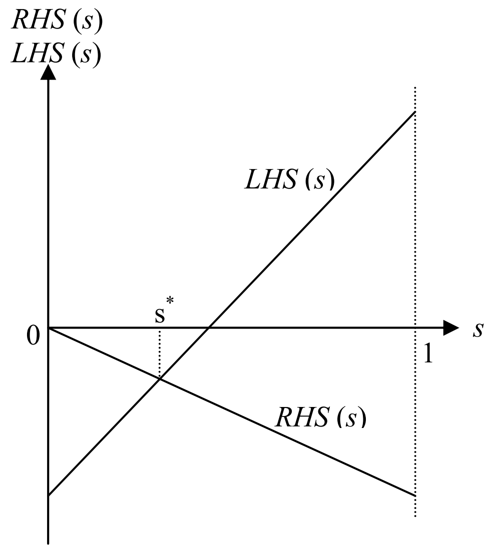


Figure 1: The existence of a steady state

## 3. Effect of Taxation and Tax Incidence

### 3.1 Effect of Taxation

We are ready to analyze the effect of taxation. Using (1), the representative household's welfare in a steady state is

$$U^* = \frac{c_1^{*\sigma}(c_2^*+\eta)^{1-\sigma}}{\rho}. \tag{8}$$

Because changes in consumption tax rates do not affect the accumulation of capital, we directly check the effect of taxation on the allocation of capital ($s^*$ and $u^*$) in two consumption sectors with two consumption goods.

Totally differentiating (7c) yields $\frac{\partial s^*}{\partial \tau_1} = -\frac{F_{\tau 1}}{F_s} > 0$ and $\frac{\partial s^*}{\partial \tau_2} = -\frac{F_{\tau 2}}{F_s} < 0$, where $F_s = s^{*(\beta-1)}(\frac{\gamma}{\rho+\delta})^{\frac{\gamma\beta}{1-\gamma}}(\frac{1}{\delta})^{\beta}(1+\frac{1-\sigma}{\sigma}\frac{1+\tau_1}{1+\tau_2}\frac{\beta}{\alpha})\beta+(1-\beta)s^{*(\beta-2)}(\frac{\gamma}{\rho+\delta})^{\frac{\gamma\beta}{1-\gamma}}(\frac{1}{\delta})^{\beta}\frac{1-\sigma}{\sigma}\frac{1+\tau_1}{1+\tau_2}\frac{\beta}{\alpha}(1-\frac{\gamma\delta}{\rho+\delta})>0,$

$F_{\tau 1} = -s^{*(\beta-1)}(\frac{\gamma}{\rho+\delta})^{\frac{\gamma\beta}{1-\gamma}}(\frac{1}{\delta})^{\beta}\frac{1-\sigma}{\sigma}\frac{1}{1+\tau_2}\frac{\beta}{\alpha}u^* < 0,$ and $F_{\tau 2} = s^{*(\beta-1)}(\frac{\gamma}{\rho+\delta})^{\frac{\gamma\beta}{1-\gamma}}(\frac{1}{\delta})^{\beta}\frac{1-\sigma}{\sigma}\frac{1+\tau_1}{(1+\tau_2)^2}\frac{\beta}{\alpha}u^* > 0.$ So an increase in the consumption tax on the necessity (luxury) good decreases (increases) the fraction of capital allocated to the production sector for the necessity good due to (7b) so that $\frac{\partial u^*}{\partial s^*} = -1 < 0$, and increases (decreases) the fraction of capital allocated to the production sector for the luxury good; thus consumption of the necessity is reduced (increased), and consumption of the luxury good is increased (reduced).

Intuitively, an increase in $\tau_1$ increases the relative price of $c_1$ to $c_2$, so the household has the incentive to use the expenditure on $c_2$ to replace that on $c_1$. Because the products $y_1$ and $y_2$ are only for the consumption of $c_1$ and $c_2$, respectively, a lower $c_1$ causes firms to produce a lower $y_1$, and thus firms will allocate a smaller fraction of capital to the $y_1$ sector. Thus we obtain a lower $u^*$ and a higher $s^*$ in the long run, and the product of $y_3$ does not change due to the fact that $k^*$ and $(1- u^* - s^*)$ are not affected.

A change in any one tax rate has an opposite effect on the two consumption goods and the household's lifetime welfare. Totally differentiating (8) yields

$$\frac{\partial U^*}{\partial \tau_1} = \frac{u(c_1^*, c_2^*)}{\rho}\left[\frac{\sigma}{c_1^*}\frac{\partial c_1^*}{\partial u^*}\frac{\partial u^*}{\partial s^*}\frac{\partial s^*}{\partial \tau_1} + \frac{1-\sigma}{c_2^*+\eta}\frac{\partial c_2^*}{\partial s^*}\frac{\partial s^*}{\partial \tau_1}\right] = \frac{u(c_1^*, c_2^*)}{\rho}\frac{F_{\tau 1}}{F_s}\frac{\sigma\alpha}{u^*}\left(1-\frac{1+\tau_2}{1+\tau_1}\right) \gtrless 0 \text{ if } \tau_1 \lessgtr \tau_2,$$

$$\frac{\partial U^*}{\partial \tau_2} = \frac{u(c_1^*, c_2^*)}{\rho}\left[\frac{\sigma}{c_1^*}\frac{\partial c_1^*}{\partial u^*}\frac{\partial u^*}{\partial s^*}\frac{\partial s^*}{\partial \tau_2} + \frac{1-\sigma}{c_2^*+\eta}\frac{\partial c_2^*}{\partial s^*}\frac{\partial s^*}{\partial \tau_2}\right] = \frac{u(c_1^*, c_2^*)}{\rho}\frac{F_{\tau 2}}{F_s}\frac{\sigma\alpha}{u^*}\left(1-\frac{1+\tau_2}{1+\tau_1}\right) \gtrless 0 \text{ if } \tau_1 \gtrless \tau_2.$$

Under $\tau_1$ >(<) $\tau_2$, it is better for the household's long-run welfare to decrease (increase) $\tau_1$ and increase (decrease) $\tau_2$ until both tax rates are equal.

In reality, a higher tax rate is usually levied on the luxury good, so the government should reduce the tax rate on the luxury good and raise that on the necessity good to the same level in order to enhance the household's lifetime welfare.

### 3.2 Tax Incidence and Quantitative Assessment

We now turn to the tax incidence exercise. To finance the government expenditure, $G$, at the same fraction of the output and in order to balance the government budget, the government chooses a combination of the two consumption tax rates that maximizes the representative household's welfare in the long run. Specifically, the government chooses the tax rate on the necessity good and the tax rate on the luxury good to maximize the social welfare (8) subject to the balanced government budget as follows

$$\tau_1\lambda_1 c_1 + \tau_2\lambda_2 c_2 = g(\lambda_1 y_1 + \lambda_2 y_2 + \lambda y_3) = G, \tag{9}$$

where $0< g <1$ is a given fraction of the government expenditure to GDP. The tax incidence exercise means that we take $g$ and any one of the two consumption tax rates as given, and determine the other tax rate endogenously to see how the household's lifetime welfare changes.

We use a simple quantitative analysis to determine the optimal tax rates. We calibrate the model in the steady state in order to reproduce key features that are representative of the U.S. economy in terms of annual frequencies. We choose our consumption tax on the necessity good as $\tau_1$ =5%, and the consumption tax on the luxury good as $\tau_2$ =20%. We assume that both consumption goods have the same share in the utility function, and so $\sigma$=0.5.

The rate of time preference is set at $\rho$=4% as used by Kydland and Prescott (1991), and the annual rate of the capital depreciation is set at $\delta$=5%. We choose shares of capital in the necessity goods sector and in the luxury goods sector of $\alpha$=0.3 and at $\beta$=0.2, respectively, due to the fact that some luxuries are expensive because they are handmade. Furthermore, we assume that the capital share in the investment goods sector is larger than that in the consumption goods sectors, and thus we choose $\gamma$=0.4.

The fraction of capital allocated to the luxury goods sector ($s$) is chosen to match the capital-output ratio that is $\frac{\lambda^* k^*}{\lambda_1^* y_1^* + \lambda_2^* y_2^* + \lambda^* y^*}$ at 3.32 (according to Cooley 1995, p21), and we obtain $s^*$=0.1191. We use (7c) and calibrate $\eta$=3.2290. Given this, we use equations (7a)-(7b) to compute the level of capital and the fraction of capital allocated to the necessity goods sector at $k^*$=54.0640 and $u^*$=0.6586. Then steady-state values of other variables are computed as follows:

$c^*_1=y^*_1=2.9206$, $c^*_2=y^*_2=1.4514$, $y^*_3=2.7032$ and the household's welfare $U^*=92.4308$. The pre-existing tax rates imply that, in the steady state, the share of government spending in output is $g=0.0684$.

We are now ready to quantify the effect of the tax incidence between $\tau_1$ and $\tau_2$ in the long run. We conduct changes in $\tau_1$ from 0 to 0.1108, and determine $\tau_2$ that finances the same fraction of the government spending in output as in the benchmark case. The results are reported in Figure 2. As $\tau_1$ increases from 0 and thus $\tau_2$ decreases from 49.29%, as expected from the above-mentioned analysis, consumption of the necessity goods decreases while consumption of the luxury goods increases. While the first effect causes welfare to decrease, the second effect causes welfare to increase. Our quantitative results indicate that the welfare maximization optimal tax mix is at $(\tau_1, \tau_2)=(8.19\%, 8.19\%)$. Similar to the theoretical results, when $\tau_1 < \tau_2$, the government should reduce the tax rate on the luxury good and raise that on the necessity good until both tax rates are the same. It is worth noting that when we change the value of σ (from 0.5 to 0.1 and 0.7), we obtain similar results so that the welfare maximization optimal tax mix is at $\tau_1=\tau_2$.

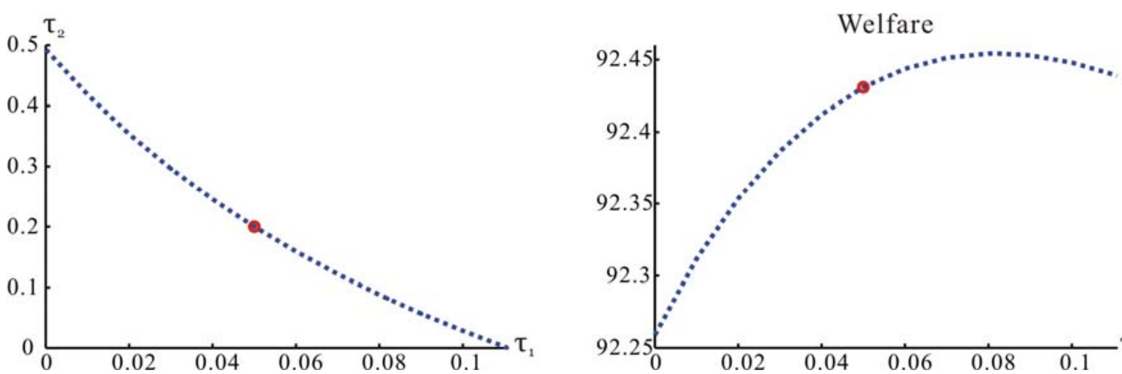


Figure 2: The results of dynamic tax incidence

Note: The dot points on the locus are the benchmarks at the pre-existing tax rates of $(\tau_1, \tau_2)=(0.05, 0.2)$.

## 4. Concluding Remarks

In this paper, we build a three-production-sector neoclassical growth model with inelastic labor supply, and examine the optimal consumption tax rates on the necessity good and the luxury good. We obtain two consumption taxes that are neutral to economic growth and that the welfare maximization optimal tax mix is where $\tau_1=\tau_2$. In reality, a higher tax rate is usually levied on the luxury good, so the government should reduce the tax rate on the luxury good and raise that on the necessity good to the same level in order to enhance the household's lifetime welfare.

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