Young workers' professional experience and access to high-skill jobs: a note

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Abstract
The implications of on-the-job search and learning-by-doing of young workers are studied in a search-matching model. The labor market is segmented in two sub-markets: that of beginners, and that of experienced workers offering higher wages. After a long enough employment spell, beginners can search for a better-paying job in the experienced sub-market. Employment instability reduces upgrading opportunities for young workers, penalizing the overall economy. Under specific conditions this phenomenon is reinforced when firms are more fussy about workers' professional experience.

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1. Introduction

The past decades have been harsh for young workers as their unemployment rate was double the rest of the populations in many developed countries. In Europe the youth unemployment rate (15 to 24 y.o.) was around twice as high as the total unemployment rate throughout the last decade (source: Eurostat). In the Euro zone, known for its low flexibility, this rate was at 16% in 2008 and almost 21% in 2010, with a peak of 42% in 2010 for Spain. In the United-States, where the labor market is flexible, in the same period, the rate also grew from 12.8% to 18.4% (source: Bureau of Labor Statistics).

Not only have young workers been penalized by economic shocks, being the first in line to be affected by the economic crisis, but they also faced a progressive change in their career path independent of the degree of flexibility of labor markets. In the literature on internal/external markets (see Kerr 1954, Dunlop 1958, and Doeringer and Piore 1971) workers habitually entered the firm at the bottom and climbed the social ladder internally. Nowadays this career path is rare. A worker’s career takes place inter-firms with the worker searching on the job and changing employers often. According to Amossé (2002), 2 out of 3 promotions implied a change of firm in 2001-2002 whereas this figure was of 1/2 in 1991-1992. In the USA, young workers hold an average of 7 jobs in 10 years (see Topel and Ward 1992), with 2/3 of their professional mobility taking place at the beginning of their career. Several studies also showed the importance of this on-the-job search process. For instance, in the USA, 50% of new contracts are job-to-job transitions (Fallick and Fleishman 2004). It concerns specifically low wage earners (Topel and Ward 1992, Charner and Fraser 1984), young workers (Skuterud 2005, Pissarides and Wadsworth 1994), and underemployed workers (Altonji and Paxson 1988). Obtaining promotion in a new job requires training in a previous job. But when job separation involuntarily occurs before the worker is trained, there is no chance for her to be promoted in the next job.

This brief article proposes a theoretical framework depicting the career path of workers and showing the consequences of job instability in the labor market. On-the-job search and learning-by-doing of young workers are introduced in a search-matching model (Pissarides 2000) where the labor market is segmented into two interdependent sub-markets: the beginners’ sub-market composed of beginners, and the experienced sub-market composed of experienced workers earning higher wages. Untrained beginners have to train themselves on the job through a learning-by-doing process before being able to search while on the job for an experienced job. The model shows that beginners’ job instability prevents them from obtaining experienced jobs thus penalizing the overall economy.

The paper is organized as follows: Section 2 sets up the analytical framework. Section 3 solves the model and presents sub-markets interaction. Section 4 analyses the impact of job instability and professional experience on the labor force structure, and Section 5 concludes.
2. The Model

We introduce on-the-job search in a search-matching model (see Pissarides 2000). The economy consists of two types of risk-neutral agents: workers and firms. Firms are infinity-lived whereas workers have a life expectancy of $1/m$, where $m$ is the labor market exit rate. Each worker who exits the labor market is replaced with a newcomer. Time is continuous and all the agents discount future payoffs at rate $r$, with $r > 0$.

2.1 A segmented labor market

The labor market is divided into two sub-markets: sub-market 2 composed of beginners, and sub-market 1 composed of experienced workers. There is a specific matching technology in each sub-market. Firms distribute themselves in the two sub-markets; each sub-market offering a different job type (2: beginners or 1: experienced). Type-1 job productivity is higher than type-2, $y_1 > y_2$, so as wages $w_1 > w_2$. This wage differentiation motivates beginners to look for a better-paying type-1 job. But only beginners which have been trained on the job in sub-market 2 are perceived as sufficiently experienced to be desirable on type-1 jobs. Trained beginners are thus the only ones searching on the job. When their search succeeds they become experienced workers. This professional path implies that the two labor sub-markets interact.

Assuming a constant labor force, workers’ flows are depicted by figure 1, where $u_1$ is experienced unemployment, $u_2$, beginners unemployment, $\ell_1$, experienced employment, $\ell_2$, untrained beginners employment, $\tilde{\ell}_2$, trained beginners employment. $p_i$, $i = \{1, 2\}$, is the rate at which a worker finds a type-$i$ job, $s_i$ is the type-$i$ job separation rate. Due to their lack of experience, beginners must train themselves before applying to type-1 job. $1/\lambda$ represents the expected average duration that a worker must face before having access to this trained stage, a beginner becomes employable by a type-1 firm at rate $\lambda$.

Figure 1: Workers’ flows
A matching function is defined for each sub-market. In sub-market 1, the tightness \( \theta_1 \) depends on the number of type-1 vacancies \( v_1 \), on that of unemployed experienced workers \( u_1 \), and on that of trained beginners \( \hat{\ell}_2 \) as they are engaged in an on-the-job search process. \( \theta_1 = \frac{v_1}{u_1 + \hat{\ell}_2} \) is the experienced sub-market tightness. The matching function, \( h_1 = h_1(v_1, (u_1 + \hat{\ell}_2)) \), is increasing in both its arguments, concave and homogeneous of degree 1. Job vacancies are filled by random sorting according to a Poisson process of rate \( q_1 \). The homogeneity of the matching function implies \( q_1 = \frac{h_1(v_1, (u_1 + \hat{\ell}_2))}{v_1} = h_1 \left( 1, \frac{1}{\theta_1} \right) = q_1(\theta_1) \), where \( q_1 \) is the rate at which an vacant type-1 job is filled. By the properties of the matching technology, \( q_1'(\theta_1) \leq 0 \). Unemployed experienced workers and trained beginners find a type-1 job at rate \( p_1 \), determined as \( p_1(\theta_1) = \theta_1 q_1(\theta_1) \) with \( p_1'(\theta_1) \geq 0 \).

In sub-market 2, the tightness \( \theta_2 \) depends on the number of type-2 job vacancies \( v_2 \) and on the number of unemployed beginners \( u_2 \), \( \theta_2 = \frac{v_2}{u_2} \). The matching function is \( h_2 = h_2(v_2; u_2) \). A vacant type-2 job is filled at rate \( q_2 = q_2(\theta_2) \), with \( q_2(\theta_2) \leq 0 \), and unemployed beginners find type-2 jobs at rate \( p_2(\theta_2) = \theta_2 q_2(\theta_2) \), with \( p_2'(\theta_2) \geq 0 \). The absolute value of the elasticity of \( q_2(\theta_2) \) is \( \eta_2(\theta_2) \in [0, 1] \).

### 2.2 Asset values

All unemployed workers produce \( d < y_2 < y_1 \) units of output at home for self-consumption. All workers, employed and unemployed, exit the labor market at rate \( m \). Each firm hires a single worker. A firm opens a job vacancy and faces a cost \( c_i \) with \( i = \{1, 2\} \) of keeping the vacancy open, with \( c_2 < c_1 \) due to differences in job productivity.

In sub-market 1, \( W_1 \) denotes the asset value of an experienced worker, and \( U_1 \) that of an unemployed experienced worker. Let \( J_1 \) be the asset value of a firm with an occupied type-1 job, and \( J_1^V \) that of a firm in a situation of vacancy. We have:

\[
\begin{align*}
ru_1 &= d + p_1 [W_1 - U_1] - mU_1 \\
rw_1 &= w_1 - s_1 [W_1 - U_1] - mW_1 \\
rJ_1 &= y_1 - w_1 - (s_1 + m)[J_1 - J_1^V] \\
rJ_1^V &= -c_1 + q_1 [J_1 - J_1^V]
\end{align*}
\]

When a firm and its employee separate at rate \( s_1 \), the worker becomes an unemployed experienced worker and the firm is in a situation of vacancy. An unemployed worker (a firm) finds a type-1 job (a worker) at rate \( p_1 (q_1) \). Here we assume that spells of unemployment of experienced workers do not downgrade workers’ skills. A firm with an occupied job produces \( y_1 \) and pays \( w_1 \) to its experienced worker.

In sub-market 2, \( W_2 \) is the asset value of an employed beginner, \( \tilde{W}_2 \) that of a trained beginner, and \( U_2 \) that of an unemployed beginner. Let \( \tilde{J}_2 \) be the asset value of a firm whose job is filled with a trained beginner, \( J_2 \) that of a firm whose job is filled with an untrained
When a beginner loses her job at rate $s_2$, she automatically loses her trained status (if she previously had it) and thus becomes an unemployed beginner. A firm offers $w_2$ to its employee and produces $y_2$. When a firm separates from its worker at rate $s_2$, it becomes a firm in a situation of vacancy. A firm employing a trained beginner faces a probability $p_1$ that its worker leaves the job for a type-1 job, whereas the firm employing an untrained beginner faces the probability $\lambda$ that its employee becomes trained.

Firms distribute themselves between the two sub-markets according to the free entry condition so that $J_{2V}^i = 0$ with $i = \{1, 2\}$.

### 3. Equilibrium and sub-markets interaction

#### 3.1 Wage setting

From now on we assume that the sub-market 2 wage is constrained by a binding mandatory minimum wage\(^1\) so that $w_2 = \bar{w}_2$. In sub-market 1, the wage is negotiated according to a Nash bargaining game, where $\beta \in [0, 1]$ is the bargaining power of workers. Combining the sharing rule of the global surplus $(1 - \beta)[W_1 - U_1] = \beta J_1$ with (1), (2), (3) and the free-entry condition leads to the wage equilibrium equation:

$$w_1 = \frac{\beta y_1 (r + s_1 + p_1 + m) + (1 - \beta)d(r + s_1 + m)}{\beta(r + s_1 + p_1 + m) + (1 - \beta)(r + s_1 + m)}$$

#### 3.2 Job creation

There is one equilibrium equation in each sub-market. In sub-market 1, the equilibrium is determined by combining (3), (4), (11) and the free-entry condition:

$$0 = -c_1 + q_1 (1 - \beta) \frac{y_1 - d}{r + m + s_1 + \beta p_1}$$

Equation (12) depends exclusively on the experienced sub-market tightness $\theta_1$. As $\theta_1$ is independent of $\theta_2$, job creation in sub-market 2 does not impact either on type-1 job creation or on experienced wage.

\(^1\)Overall Results can be extended to apply to a bargained wage.
In sub-market 2, combining (8), (9), (10), and the free-entry condition gives the sub-
market equilibrium equation for job creation:

\[ 0 = -c_2 + q_2 \frac{(y_2 - w_2)(r + s_2 + m + \lambda + p_1)}{(r + s_2 + m + p_1)(r + s_2 + m + \lambda)} \]  

Equation (13) determines the beginners’ sub-market tightness \( \theta_2 \) as a function of the ex-
perienced sub-market tightness \( \theta_1 \), \( \frac{\partial \theta_2}{\partial \theta_1} < 0 \) (see Appendix B). When the rate \( p_1 \) rises, 
trained beginners leave their type-2 job more easily, hence the value of occupied type-2 jobs 
decreases. Less jobs are thus created in sub-market 2.

3.3 Labor force structure in steady-state equilibrium

The total labor force is normalized to 1. In steady-state, inflows of workers equalize outflows 
of workers for each employment state\(^2\). These flows are given respectively for \( u_2, u_1, \ell_1, \ell_2, \hat{\ell}_2 \):

\[ u_2(\hat{p}_2 + m) = s_2(\hat{\ell}_2 + \hat{\ell}_2) + m; \quad u_1(p_1 + m) = s_1\ell_1; \quad p_2u_2 = \ell_2(s_2 + m + \lambda) \]

\[ \lambda \ell_2 = \hat{\ell}_2(s_2 + m + p_1); \quad \ell_1(s_1 + m) = p_1(u_1 + \ell_2). \]

Combining these equations gives the labor force structure in steady-
state. We have: \( \ell_2 = \frac{v_2m(s_2 + m + p_1)}{\phi}, \hat{\ell}_2 = \frac{v_2m\lambda}{\phi}, u_2 = \frac{m(s_2 + m + p_1)(s_2 + m + \lambda)}{\phi}, \ell_1 = \frac{\lambda p_2p_1(m + p_1)}{\phi(s_1 + m + p_1)}, \]

\[ u_1 = \frac{\lambda p_2p_1 s_1}{\phi(s_1 + m + p_1)}, \text{ where } \phi = m(s_2 + m + p_1)(s_2 + m + p_2 + \lambda) + \lambda p_2(m + p_1). \]

Even if tightness \( \theta_1 \) is independent of tightness \( \theta_2 \), employment in sub-market 1 depends on job creation in 
sub-market 2. Thus the employment level of trained beginners does determine the level of 
the most productive jobs in the economy.

Table 1 depicts direct impacts of tightness \( \theta \) on the labor force structure in the steady-
state equilibrium\(^3\). A rise in \( \theta_2 \) increases type-2 job creation thus raising employment in

<table>
<thead>
<tr>
<th>( \theta_1 )</th>
<th>( \ell_2 )</th>
<th>( \ell_2 \hat{u}_2 )</th>
<th>( \ell_1 )</th>
<th>( u_1 )</th>
<th>( \ell_1 + u_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 )</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

sub-market 2 (direct impact). There are thus more applicants who are likely to become 
experienced. We observe a rise in the experienced sub-market size (indirect impact).

A rise in \( \theta_1 \) increases type-1 job creation thus raising employment in the experienced 
sub-market (direct impact). However, as an increase in \( \theta_1 \) reduces \( \theta_2 \), type-2 job creation 
decreases. It is easier to find a type-1 job, but there are less on-the-job searchers applying for 
type-1 jobs, thus lowering experienced employment (indirect impact). The rise of experienced 
employment is thwarted by the reaction of the beginners’ sub-market. Here the overall impact 
on the labor force structure depends on the reaction of each employment level with respect to 
\( \theta_1 \) and \( \theta_2 \). For \( \ell_2 \) and \( \ell_2 \), \( \theta_1 \) and \( \theta_2 \) have opposite impacts, thus the effect of \( \theta \)’s reinforces each 
other. For \( u_2, \ell_1, u_1 \) and \( \ell_1 + u_1, \theta_1 \) and \( \theta_2 \) have the same impact (positive or negative). The

\(^2\)See appendix A for a summary table of the main variables and parameters of the model.

\(^3\)Detailed calculus are available from the author upon request.
overall impact therefore depends on the value of the elasticity of the probability of finding a type-2 job, \( p_2 \), with respect to \( \theta_2, (1 - \eta_2) \), which measures the strength of beginners’ sub-market reaction. Two cases should be considered:

- \((1 - \eta_2)\) is small, \( p_2 \) elasticity is weak. The matching rate of a beginner in a type-2 job varies weakly with \( \theta_2 \). Thus, the effect of \( p_1 \) always overcomes that of \( p_2 \).

- \((1 - \eta_2)\) is big, \( p_2 \) elasticity is strong. A raise in \( \theta_2 \) can lead to a sufficiently large variation of \( p_2 \) to overcome the effect of \( \theta_1 \). The effect of \( p_2 \) dominates that of \( p_1 \).

4. Employment instability and workers’ experience

4.1 Employment instability

In Europe youth unemployment is a major issue to which successive governments responded by making short fixed-term jobs more readily obtainable. These contracts are often associated with employment instability as firms are more likely to use them during periods of economic instability. Government employment policy disregards the fact that job openings, reserved for beginners, are only a first step in their career path whose final objective is to secure a stable, better-paying job. Blanchard and Landier (2002) argue that the main effect of an increase of fixed-term contracts may be high turnover of beginners, leading to higher unemployment, and may possibly reduce overall productivity and output. Introducing on-the-job search allows us to explain the negative impact of this employment instability.

Table 2 gives the comparative statics of the separation rate \( s_2 \). A rise in \( s_2 \) corresponds to a higher instability of beginners’ jobs. Beginners, trained or untrained, are more likely to lose their jobs. Job instability not only increases unemployment in sub-market 2, it also negatively affects the probability \( p_2 \) for an unemployed beginner to find a type-2 job. An increased number of unemployed workers thus face a lower chance to obtain a type-2 job. For this reason, the impact on untrained beginners employment \( \ell_2 \) is not immediate, whereas trained beginners employment \( \hat{\ell}_2 \) decreases. There are less trained workers searching for a type-1 job. As a consequence experienced employment \( \ell_1 \) is reduced. Experienced unemployment \( u_1 \) is reduced as well because fewer experienced workers face the same separation rate, \( s_1 \). When job instability increases young workers struggle to accumulate the years of successful professional experience necessary to obtain a more productive job. It is harmful for type-

This suggests that employment policies aiming for an increase in young workers job supply, by promoting short-term contracts or time limited contracts, do not have a positive impact for the most productive jobs.
During periods of crisis, experienced workers are also penalized by job instability. A rise in $s_1$ captures this job instability. The comparative statics is given in Table 3. Because

Table 3: Impact of a rise in the separation rate $s_1$

<table>
<thead>
<tr>
<th>$\theta_2$</th>
<th>$p_2$</th>
<th>$q_2$</th>
<th>$\theta_1$</th>
<th>$p_1$</th>
<th>$q_1$</th>
<th>$w_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

of sub-market interaction experienced job instability reduces $p_1$ thus increasing $p_2$. The chance for unemployed beginners to obtain a job increases. As a consequence more type-2 jobs are created, raising the number of trained on-the-job searchers. The overall impact on the experienced sub-market therefore depends on the value of $\eta_2$. A value of the elasticity $(1 - \eta_2) \in [0,1]$ that cancels both effects is defined for each variable $u_2, \ell_1, \ell_1 + u_1$ (see appendix C). $\eta'_2 = \Gamma'(r,m,s_2,p_1,\lambda)$, $\eta''_2 = \Gamma''(r,m,s_2,s_1,p_2,p_1,\lambda)$ and $\eta'''_2 = \Gamma'''(r,m,s_2,p_1)$ are respectively the value for $u_2, \ell_1$ and $\ell_1 + u_1$. Note that both situations, $(1 - \eta_2)$ weak and strong, exist and take place at the threshold of the set of definition $[0;1]$.

When $(1 - \eta_2)$ is weak, experienced employment decreases due both to the reduction of $p_1$ and the rise of $s_2$. Beginners unemployment increases because on-the-job searchers face fewer chances of getting a type-1 job, creating a larger number of on-the-job searchers exposed to the separation rate $s_2$. The size of sub-market 1 is reduced and beginners unemployment increases. When $(1 - \eta_2)$ is strong, there are more young workers facing the smaller chance $p_1$ of obtaining a type-1 job. Thus more beginners obtain a type-1 job. The impact of job instability on experienced employment remains unclear, however it decreases beginners unemployment and the size of sub-market 1 increases.

### 4.2 Professional experience

A change in the parameter $\lambda$ embodies the idea that the length of previously acquired professional experience can be a more or less important factor for experienced firms in employing a beginner. For beginners, a rise in the chance of becoming trained generates two opposite effects (see appendix C). An increase in $\lambda$ makes young workers more quickly eligible for type-1 jobs, leading to a rise in experienced employment. But the raise of $\lambda$ reduces the value of occupied type-2 jobs, the consequence being less job creation and thus less on-the-job searchers. Again, the dominant effect depends on the value of $(1 - \eta_2)$ (see table 4).

When $(1 - \eta_2)$ is weak, the predominant effect leads to a rise in the number of trained beginners (on-the-job search) thus increasing the size of the experienced sub-market. When $(1 - \eta_2)$ is strong, the predominant effect reduces job creation in the experienced sub-market leading to a cut in the size of the beginners’ sub-market.
During periods of crisis, and more generally when there is a shortage of jobs, firms can allow themselves to be more fussy about workers’ professional experience. This phenomenon corresponds to a reduction in $\lambda$ resulting in impacts which are the opposite of those presented in table 4. It is interesting to note that such a demand from firms reduces youth unemployment and fosters experienced job creation when $(1 - \eta_2)$ is strong. But when $(1 - \eta_2)$ is weak this phenomenon strongly penalizes the overall labor market by reducing youth and experienced employment.

5. Conclusion

Young workers belonging to the beginners’ sub-market must train themselves on the job before acquiring the professional experience necessary to obtain a better-paying high-skill job in the experienced sub-market. When job instability arises, due, for instance, to economic shocks, job separation increases for all types of jobs either open to beginners or to experienced workers. Moreover, job shortage allows firms to be more fussy about workers’ professional experience. In this paper, each of these three points has been individually appraised in relation to their impact on the labor market structure. Due to sub-market interactions, the accumulation of these points strongly increases youth unemployment and penalizes experienced employment when the elasticity of finding a beginner’s job with respect to the tightness of the beginners’ sub-market is weak. When this elasticity is strong the result may not hold and should be reassessed by performing a calibration of the model.

One limit of the model concerns employment instability. In order to better account for youth employment instability, our model should endogenize the job destruction process. When economic shocks occur, firms separate from their employees, but the firm does not necessarily disappear. They reduce their labor force in order to remain productive. Hence job separation would not necessarily lead to a cut in productivity. This point will be the focus of further research.

The model presented in this note highlights the importance of on-the-job search of young workers and presents the interactions of the beginners and the experienced sub-markets. Its framework can be extended to analyse the impact of several institutions such as the minimum wage and unemployment benefits. It can also serve as the baseline for research dealing with the impact on the labor force structure of formal education and of learning-by-doing of workers either in developed economies or in developing economies. These issues are the purpose of ongoing research.
A Variables and parameters list

<table>
<thead>
<tr>
<th>Variable/parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>Labor market exit rate</td>
</tr>
<tr>
<td>r</td>
<td>Rate at which agents discount future payoffs</td>
</tr>
<tr>
<td>c_i</td>
<td>Cost of keeping a type-i vacancy open</td>
</tr>
<tr>
<td>y_i</td>
<td>Productivity in a type-i job</td>
</tr>
<tr>
<td>w_i</td>
<td>Wages for a type-i job</td>
</tr>
<tr>
<td>β</td>
<td>Wage bargaining power of workers</td>
</tr>
<tr>
<td>p_i</td>
<td>Rate at which a worker finds a type-i job</td>
</tr>
<tr>
<td>q_i</td>
<td>Rate at which a type-i vacancy is filled with a worker</td>
</tr>
<tr>
<td>θ_i</td>
<td>Tightness in sub-market-i</td>
</tr>
<tr>
<td>η_2</td>
<td>Elasticity of the probability for a type-2 firm to find a worker, q_2, with respect to tightness θ_2.</td>
</tr>
<tr>
<td>s_i</td>
<td>Separation rate from a type-i job</td>
</tr>
<tr>
<td>λ</td>
<td>Rate at which untrained workers become trained/employable in type-1 firms</td>
</tr>
<tr>
<td>ℓ_2</td>
<td>Untrained beginners employment</td>
</tr>
<tr>
<td>ℓ_2̂</td>
<td>Trained beginners employment</td>
</tr>
<tr>
<td>u_2</td>
<td>Beginners unemployment</td>
</tr>
<tr>
<td>u_1</td>
<td>Experienced workers unemployment</td>
</tr>
<tr>
<td>ℓ_1</td>
<td>Experienced workers employment</td>
</tr>
</tbody>
</table>

B Relation between θ_1 and θ_2

The relation between θ_1 and θ_2 is determined by (13).

\[
\frac{\partial q_2}{\partial \theta_1} = \frac{p_2'(\theta_1) c_2(r+s_2+m+\lambda)-q_2(y_2-\bar{w}_2)}{q_2'(\theta_2) (r+s_2+m+\lambda+p_1)(y_2-\bar{w}_2)}
\]

Around the equilibrium \( c_2(r+s_2+m+\lambda) > q_2(y_2-\bar{w}_2) \). It implies \( \frac{\partial q_2}{\partial \theta_1} < 0 \).

C Comparative Statics of labor force structure

Note \( X = \{\ell_2, \ell_2^\hat{}, u_2, \ell_1, u_1, \ell_1 + u_1\} \), the effect of \( s_1 \) on \( X \) is defined such as

\[
\frac{dX}{ds_1} = \frac{\partial X}{\partial s_1} + \frac{\partial X}{\partial p_1} \frac{\partial p_1}{\partial \theta_1} + \frac{\partial X}{\partial p_2} \frac{\partial p_2}{\partial \theta_2} \frac{\partial p_1}{\partial \theta_1} \frac{\partial p_1}{\partial s_1} \]

The overall indirect impact of \( s_1 \) on the labor force structure goes through two effects via \( \theta_1 \) and \( \theta_2 \). The one that overcomes depends on the value of \((1-\eta_2)\) and thus on \( \eta_2 \).

**Proof.** The overall indirect impact of \( s_1 \) can be resumed as

\[
\frac{\partial X}{\partial \theta_1} \frac{\partial p_1}{\partial \theta_1} + \frac{\partial X}{\partial p_2} \frac{\partial p_2}{\partial \theta_2} \frac{\partial p_2}{\partial \theta_1} \frac{\partial p_1}{\partial \theta_1} \frac{\partial p_1}{\partial s_1} \]

Noting \( \mu = \frac{\partial p_2}{\partial \theta_1} \frac{\partial p_2}{\partial p_1} = \frac{p_2\lambda}{(r+m+s_2+p_1)(r+s_2+m+p_1+\lambda)} \left[ \frac{1-\eta_2}{-\eta_2} \right] < 0 \). The value of \( \eta_2 \) that cancels both effects is determined by \( \left[ \frac{\partial X}{\partial p_2} \mu + \frac{\partial X}{\partial p_1} \right] \) where \( \mu = \mu(\eta_2) \).
Both effects cancel each other for the following values of \( \eta_2 \in [0, 1] \):

**Application to \( u_2 \):**

\[
\eta_2' = \frac{(s_2 + m + p_1)(m(s_2 + m + p_1) + \lambda(m + p_1))}{(s_2 + m + p_1)(m(s_2 + m + p_1) + \lambda(m + p_1)) + s_2(r + s_2 + m + p_1)(r + s_2 + m + p_1 + \lambda)}
\]

**Application to \( \ell_1 \):**

\[
\eta_2'' = \frac{\lambda p_1(s_2 + m + p_1)}{\lambda p_1(s_2 + m + p_1) + (s_2 + m + p_2)(r + s_2 + m + p_1)(r + s_2 + m + p_1 + \lambda)}
\]

Let \( x_2 = \{s_2, \lambda\} \), the incidence of \( x_2 \) on \( X \) can be written as:

\[
\frac{dX}{dx_2} = \left(\frac{\partial X}{\partial x_2}\right)_{\text{direct}} + \left(\frac{\partial X}{\partial p_2}\frac{\partial p_2}{\partial \theta_2}\right)_{\text{indirect by } \theta_2}
\]

The impact of \( s_2 \) is either straightforward or undefinable. The overall impact of \( \lambda \) on labor force structure depends on the value of \( (1 - \eta_2) \).

\[
\frac{dX}{d\lambda} = \frac{\partial X}{\partial \lambda} + \frac{\partial X}{\partial p_2}\frac{\partial p_2}{\partial \theta_2} = \frac{\partial X}{\partial \lambda} + \frac{\lambda p_2}{(r + s_2 + m + p_1)(r + s_2 + m + p_1 + \lambda)} \left[ 1 - \eta_2 \right] \frac{\partial p_2}{\partial \lambda}
\]

Concerning \( \ell_2, u_1, \ell_1 \) and \( u_1 + \ell_1 \), the value of \( \eta_2 \), defined on \([0, 1]\), that cancels both effects is determined by:

\[
\hat{\eta}_2 = \frac{\lambda^2(s_2 + m + \lambda)}{\lambda^2(s_2 + m + \lambda) + (s_2 + m + p_2)(r + s_2 + m + p_1)(r + s_2 + m + p_1 + \lambda)}
\]

Concerning \( u_2 \), the value of \( \eta_2 \), defined on \([0, 1]\), that cancels both effects is determined by:

\[
\hat{\eta}_2 = \frac{\lambda(s_2 + m + \lambda)[m(s_2 + m + p_1) + \lambda(m + p_1)]}{\lambda(s_2 + m + \lambda)[m(s_2 + m + p_1) + \lambda(m + p_1)] + s_2 p_1(r + s_2 + m + p_1)(r + s_2 + m + p_1 + \lambda)}
\]
References


