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Sharing a polluted river network through environmental taxes

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Abstract

Gómez-Rúa (2011, SERIEs) considers a river divided into n segments. In each segment there is exactly one agent, who releases some kind of residue into the water. An environmental authority must share the total cost of cleaning the river network among all the agents. In this paper we extend the results obtained there, to the context of a river network and so we propose several rules to distribute the total cleaning-cost among the agents. Furthermore, we provide axiomatic characterizations for them using properties based on water taxes. Besides we prove that one of the rules coincides with the weighted Shapley value of a game associated to the problem.

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1. Introduction

Air and water pollution were the initial focus of many environmental policies introduced by OECD countries in the 1970s. These were motivated by a perception that natural environments were being degraded at an accelerating rate, with adverse consequences for ecosystems and human health (OECD, 2008a)

A number of countries (e.g. Australia, France, Spain) aim to manage water resources and pollutant discharges in a common, consistent framework at the river-basin level. Since the adoption of the European Union Water Framework Directive, all EU member states are required to use an integrated river basin management plan in order to achieve good water status of all water bodies by 2015.

The Water Framework Directive establishes several integrative policies to compare the costs between the cleansing of water downstream and the cost of pollution control upstream. Integrated policies also facilitate cost recovery (OECD, 2004). By giving river-basin authorities access to treatment costs that water supply operators have, it provides them with information on the costs of upstream pollution, which may be used to estimate the rates of pollutant releases water charges. In addition, river basin management not only facilitates water allocation among competing uses within the basin but also the control of inter-basin transfers (OECD, 2008b).

In Gómez-Rúa (2011) a model is developed in order to study this problem from a theoretical point of view. A river is considered which is divided into n segments. In each segment, there is an agent who discharges pollutant substances into the river. The authorities require cleansing of the river in order to return it to its natural state. To pursue this, in Gómez-Rúa (2011) several rules are proposed. It proposes to distribute the total pollutant-cleaning costs among all the agents that cause the pollution. Additionally, for each rule, an axiomatic characterization is provided by using properties based on water taxes.

This model was introduced by Ni and Wang (2007). They propose two rules to divide the total river-polluting cost among the polluters and they provide characterization result for both rules.

Dong et al. (2007) generalize the results in Ni and Wang (2007). They consider a river network divided into n segments. In each segment there is exactly one agent, who throw some kind of residue into the water. An environmental authority must share the total cost of cleaning the river network among all the agents. They propose three rules to share this cost. In particular, the Upstream Equal Sharing (UES) rule is characterized with axioms of *Additivity*, *Independence of Upstream Costs* (that ensures that no agent has any responsibility for the pollution caused in the upstream segments), *Independence of Irrelevant Costs* (which states that an agent should not bear any costs which are irrelevant to her, *i.e.*, the costs regarding upstream-downstream relationship in the network), *Efficiency* and *Upstream Symmetry* (which states that for any given downstream costs, all upstream polluters share them equally).

Efficiency, Independence of Upstream Costs and Independence of Irrelevant Costs are very appealing properties.

Additivity has been used in many different situations. For instance, in cooperative games with transferable utility (TU), the Shapley value (Shapley, 1953) is characterized with this property. Moulin (1987) and Chun (1988) used this axiom in surplus problems and in allocation problems, respectively. In bankruptcy problems and other related problems, Bergantiños and Vidal-Puga (2004) characterize up to three different rules with additivity and other properties. This axiom is also used in cost sharing problems (Moulin, 2002). Moulin and Sprumont (2005) focus on additive rules for cost sharing problems with demands. In minimum cost spanning tree problems, Bergantiños and Vidal-Puga (2009) characterize a rule and provided a detailed discussion of this property. Moreover, in this context of sharing the cost of cleaning a river, we can find real examples of water taxes that are additive (Gómez-Rúa, 2011).

Regarding the property of Upstream Symmetry, many situations exist where this axiom cannot be applied (see Gómez-Rúa (2011) The main purpose of this paper is to replace the property of Upstream Symmetry by other properties, in order to obtain rules which are more suitable to these situations.

We introduce a new property in this context, which collects the idea of both the *Independence of Upstream Costs* and *Independence of Irrelevant Costs.* We call this property *Independence of No Responsibility Costs*, and it ensures that an agent's cost share only depends on her own pollution cost as well as all downstream costs, but not on those costs associated with some other segment for which she is not liable for.

Our main result is a characterization of the set of rules satisfying Efficiency, Additivity and Independence of No Responsibility Costs. Besides this, we also characterize other rules by adding two properties to the previous ones. Both properties were introduced in Gómez-Rúa (2011) and are now generalized for the network context.

The first property takes into account the fact that in many cases residues that are dumped into the river are biodegradable. Sometimes, it is possible to know the biodegradation rate of the residues, say δ; so it seems reasonable to demand that the cost that an agent should pay for cleaning a polluted area, depends on this rate. We name this property δ-*Biodegradation rate.*

The second property follows from the fact that in many countries (such as Austria, Canada, Finland, France, Germany, Greece, Hungary, Italy, Korea, Spain, Sweden, USA, among others) there are several alternatives in the design of water tax rates. A variable component exists which depends on different factors, such as, the volume of water consumed, the pollution load, the population of the municipality, type of residue, etc. (Gago et al. 2005, 2006; OECD, 2006). Thus, each agent pays proportionally depending on these factors. This is the idea of *Proportional Tax*.

The last result of the paper is a game theoretic approach. In Gómez-Rúa (2011), we introduce a TU game, and we prove that one of the rules proposed coincide with the weighted Shapley value of that game. In this paper, we generalize this result to the new context.

The paper is organized as follows. In Section 2, we introduce the model. In Section 3, we characterize the family of rules satisfying three properties. In Section 4, we introduce two more properties in this context and we present new rules and characterization results for them. Moreover, we prove that one of the rules coincides with the weighted Shapley value of a particular cooperative game.

2. The model

We follow the model presented by Dong et al. (2007).

Consider a river network populated by a set of agents, $N = \{1, 2, ..., n\}$ and a special agent, L, called *lake.* Agents are connected to each other by a series of links. Upstream agents each exude a certain amount of pollutant to the network. The polluted river needs to be cleaned. The cost of cleaning every river link as well as the lake is known and must be shared among these agents.

Formally, let $N' = N \cup \{L\}$ be the set of agents, and let E be the set of links on N'. We assume that E is a tree, *i.e.*, all agents are connected in E and there are no cycles.

A *cost function* is a mapping $C : N \cup \{L\} \to \mathbb{R}_+$, where for each $i \in N$, $C(i) = c_i$ is the cost associated with agent i (e.g., the link cost that is associated with the link between agent i and her successor toward L) and $C(L)$ is the cost associated with L. We denote $C(N) = \sum_{i \in N} c_i$.

A *cost-sharing problem on a river network* is a triple (N', E, C).

A *solution* to a problem (N', E, C) is a vector $x = (x_1, ..., x_n, x_L) \in \mathbb{R}_+^{n+1}$ such that $\sum_{i \in N'} x_i = C(N) + C(L)$, where x_i is the cost share assigned to agent $i \in N'$.

A *method* (or *rule*) is a mapping x that assigns to each problem (N', E, C) a solution $x(N', E, C)$.

Given a tree E, the upstream-downstream relation among the agents is uniquely determined by the node L. Also, for any agent, there is a unique path that connects a sequence of downstream agents successively to L.

Now we introduce some notation related with the graph structure. Given (N', E, C), we define the following sets:

$IU(i) := \{j \in N :$ there is a path from j to L such that i is $j's$ immediate downstream agent$\}$. The agents in $IU(i)$ are called *immediate upstream agents* of i in E.

$U(i) := \{j \in N :$ there exists $h_1, h_2, ..., h_m$ in N' such that $h_1 = i$, $h_{k+1} \in IU(h_k)$ for all $1 \leq k \leq m - 1$, and $h_m = j\}$. The agents in $U(i)$ are called *upstream agents* of i in E.

$D(i) := \{j \in N'$: $i \in U(j)\}$. The agents in $D(i)$ are called *downstream agents* of i in E.

Given $i, j \in N'$, we define the set $d(i, j) := \{(k, l) \in E$ such that (k, l) is in the unique path from i to $j\}$. The *geodesic distance* from i to j is the cardinality of $d(i, j)$, *i.e.*, $|d(i, j)|$.

3. Main result

Dong, Ni and Wang (2007) characterize the UES rule with five axioms: Additivity, Efficiency, Independence of Upstream Costs, Independence of Irrelevant Costs and Upstream Symmetry. The latter states that all the upstream agents are equally liable for a given downstream pollution cost. However, there are situations where this particular axiom is not applicable. Gómez-Rúa (2011) provides further discussion on this fact.

In this section we characterize the set of rules satisfying three properties: Efficiency, Additivity and a new one that capture the ideas of both *Independence of Upstream Costs* and *Independence of Irrelevant Costs*. We call this property Independence of No Responsibility Costs.

Before presenting the main result, we formally introduce the axioms.

Efficiency (Eff) $\sum_{i\in N'} x_i = \sum_{i\in N'} c_i$.

Efficiency requires that the cost shares of the agents add up to the total cost.

Additivity (Add) For any $C^1 = (c_1^1, ..., c_n^1, c_L^1) \in \mathbb{R}_+^{n+1}$, $C^2 = (c_1^2, ..., c_n^2, c_L^2) \in \mathbb{R}_+^{n+1}$ and $i \in N'$, $x_i(C^1 + C^2) = x_i(C^1) + x_i(C^2)$, where $C^1 + C^2 = (c_i^1 + c_i^2)_{i\in N'}$.

Additivity states that dividing the total cost among agents is the same as dividing one part of the cost first and then dividing the remaining cost.

Independence of No Responsability Costs (INRC) Let $i \in N'$ and $C, C' \in \mathbb{R}_+^{n+1}$ such that $c_j = c'_j$ for all $j \in D(i) \cup \{i\}$. Then, $x_i(C) = x_i(C')$.

This property ensures that an agent's cost share, only depends on her own pollution as well as all downstream costs, but not on those costs associated with some other segment. The pollution caused by agent i cannot reach these segments, so agent i should not bear any cleanup cost of cleaning these segments.

Now we present the family of rules satisfying *Add*, *Eff* and *INRC*. These rules divide the cost of each segment j (c_j) among the agents responsible for it ($i \in U(j) \cup \{j\}$) proportionally to a weight vector $p^j \in \mathbb{R}_+^{n+1}$. Namely,

Theorem 1 *A rule x satisfies Eff, Add and INRC if and only if for each $j = 1, ..., n, L$ there exists a weight system $\left(p_i^j\right)_{i\in N'} \in \mathbb{R}_+^{n+1}$ such that $p_i^j = 0$ when $i \in N' \backslash (U(j) \cup \{j\})$, $\sum_{i\in N'} p_i^j = 1$ and*

$$x_i(C) = \sum_{j\in N'} p_i^j c_j$$

for all $C \in \mathbb{R}_+^{n+1}$ and all $i \in N'$.

Proof. *Let x be a rule defined as above. It is straightforward to prove that x satisfies Eff, Add and INRC.*

We now prove the reciprocal. Assume that x is a solution satisfying Eff, Add and INRC. For each $j \in N'$, let $1_j = (y_1, ..., y_n, y_L) \in \mathbb{R}_+^{n+1}$ be such that $y_j = 1$ and $y_i = 0$ when $i \neq j$. We define $p^j = x(1_j)$.

Let x^p be the rule induced by the weight system $\{p^j\}_{j\in N'}$. We will prove that $x = x^p$ by several claims. The claims are proved following Bergantiños and Vidal-Puga (2004).

Claim 1 *$\{p^j\}_{j\in N'}$ is a weight system.*

Proof of Claim 1. *Since x satisfies Eff, $\sum_{i\in N'} x_i(1_j) = 1$. By definition of solution, $x_i(1_j) \in \mathbb{R}_+^{n+1}$. Let $i, j \in N'$ such that $i \in N' \backslash (U(j) \cup \{j\})$. Since x satisfies INRC, $x_i(1_j) = x_i(0, ..., 0)$. Since $x(0, ..., 0) \in \mathbb{R}_+^{n+1}$ and $\sum_{l\in N'} x_l(0, ..., 0) = 0$, $x_i(0, ..., 0) = 0$.* ■

Claim 2 *Let $c_j \in \mathbb{Q}_+$ (a non-negative rational number), then*

$$x_i(0, ..., c_j, ..., 0) = c_j x_i(0, ..., 1, ..., 0).$$

Proof of Claim 2. *Let $c_j = 1/q$, where $q \in \mathbb{N}$. By Add,*
$x_i(0, ..., 1, ..., 0) = \sum_{k=1}^{q} x_i(0, ..., \frac{1}{q}, ..., 0) = q x_i(0, ..., \frac{1}{q}, ..., 0)$. Thus,

$$x_i\left(0, ..., \frac{1}{q}, ..., 0\right) = \frac{x_i(0, ..., 1, ..., 0)}{q} = c_j x_i(0, ..., 1, ..., 0). \tag{1}$$

Let $c_j \in \mathbb{Q}_+$*, say* $c_j = \frac{p}{q}$*. By Add,* $x_i\left(0,...,\frac{p}{q},...,0\right) = px_i\left(0,...,\frac{1}{q},...,0\right)$.

Then by (1), $x_i\left(0,...,\frac{p}{q},...,0\right) = \frac{p}{q}x_i(0,...,1,...,0)$.■

Claim 3 *Let* $c_j \in \mathbb{R}_+\backslash\mathbb{Q}_+$ *(a non-negative irrational number), then* $x_i(0,...,c_j,...,0) = c_jx_i(0,...,1,...,0)$.

Proof of Claim 3. *Let* $c_j \in \mathbb{R}_+\backslash\mathbb{Q}_+$*. Then, there exists* $\{b_l\}_{l=1}^{\infty}$ *such that* $b_l \in \mathbb{Q}_+$*,* $b_l < c_j$ *and* $\lim_{l\to\infty} b_l = c_j$.

Let $l \in \mathbb{N}$*. Since* $x(0,...,c_j - b_l,...,0) \in \mathbb{R}_+^{n+1}$ *and* $\sum_{i\in N'} x_i(0,...,c_j - b_l,...,0) = c_j - b_l$*,* $0 \leq x_i(0,...,c_j - b_l,...,0) \leq c_j - b_l$.

By Add, $x_i(0,...,c_j,...,0) = x_i(0,...,c_j-b_l,...,0)+x_i(0,...,b_l,...,0)$*. So,* $0 \leq x_i(0,...,c_j,...,0) - x_i(0,...,b_l,...,0) \leq c_j - b_l$.

Since $b_l \in \mathbb{Q}_+$*,* $x_i(0,...,b_l,...,0) = b_lx_i(0,...,1,...,0)$*. Then,*

$$0 \leq x_i(0,...,c_j,...,0) - b_lx_i(0,...,1,...,0) \leq c_j - b_l.$$

Thus, $0 \leq \lim_{l\to\infty}\left[x_i(0,...,c_j,...,0) - b_lx_i(0,...,1,...,0)\right] \leq \lim_{l\to\infty}\left[c_j - b_l\right]$.

So, $0 \leq x_i(0,...,c_j,...,0) - c_jx_i(0,...,1,...,0) \leq 0$.

Therefore, $x_i(0,...,c_j,...,0) = c_jx_i(0,...,1,...,0)$.■

Claim 4 *Given* $i \in N'$ *and* $C \in \mathbb{R}_+^{n+1}$*,* $x_i(c_1,...,c_n,c_L) = \sum_{j\in N'} x_i(0,...,0,c_j,0,...,0)$.

Proof of Claim 4. *It follows from the fact that* x *satisfies Add.*■

Since $x_i^p(c_1,...,c_n,c_L) = \sum_{j\in N'} p_i^j c_j$*, and by Claims 2 and 3,*

$x_i(0,...,c_j,...,0) = c_jx_i(0,...,1,...,0) = c_jp_i^j$ *for all* $j \in N'$ *and all* $c_j \in \mathbb{R}_+$*, it is clear that* $x = x^p$*.* ■

4. Other results

In this section, we provide characterizations of new rules, adding different properties based on possible and real-life taxes over pollution in Theorem 1. These properties are generalizations of the ones introduced in Gómez-Rúa (2011).

In many cases all the agents release the same kind of residues into the water and the residues are biodegradable, thus the pollution disappears over time; for instance: organic food waste, garden waste, forest residues, some industrial waste... Sometimes, it is possible to know the biodegradation rate of the residues, say δ. If it happens, the cost that an agent pays for a polluted area should depend on this biodegradation rate. We introduce a new property following this idea:

$\delta-$**Biodegradation Rate (**$\delta-$**BR)** Let $\delta \in [0,1]$. Given $j \in N'$, for any $i,k \in U(j)\cup\{j\}$ such that $|d(i,j)| \geq |d(k,j)|$, $x_i(0,...,0,c_j,0,...0) = \delta^{|d(i,j)|-|d(k,j)|}x_k(0,...,0,c_j,0,...0)$.

Notice that $\delta = 0$ means that the residue of agent i only affects its own area. In this case $\delta-$BR means that every agent pays the cost corresponding to its own area, namely $x_i(C) = c_i$ for all C and $i \in N'$. $\delta = 1$ means that the residue is non-biodegradable. In this case $\delta-$BR coincides with Upstream Symmetry (Dong et al., (2007)).

In the next theorem we study the effects of adding $\delta-$BR to the properties in Theorem 1.

Theorem 2 . *A rule x satisfies Add, Eff, INRC and BR if and only if for each $j = 1, ..., n, L$ there exists a weight system $\left(p_i^j\right)_{i \in N'} \in \mathbb{R}_+^{n+1}$ such that $p_i^j = 0$ when $i \in N' \backslash (U(j) \cup \{j\})$, $p_i^j = \delta^{|d(i,j)|-|d(k,j)|} p_k^j$ for any $i \in U(j)$, $k \in U(j) \cup \{j\}$ such that $|d(i,j)| \geq |d(k,j)|$, $\sum_{i \in N'} p_i^j = 1$ and $x_i(C) = \sum_{j \in N'} p_i^j c_j$ for all $C \in \mathbb{R}_+^{n+1}$ and all $i \in N'$.*

Proof. *It is straightforward to prove that x satisfies $\delta-$BR. We now prove the reciprocal. Let x be a rule satisfying Add, Eff, INRC and $\delta-$BR. By Theorem 1 for each $j = 1, ..., n, L$, there exists a weight system $\left(p_i^j\right)_{i \in N'} \in \mathbb{R}_+^{n+1}$ such that $p_i^j = 0$ when $i \in N' \backslash (U(j) \cup \{j\})$, $\sum\limits_{i \in N'} p_i^j = 1$ and $x_i(C) = \sum\limits_{j \in N'} p_i^j c_j$ for all $C \in \mathbb{R}_+^{n+1}$ and all $i \in N'$. We now prove that $p_i^j = \delta^{|d(i,j)|-|d(k,j)|} p_k^j$ for any $i \in U(j)$, $k \in U(j) \cup \{j\}$ such that $|d(i,j)| \geq |d(k,j)|$.*

Let $i, j, k \in N'$ such that $i \in U(j)$, $k \in (U(j) \cup \{j\})$ and $|d(i,j)| \geq |d(k,j)|$. By the proof of Theorem 1, $p^j = x(1_j)$. Since x satisfies $\delta-$BR,

$$\begin{aligned} p_i^j &= x_i(1_j) = \delta^{|d(i,j)|} x_j(1_j) = \delta^{|d(k,j)|} \delta^{|d(i,j)|-|d(k,j)|} x_j(1_j) \\ &= \delta^{|d(i,j)|-|d(k,j)|} x_k(1_j) = \delta^{|d(i,j)|-|d(k,j)|} p_k^j. \end{aligned}$$

■

In many countries, like Spain, Austria, Canada, Finland, France, Germany, Greece, Hungary, Italy, Korea, Sweden, USA, among others (OECD, 2006), there exists a difference between the rates applicable to domestic uses and those applicable to industrial ones. The taxes can be modulated considering different factors, such as pollution load, population of the cities, monthly water consumption, etc. (See Gago et al., 2006). In Gómez-Rúa (2011) we introduce a property that captures these ideas. Now, we generalize it for the context of a river network:

Proportional Tax with respect to w (PT-w) Let $w = (w_i)_{i \in N'} \in \mathbb{R}_+^{n+1}$. We say that x satisfies PT with respect to w if for any $i, j, k \in N'$ such that $i \in U(j)$, $k \in U(j) \cup \{j\}$, $\frac{x_i(0,...,0,c_j,0,...,0)}{x_k(0,...,0,c_j,0,...,0)} = \frac{w_i}{w_k}$.

This property states that the amount that each agent pays for a polluted area is given by some exogenous factor.

PT-w generalizes *Upstream Symmetry* because when $w_i = w_j$ for all $i, j \in N'$, both properties coincide.

In the next theorem we study the effects of adding *PT-w* to the properties in Theorem 1.

Theorem 3 . *A rule x satisfies Add, Eff, INRC and PT-w if and only if for each $j = 1, ..., n, L$ there exists a weight system $\left(p_i^j\right)_{i \in N'} \in \mathbb{R}_+^{n+1}$ such that $p_i^j = 0$ when $i \in N' \backslash (U(j) \cup \{j\})$, $p_i^j = \frac{w_i}{\sum_{l \in U(j) \cup \{j\}} w_l}$ for all $i \in U(j) \cup \{j\}$ and $x_i(C) = \sum_{j \in N'} p_i^j c_j$ for all $C \in \mathbb{R}_+^{n+1}$ and all $i \in N'$.*

Proof. *It is straightforward to prove that x satisfies* PT-w. *We now prove the reciprocal. Let x be a rule satisfying Add, Eff, INRC and PT-w. By Theorem 1 for each $j = 1, ..., n, L$ there exists a weight system $\left(p_i^j\right)_{i \in N'} \in \mathbb{R}_+^{n+1}$ such that $p_i^j = 0$ when $i \in N' \backslash (U(j) \cup \{j\})$,*

$\sum\limits_{i \in N'} p_i^j = 1$ *and* $x_i(C) = \sum\limits_{j \in N'} p_i^j c_j$ *for all* $C \in \mathbb{R}_+^{n+1}$ *and all* $i \in N'$. *We now prove that* $p_i^j = \frac{w_i}{\sum_{l \in U(j) \cup \{j\}} w_l}$ *for any* $i \in U(j) \cup \{j\}$.

Let $i, j \in N'$ *such that* $i \in U(j) \cup \{j\}$. *By the proof of Theorem 1,* $p^j = x(1_j)$. *Since* x *satisfies Eff and PT-w,* $\frac{1}{p_j^j} = \frac{\sum\limits_{l \in U(j) \cup \{j\}} p_l^j}{p_j^j} = \sum\limits_{l \in U(j) \cup \{j\}} \frac{x_l(1_j)}{x_j(1_j)} = \sum\limits_{l \in U(j) \cup \{j\}} \frac{w_l}{w_j} = \frac{\sum\limits_{l \in U(j) \cup \{j\}} w_l}{w_j}$.

By PT-w, for each $i \in U(j)$ $p_i^j = x_i(1_j) = \frac{w_i}{w_j} x_j(1_j) = \frac{w_i}{w_j} p_j^j = \frac{w_i}{\sum\limits_{l \in U(j) \cup \{j\}} w_l}$. ■

In Gómez-Rúa (2011), a TU game is introduced and it is proved that one of the rules proposed coincide with the weighted Shapley value of that game. Now, we generalize this result to the network context.

We then relate the solutions given by Theorem 3 with the weighted Shapley values of a TU game.

Given a problem (N', E, C) we define the TU game $\left(N', v^{E,C}\right)$ where

$$v^{E,C}(S) = \sum_{i \in S : U(i) \cup \{i\} \subset S} c_i$$

for all $S \subset N'$. Namely $v^{E,C}(S)$ represents the pollutant-cleaning costs of segments for which only agents in S are responsible. This definition implies that, if a segment i is polluted by agents that are in S but also by agents that do not belong to S, then the segment i is not taken into account in order to compute $v^{E,C}(S)$.

Theorem 4 . *Let* x^w *the solution given by Theorem 3. Then,* x^w *coincides with the weighted Shapley value of* $v^{E,C}$ *with weights given by* $w \in \mathbb{R}_{++}^N$, $\phi^w\left(N', v^{E,C}\right)$.

Proof. *Let* $w = (w_i)_{i \in N'} \in \mathbb{R}_+^{N'}$. *Let* $\{u_S\}_{S \subset N'}$ *be a family of* TU *games such that* $u_S(T) = 1$ *if* $S \cap T \neq \varnothing$ *and* $u_S(T) = 0$ *otherwise. It is well known that* $\{u_S\}_{S \subset N'}$ *is a basis for the set of all* TU *games. Kalai and Samet (1987) define the value* ϕ^{w*} *as the unique linear value satisfying that for each* $S \subset N'$, $\phi_i^{w*}(u_S) = \frac{w_i}{\sum_{k \in S} w_k}$ *if* $i \in S$ *and* $\phi_i^{w*}(u_S) = 0$ *otherwise. Moreover, they prove that for each* $w \in \mathbb{R}_+^{N'}$ *and each* TU *game* v, $\phi^{w*}(v) = \phi^w(v^*)$ *where* $v^*(S) = v(N') - v(N' \backslash S)$ *for all* $S \subset N'$.

Given (N', E, C), *for each* $j \in N'$, *let* (N', v^j) *be the* TU *game where for all* $S \subset N'$, $v^j(S) = c_j$ *if* $S \cap (U(j) \cup \{j\}) \neq \varnothing$ *and* $v^j(S) = 0$ *otherwise. Notice that* $v^j = c_j u_{\{U(j) \cup \{j\}\}}$ *for all* $j \in N'$.

Given $i \in N'$,

$$\begin{aligned} x_i^w(C) &= \sum\nolimits_{j \in N'} p_i^j c_j = \sum\nolimits_{j \in D(i) \cup \{i\}} \frac{w_i}{\sum_{k \in U(j) \cup \{j\}} w_k} c_j \\ &= \sum\nolimits_{j \in D(i) \cup \{i\}} \phi_i^{w*}\left(v^j\right) = \sum\nolimits_{j \in N'} \phi_i^{w*}\left(v^j\right) \\ &= \sum\nolimits_{j \in N'} \phi_i^w\left(v^{j*}\right) = \phi_i^w\left(\sum\nolimits_{j \in N'} v^{j*}\right). \end{aligned}$$

Let $S \subset N'$. *Then,* $v^{j*}(S) = v^j(N') - v^j(N' \backslash S) = c_j - v^j(N' \backslash S)$. *Since* $v^j(N' \backslash S) = c_j$ *when* $(N' \backslash S) \cap (U(j) \cup \{j\}) \neq \varnothing$ *and* $v^j(N' \backslash S) = 0$ *when* $(N' \backslash S) \cap (U(j) \cup \{j\}) = \varnothing$,

$$v^{j*}(S) = \begin{cases} c_j & \text{if } \{U(j) \cup \{j\}\} \subset S \\ 0 & \text{otherwise.} \end{cases}.$$

Now it is trivial to prove that for all $S \subset N'$, $v^{C,E}(S) = \sum_{j \in N'} v^{j*}(S)$. *Hence,* $x_i^w(C) = \phi_i^w\left(v^C\right)$. ■

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