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Does Defense Spending Surprise Long-Run Inflation, Economic Growth and Welfare?

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Abstract
This paper sets up an endogenous growth model to examine the effects of an expansion of defense spending share on inflation, economic growth and welfare. It is found that: (i) an increase of defense spending share will lower the inflation rate and stimulate the economic growth. (ii) from the perspective of maximum social welfare, a "large" enough redistribution of the government spending from defense sector to public sector will promote the social welfare. These can be explained why the arms race and disarmament are advocated in recent years.
1. Introduction

Scholarly debate has raised for more than two decades about the relationship between defense spending and economic performance. One of the debates builds upon a contention that defense spending may cause inflation and further inhibit economic growth. The other debates argue that defense spending usually takes away enormous economic resource from other economic activities and then deters the economic growth.

In view of the argument between defense spending and inflation, Deger and Smith (1983), find that the linkage is positive and the defense spending may further deter the economic growth. Starr et al. (1984) analyzes the relationship between defense spending and inflation by using data for the 1956-1979 era from four major Western powers and finds that it is ambiguous in the United States and United Kingdom, but mutual related in France and Germany. Vitaliano (1984) finds defense spending to have no impact upon inflation. Nourzad (1987) reexamines Vitaliano's results by using a different proxy for inflation expectations and proves that the defense spending has a positive impact on inflation. Payne (1990) finds no evidence to suggest that defense spending causes inflation by using Granger causality test. More recently, Fordham (2003) shows that defense spending may lead to a higher inflation rate by investigating data for United States. Base on the findings revealed in these empirical evidences, yet there is no agreement as to the exact nature of the relationship between defense spending and inflation.

In spite that the empirical literature studied on defense spending and inflation have been accumulated more than two decades, however, the theoretical studies are less reminded on this issue to our knowledge. Besides that, Heo (1998) addresses that if a government finances its defense spending by issuing money, the inflation condition will be expected. As a result of this matter, one of the purposes of this paper is to construct an endogenous growth model between defense sector and non-defense sector to explain the empirical findings with regard to the relationship between defense spending and inflation. In addition, most of the existing studies cooperated with endogenous growth are built upon the basis of real aspect but ignoring the role of nominal money played.\(^1\) The defect of this fact is that the observed facts described above are unable to be persuaded. In order to redeem this tarnish, this paper involves money aspect cooperating into an endogenous growth model that embraces the economic features between defense sector and non-defense sector.\(^2\)

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\(^1\) Tzeng et al. (2008) sets up a monetary endogenous growth model and explain the undetermined relationship between the defense spending and inflation. They observe that the increase of defense spending will cause unambiguous effects on the inflation and stimulate the economic growth.

\(^2\) Shieh et al. (2002) build up an endogenous growth model under the government’s resource allocation between defense and non-defense sector, they prove that the defense spending is correlated with
In addition to the relationship between defense spending and inflation, many studies have been devoted to analyzing the relationship among defense spending, economic growth, and social welfare. Benoit (1973,1978), Brumm (1997) and Murdoch et al. (1997) prove there is a positive relation between defense spending and economic growth. Deger and Smith (1983) and Faini et al. (1984) stress a negative linkage between defense spending and economic growth. Huang and Mintz (1990, 1991) point out there is no significant effect of defense spending on economic growth. Shieh (2002a,b) uses an endogenous growth model to demonstrate there exists an optimal defense spending share that maximize the economic growth and social welfare in alternative government resource allocation. Base on the findings revealed in these evidences, yet there is no agreement as to the exact nature of the relationship among defense spending, economic growth and social welfare. Hence, another purpose of this paper is to provide an explanation of how the impact of defense spending share on the three important economic performances: inflation, economic growth and social welfare.

The rest of this paper is organized as follows. Section 2 constructs the analytical framework. Section 3 examines how the steady-state effects of an expansion of defense spending share make effluence on economic performances. The conclusions are given in Section 4.

2. The model

Consider an economy consisting of a government and large number of homogeneous infinite-lived households. Households produce a single composite commodity. They can be consumed, accumulated as capital and be paid as an income tax by households. The government provides defense security and public capital by means of spending on defense capital accumulation and investing in core infrastructure, respectively.

Households’ utility $U$ comes from consumption $C$ and defense capital $S$. As indicated by van der Ploeg and Zeeuw (1990), the level of security enters into the Households’ utility function due to the fact that it provides security to the public and increases the feeling of national security by a higher level of defense capital. As a result, the representative household seeks to maximize the discounted sum of instantaneous utilities as given by:

$$\int_0^\infty U(C, S) e^{-\rho t} dt = \int_0^\infty (\ln C + \eta \ln S) e^{-\rho t} dt, \quad \eta > 0,$$

where $\rho$ is the constant rate of time preference and the parameter $\eta$ measures the balanced growth as the defense spending is covered by a lump-sum tax.

3 Accordingly, Deger and Sen (1983,1984), Zou (1995), Chang et al. (1996), Shieh et al. (2002a, b), and Tzeng et al. (2008) have taken the defense capital as a proxy for national security.
impact of the defense capital on household.\textsuperscript{4}

Based on the fact that the defense sector and non-defense public sector may have a positive impact on private output reflecting as a spin-off effect,\textsuperscript{5} we assume that output $Q$ is produced with constant returns to scale technology that uses the private capital stock $k$, public capital stock $R$, and the defense capital $S$.\textsuperscript{6} That is, the production function is assumed to take a Cobb-Douglas form:

$$Q = Q(k, R, S) = k^{1-\alpha_1} R^{\alpha_1} S^{\alpha_2}, \quad 0 < \alpha_1, \alpha_2 < 1$$

Eq.(2) implies that the both of the public capital stock and defense capital are non-excludable and non-rival.

The law of motion with real money balances is given by:

$$\frac{m}{m} = \mu - \pi$$

where $m(=M/P)$ is the real money balances with nominal money holdings $M$ and price level $P$; $\mu$ is the growth rate of the nominal money stock $(M/M)$ and $\pi$ is the rate of inflation(\(P/P\)).

Let $\theta$ and $1-\theta$ denote the fraction of government spending devoted to defense sector and non-defense sector (core infrastructure), respectively. And the government is assumed to finance its defense spending $(\theta g)$ by issuing money $(\mu m)$ and finance its public spending $(1-\theta)g$ (i.e, investment in core infrastructure) by collecting income tax revenue $(\tau Q)$. Hence the government budget constraint can thus be described as:

$$g = \mu m + \tau Q$$

$$\dot{S} = \theta g = \mu m$$

$$\dot{R} = (1-\theta)g = \tau Q$$

Equ. (4) shows the government’s budget constraint and at each instant of time, the government always balances its budget. Eq.(5) describes the linkage between the total stock of defense capital and the flow of defense spending $\theta g$ financed by issuing money. Eq.(6) describes the linkage between the total stock of public capital and the flow of core infrastructure expenditure $(1-\theta)g$ financed by collecting income tax.

\textsuperscript{4} Specifically, the main results in this study hold if the instantaneous utility function takes the form as $U(C, S) = [(CS)^{\sigma} - 1]/(1-\sigma)$, where $\sigma$ is the inverse of the elasticity of inter-temporal substitution. When $\sigma = 1$, it degenerates to the form stated in Eq. (1)

\textsuperscript{5} The spin-off effect denotes that the defense sector will give a production externality to the private sector such as infrastructure, R&D, training, education, and human capital enhancing activities. One can refer to Sandler and Harley (1995) for more detailed interpretation. In addition, the positive linkage between the public sector and private sector of spin-off effect can also be seen by Barro (1990), and Futagami et al.(1993).

\textsuperscript{6} To achieve the ongoing growth rate, the constant returns to scale technology in the growth variables in this study is necessary. This setting is also widely used in a common assumption in the endogenous growth literatures such as Barro (1990), Rebelo(1991), and Turnovsky (2000b).
revenue. Using Eqs. (5) and (6) with $S_0 / R_0 = \theta / (1 - \theta)$ initially, we have the following relation such as:

$$\frac{S}{R} = \frac{\theta}{1 - \theta} \quad (7)$$

As a result, using Eqs. (2)-(4), the budget constraint of households is given by:

$$\dot{k} + \dot{m} = (1 - \tau) k^{1 - a_1 - a_2} R^{a_1} S^{a_2} - C - \pi m \quad (8)$$

where an overdot denotes the rate of change with respect to time, and $\tau$ is a flat-rate income tax. Households choose $\{C, m, k\}_{t=0}^\infty$ in order to maximize Eq. (1) subject to Eq. (8). By letting $\lambda$ be the co-state variable associated with Eq. (8), and the transversality condition, $\lim \lambda m e^{-\rho t} = \lim \lambda k e^{-\rho t}$ given the initial real money balance $m_0$ and private capital stock $k_0$. The optimum conditions necessary for the households are:

$$H = \ln C + \eta S + \lambda [(1 - \tau) k^{1 - a_1 - a_2} R^{a_1} S^{a_2} - C - \pi m] \quad (9)$$

$$\frac{1}{C} = \lambda \quad (10a)$$

$$- \lambda \pi = -\dot{\lambda} + \lambda \rho \quad (10b)$$

$$\lambda (1 - \tau) (1 - \alpha_1 - \alpha_2) (R / k)^{a_1} (S / k)^{a_2} = -\dot{\lambda} + \lambda \rho \quad (10c)$$

$$\pi = -(1 - \tau) (1 - \alpha_1 - \alpha_2) (R / k)^{a_1} (S / k)^{a_2} \quad (10d)$$

Eq. (10a) shows that the marginal utility of consumption is equal to the sum of the shadow value of wealth. Eq. (10b) and (10c) indicate the optimal choices of real money balances and private capital stock, respectively. Eq. (10d) states the non-arbitrage condition between real money balances and holding private capital stock.

Differentiating Eq. (10a) with respect to time and substituting Eq. (10b) and (10d) into the resulting equation yields:

$$\frac{\dot{C}}{C} = (1 - \tau) (1 - \alpha_1 - \alpha_2) (R / k)^{a_1} (S / k)^{a_2} - \rho \quad (11)$$

Eq. (11) is Keynes-Ramsey rule, which means that if the net marginal capital production is large (less) than time preference, the representative household will increase (decrease) their consumption in the next period of time. And from Eq. (5) and Eq. (6), we have the growth rate of the defense capital:

$$\dot{S} / S = \theta g / S = [\theta / (1 - \theta)] (\tau Q / S) = \tau \theta^{a_1 - a_2 - 1} (1 - \theta)^{a_1 - 1} (S / k)^{a_1 - a_2 - 1} \quad (12)$$

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7 According to Mankiw (1987) and Zou (1995), assume that the defense capital stock and public capital stock are able to instantaneously reversible at any instant of time, we must have that the total government capital $(S + R)$ will change over time as well. Hence, Eq. (7) holds at any instant of time.

8 Notes that $k = Q - C - g$. 
By substituting Eq.(3) and Eqs.(5)-(7) into Eq.(8), we have the economy’s resource constraint given as:

$$\dot{k} = [(1-\theta - \tau)/(1-\theta)]k^{1-a_1-a_2} R^{a_1} S^{a_2} - C$$  \hspace{1cm} (13)

In order to solve the balance growth equilibrium, we define the following transformed variables: $x \equiv C/k$ and $y \equiv S/k$ similar to Futagami (1993), Barro and Sala-I-Martin (1995). Combining Eqs.(5)-(7),(10d), and Eqs.(11)-(13), the dynamic system with respect to transformed variables can be itemized by the following equations:

$$\dot{x} = \frac{\dot{C}}{x} - \frac{\dot{k}}{k} = \left\{\left[(\beta (1-\tau) - 1)(1-\theta) + \tau\right]\right\} \theta^{-1}(1-\theta)^{a_1-1} y^{1-\beta} + x - \rho$$  \hspace{1cm} (14)

$$\dot{y} = \frac{\dot{S}}{y} - \frac{\dot{k}}{k} = \tau \theta^{-a_1}(1-\theta)^{a_1-1} y^{-\beta} - (1-\theta - \tau) \theta^{-1}(1-\theta)^{a_1-1} y^{1-\beta} + x$$  \hspace{1cm} (15)

where $\beta = 1 - a_1 - a_2$. At the steady state, the economy is characterized by $\dot{x} = \dot{y} = 0$, and $\dot{x}$, $\dot{y}$ represent their stationary level, respectively.

3. Long-run effects and the share of defense spending

In this section, we investigate how the long run effects of inflation rate and balanced growth rate will react following a rise in the share of defense spending in the steady growth equilibrium. We denote these results as proposition 1-3, separately as the following.

**Proposition 1**

An increase in the share of defense spending ($\theta$) will lead to a lower inflation rate ($\hat{\pi}$).\(^9\)

**Proof**

From Eq.(10d), in the steady state, we have $\hat{\pi}(\theta) = -\beta (1-\tau) \theta^{-a_1}(1-\theta)^{a_1} \dot{y}^{1-\beta}(\theta)$, and by using Eq. (14) and Eq.(15) with $\dot{x} = \dot{y} = 0$, we obtain:

$$\frac{\partial \hat{y}}{\partial \theta} = \frac{\dot{y} \left\{(1-a_1) \theta \tau + \alpha_1 [\beta \dot{y}(1-\theta) (1-\tau)]\right\}}{\beta \theta (1-\theta) ((1-\beta) (1-\theta) (1-\tau) \dot{y} + \theta \tau)} > 0$$  \hspace{1cm} (16)

Differentiating Eq.(10d) with respect to $\theta$ and substituting Eq.(16) into the resulting equation yields:

$$\frac{\partial \hat{\pi}}{\partial \theta} = -\frac{\alpha_1 \beta \tau (1-\tau) \theta^{a_1-1} (1-\theta)^{a_1-1} \dot{y}^{1-\beta}}{\theta \tau + (1-\beta) (1-\theta) (1-\tau) \dot{y}} < 0$$  \hspace{1cm} (17)

The results of Eq.(17) infers that once the share of defense spending increases, there must be a lower inflation rate. The key factor for this result can be tracked by Eq.(3) due to the fact that the defense spending is financed by issuing money. To sustain the real money balance in steady state growth, the more money issued to finance the defense spending which implies a lower inflation is going to be happened.

\(^9\) The detail mathematical proof is available from the authors upon request.
The following propositions we focus are on the economic growth rate and social welfare following by an increase in the share of defense spending. Given \( \dot{x} = \dot{y} = 0 \) implies that \( C, m, k, R, S, \) and \( Q \) all grow at same rate. Let \( \dot{y} \) be the steady-state economic growth rate that:

\[
\frac{\dot{C}}{C} = \frac{\dot{m}}{m} = \frac{\dot{k}}{k} = \frac{\dot{R}}{R} = \frac{\dot{S}}{S} = \dot{y}
\]

(18)

holds in the steady-state growth equilibrium.

**Proposition 2**

An increase in the share of defense spending \( (\theta) \) stimulates the balanced economic growth rate \( \dot{y} \).

**Proof**

From Eq.(10a) and (10b), in the steady state, we have \( \dot{C}/C = -[\dot{\pi}(\theta) + \rho] \), and by using Eq.(17) and (18) we obtain:

\[
\frac{\partial \dot{y}}{\partial \theta} = \frac{\partial \left( \frac{\dot{C}}{C} \right)}{\partial \theta} = -\frac{\partial \dot{\pi}}{\partial \theta} = \frac{\alpha_{2} \tau (1-\tau) \theta^{-\alpha_{1}} (1-\theta)^{\alpha_{1}-1} \dot{y}^{1-\beta}}{\theta \tau + (1-\beta)(1-\theta)(1-\tau)\dot{y}} > 0
\]

(19)

Eq.(19) shows that the increase in the share of defense spending will stimulate the balanced economic growth rate.\(^{10}\) This result also implies that the increase in the share of public spending will deteriorate the economic growth rate. On the other hand, if the government pursues a higher economic growth rate, spending on public spending will be in vain.

**Proposition 3**

An enough large share of defense spending \( (\theta) \) deteriorates the social welfare \( (W) \).

**Proof**

To explore the effect of the share of the defense spending on the social welfare, we follow the procedure proposed by Greiner and Hanusch (1998) and deal with the balanced growth path with a given initial private capital \( k_{0} \) along the stationary growth equilibrium. Furthermore, both private consumption and defense capital stock growth at the same rate \( \dot{y} \) which are affected by \( \theta \) as well, hence we describe the time path of private consumption and the defense capital stock as:

\[
C_{t} = C_{0}e^{\dot{y}t}
\]

(20a)

\[
S_{t} = S_{0}e^{\dot{y}t}
\]

(20b)

\(^{10}\) Benoit (1973) finds a positive relation between defense spending and economic growth by using data for 44 less-developed countries during 1950-1965 period. This is the famous Benoit Hypothesis in defense economic field. For a more detail review of the Benoit Hypothesis, please see Sandler(1995) and Ram (1995).
where \( C_0 \) and \( S_0 \) are endogenously determined by the economy’s structure and straightforward to be solved by rearranging Eqs (7), (11)-(13), and (18):\(^\text{11}\)
\[
C_0 = \frac{[1 - \theta - \tau(\hat{\gamma}(\theta) + \rho)]}{\beta(1 - \tau)} - \hat{\gamma}(\theta) \]
\[
S_0 = \frac{[\theta^{\alpha_1} (\hat{\gamma}(\theta) + \rho)]}{\beta(1 - \tau)} - \hat{\gamma}(\theta) \]
\(^{11}\)

That is, a change in \( \theta \) will affect the \( C_0 \) and \( S_0 \). Substituting Eqs. (21a) and (21b) into Eq. (1) and integrating the household’s welfare over an infinite planning horizon, the social welfare function \( W(\theta) \) can be written as:
\[
W(\theta) = \int^\infty_0 [\ln C_0(\theta) + \eta \ln S_0(\theta) + (1 + \eta) \hat{\gamma}(\theta) \tau] e^{-\rho t} \]
\(^{22}\)

Differentiation equation (22) with respect to \( \theta \) yield:
\[
\frac{\partial W}{\partial \theta} = \frac{1}{\rho} \left[ \frac{1}{C_0} \frac{\partial C_0}{\partial \theta} + \frac{\eta}{S_0} \frac{\partial S_0}{\partial \theta} + 1 + \eta \hat{\gamma} \right] \]
\[
\frac{1}{\rho} \frac{\partial C_0}{\partial \theta} = \frac{1}{\rho} \left[ -\tau(\rho + \hat{\gamma}) + \Omega(\hat{\gamma}/\theta) \right] < 0 \]
\[
\frac{1}{\rho} \frac{\partial S_0}{\partial \theta} = \frac{1}{\rho} \left[ \frac{\eta}{(\rho + \hat{\gamma})(1 - \beta)} \frac{\partial \hat{\gamma}}{\partial \theta} - \frac{\alpha_1 \eta}{\theta(1 - \theta)(1 - \beta)} \right] < 0 \]
\[
\frac{1 + \eta \hat{\gamma}}{\rho^2} < 0 \]

where \( \Omega = (1 - \theta)(1 - \tau - \theta - \beta(1 - \tau)(1 - \tau)) \).\(^\text{12}\)

Eq. (23) reveals that the effect of a rise in \( \theta \) on social welfare includes three distinct terms of components. We can see that the second and third terms are negative effect to the social welfare as \( \theta \) increases. Moreover, the first term is undetermined but close to negative infinite if \( \theta \) approximates to unity. That is, we have the negative effect on social welfare as long as we have approximately large enough of the share of defense spending. This result also implies that to attain the maximization of social welfare, a large enough redistribution of the government spending from defense sector to public sector will promote the social welfare.

4. Conclusion

The efficiency of a government spending allocation on economic performance has been a long controversial issue. The general conventional viewpoint outlines that

\(^\text{11}\) Notes that \( y = s/k \), and then from Eq.(13)and (18), we have:
\[
k/k = \hat{\gamma}(\theta) = [(1 - \theta - \tau(\theta^{a_1} + (1 - \theta) a_2)] \hat{\gamma}^{a_1 + a_2} - C_0/k_0 = \hat{C}/C = [\beta(1 - \tau)(1 - \theta)^{a_0} - \rho]
\]

\(^\text{12}\) Notes that when \( \theta \to 1 \), \( \Omega \) will approach to zero.
allocating the government spending from defense spending to public spending (especially the core infrastructures) will stimulate the economic growth. In this paper, we construct an endogenous growth model to analyze the government spending allocation in terms of the inflation, economic growth and social welfare following a rise of defense spending share. Given that the defense spending is financed by issuing money, and the public spending is financed by collecting income tax revenue, it is found that an increase in the share of defense spending will lower the inflation rate and raise the economic growth. Specifically, if the government’s objective is to achieve the maximum of economic growth and minimum of inflation rate, it should not devote too much spending allocated on public spending. This result is also consistent with the famously Benoit Hypothesis. We also find that from the perspective of maximum social welfare, a “large” enough redistribution of the government spending from defense sector to public sector will promote the social welfare. In this paper, our findings may also be an explanation of why in view of economic performances, those arms race and disarmament issues are advocated in recent years.
References


