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Comparing the small sample properties of two break Lagrange Multiplier unit root tests

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Abstract

In this note, we examine the size and power properties and the break date estimation accuracy of the Lee and Strazicich (LS, 2003) two break endogenous unit root test, based on two different break date selection methods: minimising the test statistic and minimising the sum of squared residuals (SSR). Our results show that the performance of both Models A and C of the LS test are superior when one uses the minimising SSR procedure.

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1. Introduction

One area of research that has become famous in applied econometrics is the unit root null hypothesis. The main reason for this is because the unit root null hypothesis is used to examine both economic theories, such as purchasing power parity, and financial theories, such as the efficient market hypothesis. One very appealing test in this regard has been the two endogenous structural break test of Lee and Strazicich (LS, 2003). With the structural break unit root test, an issue at the forefront is the method of estimating the structural breaks. The different approaches to estimating the structural break dates have implications for the size and power properties of the respective unit root tests, which in turn dictates the appeal of the test.

The LS test uses a grid search procedure to estimate the break date. Our goal in this note is to examine whether using a different approach, namely the minimal sum of squared residuals procedure, improves the size and power properties of the LS test.

The balance of this note is organised as follows. In the next section, we describe the LS test and discuss the two approaches to estimating break dates. In section 3, we discuss our simulation design. In section 4, we present our findings, and in the final section we provide some concluding remarks.

2. The LS unit root test

The LS test, based on the Lagrange Multiplier (LM) principle, is a generalisation of the Schmidt and Phillips (1992) and Lee and Strazicich (2004) tests and allows for breaks under the null and alternative hypotheses for trending data. Their test allows for two endogenous breaks in the level and trend, and begins with the following data generating process:

$$y_t = \delta' Z_t + e_t, \quad e_t = \beta e_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iidN(0, \sigma^2) \tag{1}$$

where Z_t is a vector of exogenous variables:

$$\text{Model A:} \qquad Z_t = [1, t, D_{1t}, D_{2t}]\,,\ \delta = [\mu, \gamma, d_1, d_2] \tag{2}$$

$$\text{Model C:} \qquad Z_t = [1, t, D_{1t}, D_{2t,} DT_{1t}, DT_{2t}]\,,\ \delta = [\mu, \gamma, d_1, d_2, d_3, d_4] \tag{3}$$

with

$$D_{jt} = \begin{cases} 1 & t \geq T_{Bj}+1 \\ 0 & \text{otherwise} \end{cases}, j = 1, 2, \tag{4}$$

$$DT_{jt} = \begin{cases} t - T_{Bj} & t \geq T_{Bj}+1 \\ 0 & \text{otherwise} \end{cases}, j = 1, 2 \tag{5}$$

where T_{Bj} denotes the break date.

The null and alternative hypotheses are:

$$H_0 : \beta - 1 = \phi = 0 \quad H_1 : \beta - 1 = \phi < 0 \tag{6}$$

The test procedure is as follows:

$$\Delta y_t = \delta' \Delta Z_t + \phi \tilde{S}_{t-1} + u_t, \tag{7}$$

where $\tilde{S}_t = y_t - \tilde{\psi}_x - Z_t \tilde{\delta}$, $t = 2, \ldots, T$; $\tilde{\delta}$ are coefficients in the regression of Δy_t on ΔZ_t; $\tilde{\psi}_x$ is given by $y_1 - Z_1 \tilde{\delta}$ where y_1 and Z_1 denote the first observation of y_t and Z_t.

The t-statistic $\tilde{\tau}$ is used to test the null hypothesis of $\phi = 0$. In order to estimate the unknown break dates, LS use a grid search procedure. The break dates are associated with these dates for which the value of the test statistic is minimal:

$$LM_\tau = \inf_\lambda \tilde{\tau}(\lambda), \quad \lambda = (\lambda_1 = T_{B1}/T, \lambda_2 = T_{B2}/T). \tag{8}$$

We compare the performance of the LM test as described in LS (2003) with the LM test using an alternative method to estimate the break dates for which the break dates are chosen according to the minimal sum of squared residuals (SSR):

$$LM_\tau^* = \tilde{\tau}(\hat{\lambda}) \quad \text{with } \hat{\lambda} = \arg\min_\lambda SSR(\lambda). \tag{9}$$

3. Simulation design

The simulations are based on 5000 replications in samples of $T = 100$ observations. We generate 150 observation and discard the first 50 to avoid any effect from initial conditions. The data is simulated according to (1), (2) and (3), further assuming $\varepsilon_t \sim iidN(0,1)$. The simulations are conducted using GAUSS 8.0. To assess the size and power properties, we assume $\beta = 1$ and $\beta = 0.9$, respectively. We assume that the magnitude of the first break and second break is equal, i.e. for level breaks $d_1 = d_2$ and for slope breaks $d_3 = d_4$. For Model A, the level break size varies over 0, 2, 3, 5, 10, and 20. For Model C, we judge the performance for all combinations of the values $\{0, 2, 5, 10\}$ for either level and slope breaks. The breaks are assumed to occur at $T_B = (T_{B1} = 40, T_{B,2} = 60)$. The trimming factor is 0.2, i.e. we exclusively search for breaks in the interval $[0.2T, 0.8T]$.

For test decision, the critical values derived under the assumption of no break, that is, a break size of zero is used. We also judge the properties of the LS test by using the critical values of the test when the break dates are exogenously given, namely the critical values of $\tilde{\tau}(\lambda^c)$ where λ^c is the correct break fraction. These tests are denoted $\widetilde{LM}_\tau$ and $\widetilde{LM}^*_\tau$, respectively.

We use the set of critical values for known break dates because of the following reason. If with increasing break size the probability of detecting the true break date goes to 1, $\lim_{\text{break size}\to\infty} P(\hat{T}_B = T'_B) = 1$,[1] the critical values (and distribution) for the endogenous break test converges to that for the exogenous break test. And if additionally the test is invariant to the break size, as is shown by Lee and Strazicich (2003) for the LS test, it implies the equivalence of the critical values of the endogenous break test assuming no break and the exogenous break test. When the difference between these two sets of critical values is large and the break date estimation accuracy increases with the break size, this leads to tests with unstable size.

4. Results

The critical values for $\tilde{\tau}(\lambda^c)$, LM_τ and LM^*_τ are tabulated in Table I. It can be seen both for Model A and C that the critical values for the endogenous break test are absolutely higher than the critical values of the exogenous break test, but that the difference is smaller for LM^*_τ. In Table 2, we report the size and power properties and the break date estimation accuracy of these tests for Model A. Because of the difference between the critical values of LM_τ,

[1] A situation in which one always identifies the break date correctly, i.e. $P(\hat{T}_B = T'_B) = 1$, is like knowing the break date.

LM_τ^* and $\tilde{\tau}(\lambda^c)$ and the fact that $P(\hat{T}_B = T'_B)$ increases with the level break magnitude, the endogenous break tests LM_τ and LM_τ^* become conservative. The LM_τ^* test is already conservative for medium sized breaks because of its high accuracy in selecting the break date. When using the $\arg\min_\lambda SSR(\lambda)$ procedure, the probability of detecting the break dates simultaneously is about 40 per cent for medium sized breaks and 100 per cent for large breaks.

For the LM_τ test, one has the impression of an invariant (to the break size) test. But, for large breaks LM_τ is also conservative. This result is mirrored in the test properties when using the critical values of $\tilde{\tau}(\lambda^c)$ for test decision. So, we observe large oversizing for $\widetilde{LM}_\tau$ in the case of small and medium sized breaks and an empirical size close to the nominal one for large breaks. The $\widetilde{LM}_\tau^*$ test rejects the null hypothesis in approximately 11 per cent of the cases for small breaks. For medium and large sized breaks the size of the test is around the nominal 5 per cent.

For Model A, the LM_τ^* and $\widetilde{LM}_\tau^*$ tests are characterised by break date estimation accuracy, while the LM_τ test has slightly more stable rejection frequency for empirically relevant break sizes. The power of the three tests do not differ considerably.

The size, power, and break date estimation accuracy results for Model C are reported in Table 3. Using the selection method of minimising LM_τ one is not able to identify the break date accurately. So, the prerequisite of the convergence of the critical values of the endogenous break test to that of the exogenous break test with increasing break magnitude is not fulfilled and, as a result, the use of $\widetilde{LM}_\tau$ is not recommended. But the test size of LM_τ varies a lot with the break size, especially in the case of slope breaks.

In contrast, the break date selection accuracy is much superior when one uses the selection method based on the minimising SSR procedure. For example, the estimation accuracy ranges from 70 per cent in the case of medium sized breaks to 100 per cent in the case of large sized breaks. The LM_τ^* test which is based on this selection approach is very conservative. But using the critical values of $\tilde{\tau}(\lambda^c)$ the empirical size is very close to the nominal size. The power of the LM_τ test is mostly higher than for the $\widetilde{LM}_\tau^*$ test. This is only due to oversizing, which can be seen in those cases of level and slope breaks where we observe rejection frequencies of around 5 per cent under the null hypothesis.

For Model C, the $\widetilde{LM}_\tau^*$ test is superior to the LM_τ test. Taken on the whole, when using the LS unit root test, the practitioner seems to be better off by using the $\arg\min_\lambda SSR(\lambda)$ break date selection criteria than the $\arg\min_\lambda LM_\tau(\lambda)$ criteria.

We do not emphasise on Model B, which is referred to as a mixed model, because it is hardly used in applied work. The most commonly used models are A and C. However, at the suggestion of a referee of this journal, we also consider the performance of the mixed model. Like with the results from models A and C, we find that the LS test of Model C has better statistical properties when the break date selection is based on the SSR procedure. Detailed results are available upon request.

5. Concluding remarks

The Lee and Strazicich (2003) endogenous two break unit root test is widely used in applied economics for theoretical evaluations. Given this popularity, and the implications of the break date selection criterions on the performance of the test itself, our goal in this note was to compare the size, power, and break date estimation accuracy properties of the LS test based on two break date selection procedures, namely minimising LM_τ and SSR. We find that generally Models A and C of LS have better statistical properties when the break date selection is based on the SSR procedure. Moreover, our findings suggest that using the $\arg\min_\lambda SSR(\lambda)$ procedure, one is able to estimate break dates very accurately - 100 per cent estimation accuracy for large breaks when using SSR compared with about 40 per cent accuracy under the $\arg\min_\lambda LM_\tau(\lambda)$ procedure.

References

Lee, J and M. Strazicich (2003) "Minimum Lagrange Multiplier Unit Root Test With Two Structural Breaks" *The Review of Economics and Statistics* **85**, 1082-1089.

Lee, J and M. Strazicich (2004) "Minimum LM Unit Root Test With One Structural Break" Working Papers 04-17, Department of Economics, Appalachian State University.

Schmidt, P and P.C.B. Phillips (1992) "LM Tests for a Unit Root in the Presence of Deterministic Trends " *Oxford Bulletin of Economics and Statistics* 54, 257-287.

Table I: Critical values of $\tilde{\tau}(\lambda^c)$, LM_τ and LM_τ^* for $T = 100$ based on 5000 replications; $\lambda^c = (0.4, 0.6)$

Test statistic	Selection method	Model A			Model C		
		1%	5%	10%	1%	5%	10%
$\tilde{\tau}(\lambda^c)$	exogenous	-3.699	-3.090	-2.802	-4.742	-4.171	-3.898
LM_τ	$\arg\min_\lambda LM_\tau(\lambda)$	-4.515	-3.862	-3.501	-5.882	-5.283	-4.993
LM_τ^*	$\arg\min_\lambda SSR(\lambda)$	-4.329	-3.651	-3.280	-5.626	-5.006	-4.674

Table II: 5 percent rejection frequency and probability of detecting the true break dates simultaneously, $T = 100$, $\lambda^c = (0.4, 0.6)$, 5000 replications; Model A

		$\arg\min_\lambda LM_\tau(\lambda)$			$\arg\min_\lambda SSR(\lambda)$		
β	$d_1 = d_2$	LM_τ	$\widetilde{LM}_\tau$	$P(\hat{T}_B = T_B)$	LM_τ^*	$\widetilde{LM}_\tau^*$	$P(\hat{T}_B = T_B)$
1	0	0.050	0.178	0.001	0.050	0.123	0.001
1	2	0.052	0.201	0.026	0.049	0.123	0.102
1	3	0.060	0.206	0.088	0.040	0.101	0.422
1	5	0.071	0.238	0.198	0.014	0.054	0.963
1	10	0.036	0.227	0.307	0.010	0.049	1.000
1	20	0.007	0.076	0.410	0.013	0.052	1.000
0.9	0	0.205	0.585	0.001	0.197	0.431	0.001
0.9	2	0.192	0.550	0.033	0.170	0.375	0.099
0.9	3	0.187	0.544	0.133	0.141	0.339	0.405
0.9	5	0.145	0.511	0.331	0.066	0.225	0.957
0.9	10	0.048	0.373	0.590	0.053	0.210	1.000
0.9	20	0.034	0.222	0.800	0.060	0.220	1.000

Table III: 5 percent rejection frequency and probability of detecting the true break dates simultaneously, $T = 100$, $\lambda^c = (0.4, 0.6)$, 5000 replications; Model C

			$\arg\min_\lambda LM_\tau(\lambda)$			$\arg\min_\lambda SSR(\lambda)$		
β	$d_1 = d_2$	$d_3 = d_4$	LM_τ	$\widetilde{LM}_\tau$	$P(\hat{T}_B = T_B)$	LM^*_τ	$\widetilde{LM}^*_\tau$	$P(\hat{T}_B = T_B)$
1	0	0	0.050	0.434	0.001	0.050	0.252	0.001
1	0	2	0.011	0.193	0.006	0.010	0.084	0.113
1	0	5	0.032	0.351	0.001	0.005	0.058	0.228
1	0	10	0.372	0.854	0.000	0.006	0.060	0.232
1	2	0	0.058	0.457	0.001	0.052	0.226	0.071
1	2	2	0.027	0.282	0.000	0.009	0.073	0.603
1	2	5	0.102	0.534	0.000	0.007	0.063	0.699
1	2	10	0.508	0.932	0.000	0.007	0.065	0.694
1	5	0	0.064	0.527	0.014	0.008	0.074	0.921
1	5	2	0.086	0.522	0.000	0.004	0.058	0.996
1	5	5	0.289	0.816	0.000	0.004	0.059	0.998
1	5	10	0.752	0.989	0.000	0.005	0.054	0.999
1	10	0	0.039	0.555	0.027	0.005	0.051	1.000
1	10	2	0.176	0.832	0.001	0.005	0.053	1.000
1	10	5	0.599	0.978	0.000	0.004	0.054	1.000
1	10	10	0.938	0.999	0.000	0.006	0.055	1.000
0.9	0	0	0.105	0.618	0.001	0.102	0.378	0.001
0.9	0	2	0.028	0.320	0.005	0.025	0.166	0.105
0.9	0	5	0.075	0.534	0.001	0.015	0.140	0.265
0.9	0	10	0.491	0.935	0.000	0.016	0.125	0.246
0.9	2	0	0.106	0.626	0.003	0.091	0.346	0.074
0.9	2	2	0.054	0.440	0.001	0.021	0.146	0.586
0.9	2	5	0.184	0.715	0.000	0.017	0.139	0.681
0.9	2	10	0.649	0.975	0.000	0.018	0.127	0.675
0.9	5	0	0.096	0.653	0.021	0.020	0.130	0.923
0.9	5	2	0.137	0.677	0.000	0.013	0.116	0.992
0.9	5	5	0.421	0.918	0.000	0.012	0.118	0.996
0.9	5	10	0.868	0.997	0.000	0.014	0.114	0.995
0.9	10	0	0.053	0.629	0.064	0.014	0.113	1.000
0.9	10	2	0.251	0.916	0.003	0.015	0.121	1.000
0.9	10	5	0.733	0.996	0.000	0.012	0.118	1.000
0.9	10	10	0.982	1.000	0.000	0.013	0.125	1.000