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Analysis of the Tail Dependence Structure in the Global Markets: A Pair Copula Construction Approach

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Abstract

In this paper we estimated pair copula constructions (PCC) for three sets of markets: developed, Latin emerging and Asia-Pacific emerging. To that, we used daily prices from January 2003 to November 2011, totaling 1872 observations. After, we estimated the lower and upper tail dependence for each bivariate relationship, comparing the three sets of markets. The results allow concluding that there are some discrepancies in the dependence of the lower and upper tails. The Asia-Pacific markets obtained the great general absolute and tail dependences. The results reinforced the need for a correct risk management in the case of international portfolios due to the fact that the tails generally represent extreme events, as crisis for example, which can lead to deep losses periods occasioned by eventual contagions caused by the tail dependence among the markets. Further, the diversification must be done properly, with optimization process that consider this dependence in the extreme values.

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1. Introduction

Since the introduction of the mathematical theory of portfolio selection and of the Capital Asset Pricing Model (CAPM), the issue of dependence has always been of fundamental importance to financial economics. In the context of international diversification, there is the need for minimizing the risk of specific assets through optimal allocation of resources. Therefore, it is necessary to understand the multivariate relationship between different markets. Thus we need a statistical model able to measure the temporal dependence between shocks of different countries.

The global extent of extreme shocks and the potential damaging consequences continuously attract attention among economists and policymakers. These extreme dependences, which represent the tails of the probability function, beyond any fundamental link, has long been an issue of interest to academics, fund managers and traders, as it has important implications for portfolio allocation and asset pricing.

An inappropriate model for model this extreme dependence can lead to suboptimal portfolios and inaccurate assessments of risk exposures. Traditionally, correlation is used to describe dependence between random variables, but recent studies have ascertained the superiority of copulas to model dependence, as they offer much more flexibility than the correlation approach, because a copula function can deal with non-linearity, asymmetry, serial dependence and also the well-known heavy-tails of financial assets marginal and joint probability distribution.

A copula is a function that links univariate marginals to their multivariate distribution. Since it is always possible to map any vector of random variables into a vector with uniform margins, we are able to split the margins of that vector and a digest of the dependence, which is the copula. Despite the literature on copulas is consistent, the great part of the research is still limited to the bivariate case. Thus, construct higher dimensional copulas is the natural next step, even this do not being an easy task. Apart from the multivariate Gaussian and Student, the selection of higher-dimensional parametric copulas is still rather limited (Genest *et al.*, 2009).

The developments in this area tend to hierarchical, copula-based structures. It is very possible that the most promising of these is the pair-copula construction (PCC). Originally proposed by Joe (1996), it has been further discussed and explored in the literature for questions of inference and simulation (Bedford and Cooke, 2001; Bedford and Cooke, 2002; Kurowicka and Cooke, 2006; Aas *et al*, 2009). The PCC is based on a decomposition of a multivariate density into bivariate copula densities, of which some are dependency structures of unconditional bivariate distributions, and the rest are dependency structures of conditional bivariate distributions.

In this sense, this paper aims to estimate and compare the tail dependence structure existing in the global markets. To that, we collected data from developed markets (U.S., Germany, England and Japan), Latin (Argentina, Brazil, Mexico and Chile) and Asia-Pacific (China, Hong Kong, Indonesia and Singapore) emerging markets in the period from January, 2003 to November, 2011. This period was chosen due to the need to consider the international market openness without give too much importance to very past information. For each set of markets, we estimated a PCC, in order to estimate the diverse bivariate tail dependences.

The sequence of this paper is structured on the following way: Section 2 briefly expose the background about copulas and PCC; Section 3 presents the material and methods of the study, presenting the data and the procedures to achieve the objective of the paper; Section 4 presents the found results and their discussion; Section 5 expose the conclusions of the paper.

2. Background

This section is subdivided on: i) Copulas, which briefly explain about definition and properties of this class of function; ii) Pair Copula Construction, which succinctly expose the concepts of this construction.

2.1 Copulas

Dependence between random variables can be modeled by copulas. A copula returns the joint probability of events as a function of the marginal probabilities of each event. This makes copulas attractive, as the univariate marginal behavior of random variables can be modeled separately from their dependence (Kojadinovic and Yan, 2010).

The concept of copula was introduced by Sklar (1959). However, only recently its applications have become clear. A detailed treatment of copulas as well as of their relationship to concepts of dependence is given by Joe (1997) and Nelsen (2006). A review of applications of copulas to finance can be found in Embrechts *et al.* (2003) and in Cherubini *et al.* (2004).

For ease of notation we restrict our attention to the bivariate case. The extensions to the *n*-dimentional case are straightforward. A function $C : [0,1]^2 \rightarrow [0,1]$ is a *copula* if, for $0 \leq x \leq 1$ and $x_1 \leq x_2$, $y_1 \leq y_2$, (x_1, y_1), $(x_2, y_2) \in [0,1]^2$, it fulfills the following properties:

$$C(x,1) = C(1,x) = x, \quad C(x,0) = C(0,x) = 0. \tag{1}$$

$$C(x_2, y_2) - C(x_2, y_1) - C(x_1, y_2) + C(x_1, y_1) \geq 0. \tag{2}$$

Property (1) means uniformity of the margins, while (2), the *n-increasing property* means that $P(x_1 \leq X \leq x_2, y_1 \leq Y \leq y_2) \geq 0$ for (X,Y) with distribution function *C*.

In the seminal paper of Sklar (1959), it was demonstrated that a Copula is linked with a distribution function and its marginal distributions. This important theorem states that:

(i) Let *C* be a copula and F_1 and F_2 univariate distribution functions. Then (3) defines a distribution function *F* with marginals F_1 and F_2.

$$F(x,y) = C\big(F_1(x), F_2(y)\big), \ (x,y) \in R^2. \tag{3}$$

(ii) For a two-dimensional distribution function *F* with marginals F_1 and F_2, there exists a copula *C* satisfying (3). This is unique if F_1 and F_2 are continuous and then, for every $(u,v) \in [0,1]^2$:

$$C(u,v) = F\big(F_1^{-1}(u), F_2^{-1}(v)\big). \tag{4}$$

In (4), F_1^{-1} and F_2^{-1} denote the generalized left continuous inverses of F_1 and F_2. However, as Frees and Valdez (1998) note, it is not always obvious to identify the copula. Indeed, for many financial applications, the problem is not to use a given multivariate distribution but consists in finding a convenient distribution to describe some stylized facts, for example the relationships between different asset returns.

2.2 Pair Copula Construction

The PCC is a very flexible construction, which allows for the free specification of *n(n−1)/2* copulas. This construction was proposed by the seminal paper of Joe (1996), and it has been discussed in detail, especially, for applications in simulation and inference. Similar to the NAC, the PCC is hierarchical in nature. The modeling scheme is based on a decomposition of a multivariate density into *n(n−1)/2* bivariate copula densities, of which the first *n−1* are dependency structures of unconditional bivariate distributions, and the rest are dependency structures of conditional bivariate distributions (Aas and Berg, 2011).

The PCC is usually represented in terms of the density. The two main types of PCC that have been proposed in the literature are the C (canonical)-vines and D-vines. In the present paper we focus on the D-vine estimation, which accordingly to Aas *et al*. (2009) has the density as in formulation (5)

$$f(x_1,\cdots,x_n) = \prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=i}^{n-j} c \begin{Bmatrix} F(x_i|x_{i+1},\cdots,x_{i+j-1}), \\ F(x_{i+j}|x_{i+1},\cdots,x_{i+j-1}) \end{Bmatrix}. \quad (5)$$

In (5), $x_1,\cdots,x_n$ are variables; f is the density function; $c(\cdot,\cdot)$ is a bivariate copula density and the conditional distribution functions are computed, accordingly to Joe (1996), by formulation (6).

$$F(x|\boldsymbol{v}) = \frac{\partial C_{x,v_j|\boldsymbol{v}_{-j}}\{F(x|\boldsymbol{v}_{-j}),F(v_j|\boldsymbol{v}_{-j})\}}{\partial F(v_j|\boldsymbol{v}_{-j})}. \quad (6)$$

In (6) $C_{x,v_j|\boldsymbol{v}_{-j}}$ is the dependency structure of the bivariate conditional distribution of *x* and v_j conditioned on $\boldsymbol{v}_{-j}$, where the vector $\boldsymbol{v}_{-j}$ is the vector $\boldsymbol{v}$ excluding the component v_j.

Thus, the conditional distributions involved at one level of the construction are always computed as partial derivatives of the bivariate copulas at the previous level (Aas and Berg, 2011). Since only bivariate copulas are involved, the partial derivatives may be obtained relatively easily for most parametric copula families. It is worth to note that the copulas involved in (5) do not have to belong to the same family. Hence, we should choose, for each pair of variables, the parametric copula that best fits the data.

3. Empirical Method

We collected daily prices from January 2003 to November 2011, totaling 1872 observations of S&P500 (U.S.), DAX (Germany), FTSE100 (England), Nikkei225 (Japan), which represents the developed markets (set 1); Merval (Argentina), Ibovespa (Brazil), IPC (Mexico), IPSA (Mexico), which are the emerging Latin markets; SSEC (China), HSI (Hong Kong), JKSE (Indonesia) and STI (Singapore), which compose the Asia-Pacific emerging markets (set 3). This period was chosen due to the need to consider the international market openness without give too much importance to very past information, which could cause some bias to the found results.

Firstly, in order to avoid non-stationarity issues we calculated the log-returns of the assets by formulation (7).

$$r_t = \ln P_t - \ln P_{t-1}. \quad (7)$$

In (7), r_t is the log-return at period *t*; P_t is the price at period *t*.

Before estimate the PCCs, for each set of assets we modeled their marginal. Initially, we used a vector autoregressive model (VAR) to obtain the estimated returns and residuals of each set. The mathematical form of the VAR(p) model used is represented by (8).

$$\boldsymbol{r}_t = \boldsymbol{\phi}_0 + \boldsymbol{\Phi}_1\boldsymbol{r}_{t-1} + \cdots \boldsymbol{\Phi}_p\boldsymbol{r}_{t-p} + \boldsymbol{a}_t. \quad (8)$$

In (8), $\boldsymbol{r}_t$ is a *k*-dimensional vector of the log-returns at period *t*; $\boldsymbol{\phi}_0$ is a *k*-dimensional vector of constants; $\boldsymbol{\Phi}_i$, *i=1,...,p* are *k x k* matrixes of parameters; $\{\boldsymbol{a}_t\}$ is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix $\boldsymbol{\Sigma}$.

Subsequently, to consider the well-known conditional heteroscedastic behavior of financial assets, using the residuals of the VAR applied to each set of returns, we used estimated a copula-based GARCH model, with skew-t innovations to fit the asymmetry of the returns, as represented in (9).

$$h_{i,t}^2 = c_i + b_i h_{i,t-1}^2 + a_i \varepsilon_{i,t-1}^2. \quad (9)$$

Where $h_{i,t}^2$ is the conditional variance of asset I in period t; a_i, b_i and c_i are parameters; $\varepsilon_{i,t} = h_{i,t} z_{i,t}, z_{i,t} \sim skewed - t(z_i|\phi_i)$. ϕ is the asymmetry parameter. Further, the model was estimated with a student copula as multivariate distribution.

After model the marginal, we estimated a PCC for each set of returns. To that, we standardized the residuals of the VAR-GARCH approach into pseudo-observations $\boldsymbol{U}_j = (U_{1j}, \dots, U_{ij})$ through the ranks as $U_{ij} = R_{ij}/(n+1)$. Subsequently, we ordered the variables by the decreasing order of the sum of the non-linear dependence with the other variables in the set by the Kendall's tau. Subsequently, to choose the copula that best fits each bivariate pair of variables we employed the AIC criterion.

To validate the choice of a D-vine PCC, we compared each model with their counterpart C-vine by the test proposed by Clark (2007). This test allows comparing non-nested models. For this let C_1 and C_2 be two competing vine copulas in terms of their densities and with estimated parameter sets θ_1 and θ_2. The null hypothesis of statistical indifference of the two models is:

$$H_0: P(m_i > 0) = 0.5\,, m_i = \log\left[\frac{C_1(u_i|\theta_1)}{C_2(u_i|\theta_2)}\right],\ \forall_i = 1, \cdots, n. \quad (10)$$

We used the fitted PCCs in order to estimate each bivariate tail dependence index. Given the estimated bivariate copula C, the lower and upper tail dependence are represented by formulations (11) and (12), respectively.

$$\lambda_L = \lim_{u \to 0^+} \frac{C(u,u)}{u}. \quad (11)$$

$$\lambda_U = \lim_{u \to 1^-} \frac{1 - 2u + C(u,u)}{1-u}. \quad (12)$$

4. Results and discussion

We first calculated the log-returns of each asset for the studied period. The Figures 1, 2 and 3 exhibit the plots of these returns for each set of markets. These Figures elucidate that the developed markets has less oscillation than the emerging ones, as expected due to their economic solidity and financial liquidity. It should be noted also that there was clear vestiges of turbulence periods during the well-known financial crisis, as pointed by the volatility clusters. The most noted clusters occurred around the observations 1200 to 1400, representing the sub-prime crisis of 2007/2008.

In order to complementing this initial visual analysis, we present in Table 1 the descriptive statistics of the markets during the analyzed period. The results in Table 1 confirm that the developed markets tend to have less oscillation than the emerging ones. The mean of the log-returns in these developed markets is also slightly smaller, although no one of the calculated means was significantly different of zero. Further, all markets had leptokurtic log-returns, and, with exception of Mexico and Chile, there was a predominance of negative skewness. These results reinforce the use of a skew-t distribution to model the innovations of the log-returns.

Subsequently, to choose the order of the variables in the PCC construction, we estimated the dependence matrix for each set of returns by the Kendall's tau approach. Table 2 presents the results of these dependence matrixes. The chosen criteria was order the assets conform the absolute sum of their calculated Kendall's tau to the others.

The results in Table 2 indicate that, in a general way, the Asia-Pacific markets are more dependent with the others, if compared to the remaining sets. The negative signal in the calculated Kendall's tau only appeared in the Latin markets, for the bivariate cases of Brazil/Argentina and Brazil/Mexico. This result corroborate with the increasing in the globalization of the financial markets, as pointed by the predominance of positive dependence among the log-returns.

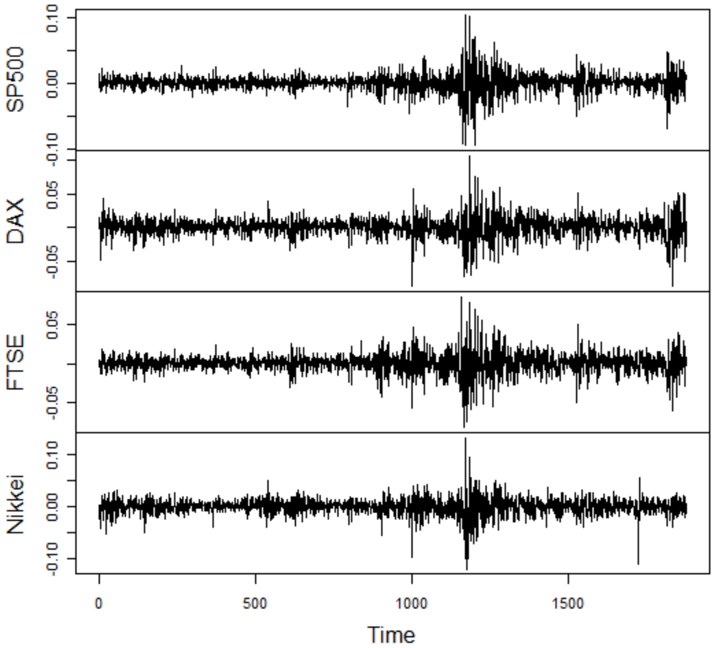

Figure 1. Daily log-returns of the developed markets (set 1) during the period from January 2003, to November 2011.

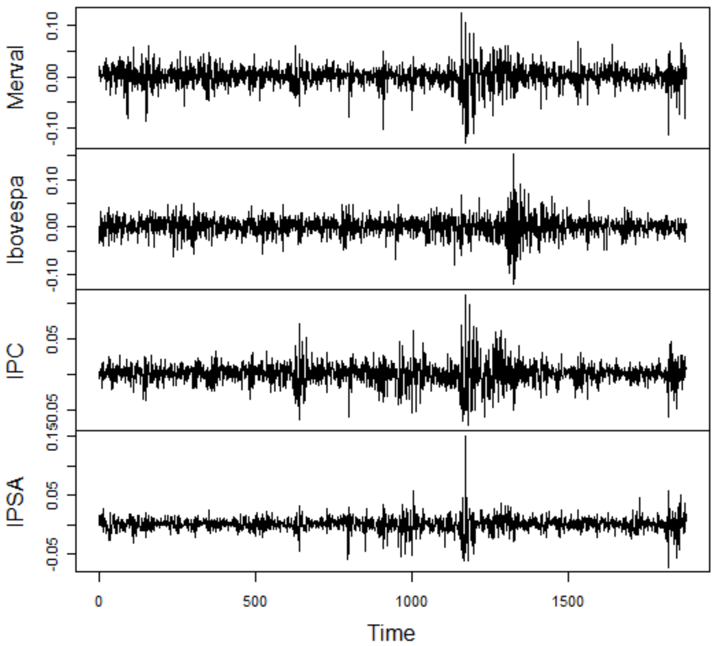

Figure 2. Daily log-returns of the Latin emerging markets (set 2) during the period from January 2003, to November 2011.

With the results contained in Table 2 we decided the order of the variables in the PCCs. For set 1: DAX, FTSE100, Nikkei225 and S&P500; for set 2: IPC, Merval, IPSA and Ibovespa; for set 3: HIS, STI, JKSE and SSEC. After, we modeled the marginal of the assets through the VAR-copula based GARCH procedure explained in the subsection 2.3 of the current paper. The results of the estimation of these model were omitted due to lack of space, beyond are not the principal scope of the study. The residuals of the previous modeling were standardized and utilized to estimate the PCCs. The results of the estimation of the dependence structure of each set of markets are presented in Table 3.

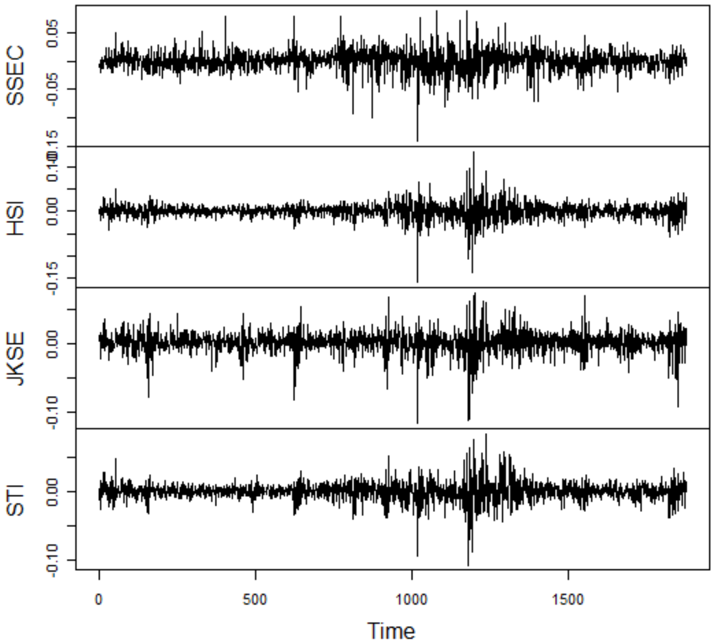

Figure 3. Daily log-returns of the Asia-Pacific emerging markets (set 3) during the period from January 2003, to November 2011.

Table 1. Descriptive statistics of the daily log-returns of the studied markets during the period from January 2003, to November 2011.

Statistic	Minimum	Maximum	Mean	St. Dev.	Skewness	Kurtosis
Set 1						
S&P500	-0.0947	0.1042	0.0001	0.0142	-0.3695	9.1437
DAX	-0.0883	0.1068	0.0003	0.0149	-0.2687	5.3675
FTSE100	-0.0818	0.0847	0.0001	0.0131	-0.1855	5.7954
Nikkei225	-0.1211	0.1323	-0.0001	0.0167	-0.6222	8.6198
Set 2						
Merval	-0.1295	0.1249	0.0009	0.0205	-0.5819	5.4675
Ibovespa	-0.1210	0.1547	0.0011	0.0200	-0.1724	4.8367
IPC	-0.0726	0.1111	0.0009	0.0152	0.1385	5.5279
IPSA	-0.0621	0.1502	0.0008	0.0117	0.5051	19.4637
Set 3						
SSEC	-0.1597	0.1341	0.0004	0.0176	-0.3919	12.4328
HSI	-0.1063	0.0835	0.0004	0.0135	-0.3978	8.1862
JKSE	-0.1147	0.0736	0.0011	0.0161	-0.8622	7.2178
STI	-0.1417	0.0903	0.0004	0.0192	-0.3907	4.5633

The results contained in Table 3 indicate that there is a clear predominance of the Student and BB7 copulas in the bivariate relationships among the three sets of studied markets. Gumbel and BB1 copulas also appeared as having the best fit to some data. These copulas assign, in certain degree, importance to the tails of the joint probability distribution. This fact clarify that there is more dependence among the markets in extreme events than the normally expected. This corroborate with the studies that appoint to an increase of the dependence between markets in periods of great shocks.

Table 2. Kendall's Tau dependence matrixes of each set of daily log-returns of the studied markets during the period from January 2003, to November 2011.

Developed markets				
	S&P500	**DAX**	**FTSE100**	**Nikkei225**
S&P500	1.0000	0.0171	0.0034	0.0890
DAX	0.0171	1.0000	0.6645	0.2825
FTSE100	0.0034	0.6645	1.0000	0.2658
Nikkei225	0.0890	0.2825	0.2658	1.0000
Sum	0.1095	0.9641	0.9337	0.6373
Latin markets				
	Merval	**Ibovespa**	**IPC**	**IPSA**
Merval	1.0000	-0.0386	0.3586	0.3091
Ibovespa	-0.0386	1.0000	-0.0148	0.0043
IPC	0.3586	-0.0148	1.0000	0.3465
IPSA	0.3091	0.0043	0.3465	1.0000
Sum	0.7063	0.0577	0.7199	0.6599
Asia-Pacific markets				
	SSEC	**HSI**	**JKSE**	**STI**
SSEC	1.0000	0.2458	0.1360	0.1836
HIS	0.2458	1.0000	0.3686	0.5180
JKSE	0.1360	0.3686	1.0000	0.3792
STI	0.1836	0.5180	0.3792	1.0000
Sum	0.5654	1.1324	0.8838	1.0808

Regarding to the differences of the estimated PCCs, the results in Table 3 emphasizes that in the developed markets dependence structure, the student copula was predominant, while BB7 copula obtained the best fit in the most of bivariate relationships. Again, in a general form, the Asia-Pacific markets presented the bigger dependence. Further, all the PCCs rejected the null hypothesis of the Clark test, which states that there is no distinction in the fit of the utilized D-vine approach and the C-vine construction.

The results in Table 3 fundamentally emphasize the need for a properly estimation of the dependence structure of financial assets. This procedure allied with a precise estimation of the marginal of the log-returns should lead to a trustable prediction of the dynamic risk of a portfolio. Subsequently, we calculated the lower and upper tail dependence index for each bivariate relationship through formulations (11) and (12). The results are presented in Table 4.

Firstly, the results in Table 4 indicate that some relationships exhibited more importance in its structures regarding to the dependence in the tails than that showed by the absolute Kendall's Tau measure. This is the case, for example, of IPC,IPSA|Merval and STI,SSEC|JKSE. Further, there are discrepancies in the dependence of the lower and upper tails for some relationships, as is the case of Nikkei225,S&P500; DAX,Nikkei225|FTSE100 and IPC,IPSA|Merval. The bigger values for the tail dependences was obtained, in each set of markets, for the countries with more absolute dependence: DAX and FTSE100 (developed); IPC and Merval (Latin); HSI and STI (Asia-Pacific).

Regarding to comparisons among the three sets of markets, the predominance of the Asia-Pacific markets obtained for the Kendall's Tau measure was maintained in the tails dependence. This set presented the great means for both lower and upper tails. The developed and the Latin emerging countries showed similar general tail dependences. These results reinforced the need for a concise estimation of the dependence, beyond the importance of the risk management caused by the fact that the tails generally represent extreme events (especially losses).

Table 3. Pair Copula Constructions for each set of markets during the period from January 2003, to November 2011.

Developed markets			
Pair	**Copula**	**Parameter 1**	**Parameter 2**
DAX,FTSE100	Student	0.7705	4.4751
FTSE100,Nikkei225	Student	0.2116	3.9106
Nikkei225,S&P500	BB7	1.2355	0.2081
DAX,Nikkei225|FTSE100	Gumbel	1.1038	-
FTSE100,S&P500|Nikkei225	Student	0.0606	4.2582
DAX,S&P500|FTSE100,Nikkei225	Student	0.0415	5.9611
Clark test			739 (0.0000)
Latin emerging markets			
Pair	**Copula**	**Parameter 1**	**Parameter 2**
IPC,Merval	BB7	1.4136	0.7968
Merval, IPSA	BB7	1.3288	0.6283
IPSA, Ibovespa	Student	0.0044	13.8539
IPC,IPSA|Merval	BB1	1.1918	1.1994
Merval,Ibovespa|IPSA	Gumbel (rotated 90º)	-1.0302	-
IPC,Ibovespa|Merval,IPSA	Student	-0.0080	20.1479
Clark test			907 (0.0003)
Asia-Pacific emerging markets			
Pair	**Copula**	**Parameter 1**	**Parameter 2**
HSI, STI	BB7 (rotated 180º)	2.1605	1.2245
STI, JKSE	BB7	1.5307	0.8327
JKSE,SSEC	Student	0.2076	6.2932
HSI,JKSE|STI	Student	0.2063	10.5908
STI,SSEC|JKSE	Gumbel (rotated 180º)	1.1447	-
HSI,SSEC|STI,JKSE	BB7 (rotated 180º)	1.1163	0.2604
Clark test			882 (0.0260)

Table 4. Lower and Upper bivariate tail dependence for each set of markets during the period from January 2003, to November 2011.

Pair	**Lower Tail**	**Upper Tail**
DAX,FTSE100	0.4141	0.4141
FTSE100,Nikkei225	0.1350	0.1350
Nikkei225,S&P500	0.0357	0.2476
DAX,Nikkei225|FTSE100	0.0000	0.1262
FTSE100,S&P500|Nikkei225	0.0807	0.0807
DAX,S&P500|FTSE100,Nikkei225	0.0393	0.0393
Mean	0.1175	0.1738
Pair	**Lower Tail**	**Upper Tail**

IPC,Merval	0.4190	0.3671
Merval, IPSA	0.3318	0.3152
IPSA, Ibovespa	0.0016	0.0016
IPC,IPSA|Merval	0.0491	0.2177
Merval,Ibovespa|IPSA	0.0000	0.0000
IPC,Ibovespa|Merval,IPSA	0.0001	0.0001
Mean	0.1336	0.1502
Pair	**Lower Tail**	**Upper Tail**
HSI, STI	0.6217	0.5677
STI, JKSE	0.4350	0.4273
JKSE,SSEC	0.0634	0.0634
HSI,JKSE|STI	0.0177	0.0177
STI,SSEC|JKSE	0.1678	0.1678
HSI,SSEC|STI,JKSE	0.1393	0.0698
Mean	0.2408	0.2189

4. Conclusions

In this paper we estimated and compared compare the tail dependence structure existing in the global markets. To that, we used data from developed markets (U.S., Germany, England and Japan), Latin (Argentina, Brazil, Mexico and Chile) and Asia-Pacific (China, Hong Kong, Indonesia and Singapore) emerging markets in the period from January 2003, to November 2011.

We first estimated the marginal of the assets through a copula based multivariate GARCH model for each set of markets. Subsequently, we standardized the residuals of the GARCH models and estimated the PCCs. The results evidenced that the Student copula predominated in the bivariate relationships of the developed markets, while the BB7 copula was the most present in the relationships of the emerging markets. Gumbel and BB1 copula also appeared in the PCCs. This fact clarify that there is more dependence among the markets in extreme events than the normally expected, once that these copulas assign, in certain degree, importance to the tails of the joint probability distribution. Thus, this dependence structure estimation reinforced the need for a properly estimation of the dependence structure of financial assets, independently of its economic stage.

After, based on the estimated copulas, we calculated the lower and upper tail dependence for each bivariate relationship. Some relationships exhibited more importance regarding to the dependence in the tails than that showed by the absolute Kendall’s Tau measure, while others presented discrepancies in the dependence of the lower and upper tails. In each set, the more relevant tail dependences were obtained by the countries with more absolute dependence. The Asia-Pacific markets obtained the great general absolute and tail dependences.

These results reinforced the need for a correct risk management in the case of international portfolios due to the fact that the tails generally represent extreme events, as crisis for example, which can lead to deep losses periods occasioned by eventual contagions caused by the tail dependence among the markets. Further, the diversification must be done properly, with optimization process that consider this dependence in the extreme values.

Finally, we suggest for future research that the PCC procedure be used for others financial applications, as the optimal allocation in a portfolio based on the tail dependence indexes obtained through the parameters of the estimated copulas, beyond the absolute non-linear dependence measures.

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