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Updating Choquet capacities: a general framework

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Abstract

Several updating rules for Choquet capacities have been proposed in the literature. We propose to study them in a general framework consisting of extending the definition of conditional expectations to the Choquet case. Then we adopt several relations between unconditional and conditional expectations and observe which updating rule is able to resolve such and such a relation. It allows to specify which updating rule should be used depending on the decision context.

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1. Introduction

Choquet beliefs, or capacities, generalizes the notion of probability by relaxing additivity. Since a number of economic problems involve not only uncertainty, but also sequential resolution of it, we have to integrate sequential arrivals of information in the decision process. In the Choquet framework, we have to specify an updating rule, saying how to calculate updated capacities. Several updating rules have been proposed in the literature. For instance, Gilboa and Schmeidler (1993) identify f-Bayesian updating rules[1] and Eichberger et al. (2007) axiomatically characterize the Full Bayesian updating rule. In these works, updating rules are obtained by specifying a link between unconditional and conditional preferences. But the study of the update of Choquet capacities is not limited to axiomatic approaches. For instance, Jaffray (1992) and Denneberg (1994) adopt a statistical approach, in the sense that capacities are rather seen as a way of representing objective but imprecise information.

The core of our approach is to consider several relations between unconditional and conditional expectations. Hence a behavioral characterization of updating rules is implicitly made. These relations are equivalent in the additive case and then each of them may serve as a definition of the conditional expectation, that is obtained thanks to the Bayes update rule. Nevertheless, for non-additive measures, the equivalence does not hold and one get different updating rules. Then the definition of conditional expectations may be used to update Choquet capacities. Furthermore, linking unconditional and conditional expectations involves a specific way of resolving a decision problem and therefore our approach allows to determine which updating rule should be used depending on the decision context (insurance, financial...).

The paper is organized as follows. In the next section, we introduce our set-up and notations. In section 3, we present our results. Finally, section 4 concludes. Proofs are relegated to the appendix.

2. Set-up, notations and definitions

Let S be a finite state space and let Σ be its power set. For all $A \in \Sigma$, the event $S - A$ is denoted by A^c. The objects of choice are bounded Σ-measurable random variables of the form $X : S \to \mathbb{R}$. We denote by $\mathscr{A}$ the set of such functions. The characteristic function of an event $A \in \Sigma$ is the binary random variable 1_A such that $1_A(s) = 1$ when $s \in A$ and $1_A(s) = 0$ otherwise. Two random variables X and Y are said to be comonotonic if $[X(s) - X(s')][Y(s) - Y(s')] \geq 0$ for all s and s' in S and antimonotonic if $\geq$ is replaced by $\leq$.

In this paper, we assume that the DM's preferences are represented by a Choquet expectation noted $I(.)$ (see for instance Chateauneuf 1991). This model assumes the DM's beliefs be represented by a Choquet capacity, i.e. a set function $\nu : \Sigma \to \mathbb{R}$ such that (i) $\nu(.)$ is normalized, *i.e.* $\nu(S) = 1$ and $\nu(\emptyset) = 0$, and (ii) $\nu(.)$ is monotonic with respect to set inclusion, i.e. $A \subseteq B \Rightarrow \nu(A) \leq \nu(B)$ for all $A, B \in \Sigma$. The conjugate capacity of $\nu(.)$, noted $\bar{\nu}(.)$, is defined by $\bar{\nu}(A) = 1 - \nu(A^c)$ for all $A \in \Sigma$. We note $\nu(.|E)$ the conditional capacity for $\nu(.)$ given any $E \in \Sigma$ such that $\nu(.|E)$ is a Choquet capacity, $\nu(E|E) = 1$ and $\nu(E^c|E) = 0$. The Choquet integral w.r.t. $\nu(.)$ of any $X \equiv (x_i, A_i)_{i=1}^n$[2] taking n distinct values, w.l.o.g. $x_1 \leq \ldots \leq x_n$, is defined by:

$$I(X) = \sum_{i=1}^{n} x_i \cdot [\nu(A_i \cup \ldots \cup A_n) - \nu(A_{i+1} \cup \ldots \cup A_n)] + x_n \cdot \nu(A_n)$$

[1] See also Horie (2007).

[2] That is, X is measurable w.r.t. the partition $\{A_1, \ldots, A_n\}$ of S.

Let $I(X/1_E = 1)$ and $I(X/1_E = 0)$ be conditional Choquet expectations of X on E and E^c. They use $\nu(.|E)$ and $\nu(.|E^c)$, respectively. Hence 1_E designates the information function, since it can take two values, 1 or 0, depending on whether E is observed or if it is E^c. Then $I(X/1_E)$ designates the binary random variable yielding conditional Choquet expectations of X.

An updating rule is a formula associating, for each $E \in \Sigma$, a specific $\nu(.|E)$ to the capacity $\nu(.)$. The value of each conditional expectation is crucially dependent on the updating rule used by the DM. Three updating rules are commonly used to update Choquet capacities:

- The Full Bayesian (FB) update rule of $\nu(.)$ conditional on any $E \in \Sigma$ such that $E \neq \emptyset$ is given by:
$$\nu(A|E) = \frac{\nu(A \cap E)}{1 + \nu(A \cap E) - \nu(A \cup E^c)}, \quad A \in \Sigma$$
- The Naive Bayesian (NB) update rule of $\nu(.)$ conditional on any $E \in \Sigma$ such that $\nu(E) > 0$ is given by:
$$\nu(A|E) = \frac{\nu(A \cap E)}{\nu(E)}, \quad A \in \Sigma$$
- The Dempster-Shafer (DS) update rule of $\nu(.)$ conditional on any $E \in \Sigma$ such that $\bar{\nu}(E) > 0$ is given by:
$$\nu(A|E) = \frac{\nu((A \cap E) \cup E^c) - \nu(E^c)}{1 - \nu(E^c)}, \quad A \in \Sigma$$

The following assumption is made throughout:

Assumption 1 (Conditioning event)**.** *There exists at least one event $E \in \Sigma$ such that $\nu(.|E)$ is well defined for each updating rule.*

3. Results

We begin this section by examining some relations between comotonicity of the information with the valued random variable and updating. The comonotonic link between the valued asset and the information function is often encountered in insurance economics, and more generally cover a wide range of applications. Consider the binary random variable

$$I(1_A/1_E) \equiv \begin{pmatrix} I(1_A/1_E = 1) & if \quad s \in E \\ I(1_A/1_E = 0) & if \quad s \in E^c \end{pmatrix}$$

where $I(1_A/1_E = 1) = \nu(A|E)$ and $I(1_A/1_E = 0) = \nu(A|E^c)$. It expresses the DM's willingness to pay for "betting on event A", but it may take two distinct values, depending on the information function 1_E. In the additive case, the relation

$$I(1_A) = I[I(1_A/1_E)] \tag{1}$$

is straightforwardly satisfied if and only if conditional probabilities are calculated by Bayesian updating. It means that the value of 1_A is not affected by information available at time 1. The following proposition states the corresponding updating rule for the non-additive case.

Proposition 1. *The solution of relation (1) is given by the NB rule when 1_A and 1_E are comonotonic and by the DS rule when 1_A and 1_E are antimonotonic.*

The general case where 1_A and 1_E are neither comonotonic nor antimonotonic cannot be resolved by relation (1). Indeed, one should obtain one equation for two unknowns, $I(1_A/1_E = 0)$ and $I(1_A/1_E = 1)$, which are not necessarily equal to 0 nor to 1 as in the two previous cases and cannot be ranked, in general, to compute the Choquet integral. That is why the characterization of $\nu(.|E)$ is only partial in the non-additive case. If we consider arbitrary random variables instead of characteristic functions, relation (1) becomes

$$I(X) = I[I(X/1_E)]$$

Statement of proposition 1 may be enlarged to any $X \in \mathscr{A}$ with 1_E and is not only valid for characteristic functions (see notably Chateauneuf et al. 2001). That is, if X is comonotonic with the information function 1_E, $I(X) = I[I(X/1_E)]$ holds if and only if $\nu(.|E)$ is given by the NB rule, and $\nu(.|E^c)$ is given by the DS rule. Hence this way of updating Choquet capacities may be used to resolve a wide range of economic situations involving sequential resolution of uncertainty.

In the additive case, an equivalent definition to relation (1) is

$$I[1_A - I(1_A/1_E)] = 0, \tag{2}$$

Nevertheless, the equivalence does not holds for non-additive measures, and we obtain a different (partial) characterization of $\nu(.|E)$ for non-additive measures. Relation (2) expresses the fact that $I(1_A/1_E)$ nullifies the value of "betting on A" (1_A). Hence it corresponds to the maximal price that the DM is ready to put in 1_A. This price then may take two values depending on E or E^c.

Proposition 2. *The solution of relation (2) is given by the FB rule when 1_A and 1_E are comonotonic or antimonotonic.*

As for relation (1), the general case where 1_A and 1_E are not comonotonic neither nor antimonotonic random variables cannot be solved by relation (2). But contrarily to relation (1), this one cannot be hold for more general random variables. It does not hold for more structured random variables, i.e., for any X such that the range of X contains more than three outcomes. Therefore, the FB rule cannot resolved, in general, equations of the form $I[X - I(X/1_E)] = 0$, even for comonotonic or antimonotonic X and 1_E.

Finally, an other additively equivalent relation between 1_A and $I(1_A/1_E)$ is

$$I[I(1_A/1_E) - 1_A)] = 0 \tag{3}$$

Proposition 3. *The solution of relation (3) is given by the FB rule applied to $\bar{\nu}(.)$ when 1_A and 1_E are comonotonic or antimonotonic.*

Again, the general case cannot be resolved hence it can be applied only when the valued random variable 1_A and the information function 1_E are comonotonic. Furthermore, as for the previous one, the equality does not hold true for more general functions.

In order to generalize our approach of conditioning, it may be relevant to consider only the restriction of any X to the conditioning event E:

$$I(X1_E) = I[I(X/1_E)1_E]$$

Firstly, note that relation (1) becomes:

$$I[1_A 1_E] = I[I(1_A/1_E)1_E] \tag{4}$$

Proposition 4. *The solution of relation (4) is given by the NB rule.*

Secondly, relation (2) is now expressed as:

$$I\left[[1_A - I(1_A/1_E)]1_E\right] = 0 \tag{5}$$

Proposition 5. *The solution of relation (5) is given by the FB rule.*

Proof See Denneberg (1994), proposition 2.2. □

Finally, relation (3) may be rewritten as:

$$I\left[[I(1_A/1_E) - 1_A]1_E\right] = 0 \tag{6}$$

Proposition 6. *The solution of relation (6) is given by the FB rule applied to $\bar{\nu}(.)$.*

Contrarily to the three first relations, conditional capacities obtained here can always be defined. Proposition 6 shows that relation (4) holds if and only if $\nu(.|E)$ is given by the NB rule and proposition 4 shows that relation (5) holds if and only if $\nu(.|E)$ is given by the FB rule. Nevertheless, it does not show that these relations are satisfied for more general random variables, i.e. random variables that are not necessarily characteristic functions. Indeed, the relation

$$I(X1_E) = I[I(X/1_E)1_E]$$

is satisfied by the NB rule only when $X \geq 0$. Further, relations

$$I[[X - I(X/1_E)]1_E] = 0$$
$$I[[I(X/1_E) - X]1_E] = 0$$

are satisfied by the FB rule applied to $\nu(.)$ and $\bar{\nu}(.)$, respectively, only when the range of X contains less than four distinct outcomes.

In order to generalize our approach of conditioning to *any* random variable X, let us define $\nu_E(.)$ as the restriction of $\nu(.)$ to $\sigma(E)$, that is the sigma-algebra generated by E:

$$\forall A \in \Sigma,\ \nu_E(A) := \nu(A \cap E)$$

Then $I_E(.)$ denotes the Choquet integral w.r.t. $\nu_E(.)$, that is, for $X \equiv (x_i, A_i)_{i=1}^n$ such that $x_1 \leq \ldots \leq x_n$,

$$I_E(X) = \sum_{i=1}^{n} x_i \cdot [\nu((A_i \cup \ldots \cup A_n) \cap E) - \nu((A_{i+1} \cup \ldots \cup A_n) \cap E)] + x_n \cdot \nu(A_n \cap E)$$

Further, In the non-additive case, $I_E(X) \neq I(X1_E)$ in general. Nevertheless, one have $I_E(X) = I_E(X1_E)$.

Theorem 1. *Assume that assumption 1 holds. The following statements are equivalent:*

(i) For any $X \in \mathscr{A}$ and any $E \in \Sigma$, $I_E(X) = I_E[I(X/1_E)]$;

(ii) For any $X \in \mathscr{A}$ and any $E \in \Sigma$, $I_E[X - I(X/1_E)] = 0$;

(iii) For any $E \in \Sigma$ such that $\nu(E) > 0$, the conditional capacity $\nu(.|E)$ is well defined and it is given by the NB rule.

Notice that theorem 1 is a representation result, since relations $I_E(X) = I_E[I(X/1_E)]$ and $I_E[X - I(X/1_E)] = 0$ work for any $X \in \mathscr{A}$ and then the NB rule characterizes conditional expectations $I(X/1_E)$. A direct implication of theorem 1 is that the NB rule is the unique updating rule allowing to a Choquet DM to satisfy both statements (i) and (ii). As well as a Bayesian DM, a "naive" Choquet DM is able to resolve dynamic decision problems in a consistent way, as described by these relations. Contrarily to relation (1), statement (i) only allows the use of backward induction on $\sigma(E)$. Finally, the relation

$$I_E[I(X/1_E) - X] = 0$$

is resolved by the DS rule applied to $\bar{v}(.)$.

Proposition 7. *For any $X \in \mathscr{A}$ and any $E \in \Sigma$ such that $v(E) > 0$, we have $I_E[I(X/1_E) - X] = 0$ if and only if, for any $A \in \Sigma$,*

$$v(A|E) = \frac{\bar{v}((A \cap E) \cup E^c) - \bar{v}(E^c)}{1 - \bar{v}(E^c)} \tag{7}$$

4. Conclusion

We summarize the previous results in the table 1. Each result gives a characterization of the conditional capacity. FB/C and DS/C refer, respectively, to the FB rule and the DS rule applied to the conjugate capacity.

Table 1: Summary of the results

	Definition of conditional expectations	Updating rule	Condition
1	$I(X) = I[I(X/1_E)]$	NB or DS	X and 1_E co. or anti.
2	$I[X - I(X/1_E)] = 0$	FB	X and 1_E co. or anti.,$\lvert X(S)\rvert \leq 3$
3	$I[I(X/1_E) - X] = 0$	FB/C	X and 1_E co. or anti., $\lvert X(S)\rvert \leq 3$
4	$I(X1_E) = I[I(X/1_E)1_E]$	NB	$X \geq 0$
5	$I[[X - I(X/1_E)]1_E] = 0$	FB	$\lvert X(S)\rvert \leq 3$
6	$I[[I(X/1_E) - X]1_E] = 0$	FB/C	$\lvert X(S)\rvert \leq 3$
7	$I_E(X) = I_E[I(X/1_E)]$	NB	None
8	$I_E[X - I(X/1_E)] = 0$	NB	None
9	$I_E[I(X/1_E) - X] = 0$	DS/C	None

We have studied how updating rules for Choquet preferences could be characterized, depending of the link between unconditional and conditional expectations. We have firstly considered situations in which the valued random variable and the information function are comonotonic (relations 1, 2 and 3). Such situations are rather common, notably in insurance economics. Then we have tried to enlarge the approach to non-necessarily comonotonic functions. In relations 4, 5 and 6, only consequences yielded by the valued r.v. X on E are taking into account, whereas consequences outside of E take a null value. Finally, we have restrict the capacity to the conditioning event. Then relations 7, 8 and 9 may hold for any $X \in \mathscr{A}$. It allows to obtain a characterization, in terms of preference, of the NB rule. Restricting the capacity to conditioning events may be relevant in applications, where individual beliefs are often defined only on events directly involved by the decision problem.

Appendix: Mathematical appendix and proofs

Proof of proposition 1 By definition, $I(1_A) = \nu(A)$.

(i) If 1_A and 1_E are comonotonic, then $A \subset E$ or $E \subset A$. We consider the non-trivial case where $A \subset E$. Then,

$$I(1_A/1_E) = \begin{pmatrix} I(1_A/1_E = 1) & if \quad s \in E \\ 0 & if \quad s \in E^c \end{pmatrix}$$

hence, relation (1) yields $\nu(A) = I(1_A/1_E = 1)\nu(E)$ that is equivalent to the NB rule.

(ii) If 1_A and 1_E are antimonotonic, then $E^c \subset A$ or $A \subset E^c$. We consider the non-trivial case where $E^c \subset A$. Then,

$$I(1_A/1_E) = \begin{pmatrix} I(1_A/1_E = 1) & if \quad s \in E \\ 1 & if \quad s \in E^c \end{pmatrix}$$

hence, since $1_A = 1_{(A\cap E)\cup E^c}$, relation (1) yields $\nu((A \cap E) \cup E^c) = I(1_A/1_E = 1) + [1 - I(1_A/1_E = 1)]\nu(E^c)$ that is equivalent to the DS updating rule. □

Proof of proposition 2 As in proof of proposition 1, we consider two cases: $A \subset E$ and $E^c \subset A$. In both cases,

$$1_A - I(1_A/1_E) = \begin{pmatrix} -I(1_A/1_E = 1) & if \quad s \in A^c \cap E \\ 0 & if \quad s \in E^c \\ 1 - I(1_A/1_E = 1) & if \quad s \in A \cap E \end{pmatrix} \tag{8}$$

hence $I[1_A - I(1_A/1_E)] = I(1_A/1_E = 1)[\nu(A \cup E^c) - 1] + [1 - I(1_A/1_E = 1)]\nu(A \cap E)$. Under relation (2), this is equal to zero, and then it is resolved by the FB updating rule. □

Proof of proposition 3 With the same method as in the proof of proposition 2, we consider two cases: $A \subset E$ and $E^c \subset A$. In both cases,

$$I(1_A/1_E) - 1_A = \begin{pmatrix} I(1_A/1_E = 1) - 1 & if \quad s \in A \cap E \\ 0 & if \quad s \in E^c \\ I(1_A/1_E = 1) & if \quad s \in A^c \cap E \end{pmatrix} \tag{9}$$

hence

$$I[I(1_A/1_E) - 1_A] = I(1_A/1_E = 1)[1 - \nu(A^c \cup E^c) + \nu(A^c \cap E)] - 1 + \nu(A^c \cup E^c) \tag{10}$$

and then relation (3) implies $I(1_A/1_E = 1) = [1 - \nu(A^c \cup E^c)]/[1 - \nu(A^c \cup E^c) + \nu(A^c \cap E)]$ that corresponds to the FB rule applied to $\bar{\nu}(.)$. □

Proof of proposition 4 First observe that $I(1_A 1_E) = \nu(A \cap E)$. Further,

$$I(1_A/1_E) = \begin{pmatrix} I(1_A/1_E = 0) & if \quad s \in E^c \\ I(1_A/1_E = 1) & if \quad s \in E \end{pmatrix} \tag{11}$$

Then,

$$I[I(1_A/1_E)1_E] = I(1_A/1_E = 1)\nu(E) \tag{12}$$

and then relation (4) entails that $\nu(A \cap E) = I(1_A/1_E = 1)\nu(E)$ that is equivalent to the NB rule. □

Proof of proposition 6 We have:

$$I(1_A/1_E) - 1_A \equiv \begin{pmatrix} I(1_A/1_E=1)-1 & if & s\in A\cap E \\ I(1_A/1_E=0)-1 & if & s\in A\cap E^c \\ I(1_A/1_E=1) & if & s\in A^c\cap E \\ I(1_A/1_E=0) & if & s\in A^c\cap E^c \end{pmatrix} \tag{13}$$

Then,

$$I\left[[I(1_A/1_E)-1_A]1_E\right] = I(1_A/1_E=1)[1+\nu(A^c\cap E)-\nu(A^c\cup E^c)]-1+\nu(A^c\cup E^c)$$

that is equal to 0 if and only if $\nu_E(A)$ is given by the FB updating rule applied to $\bar{\nu}(.)$. □

The two following properties of Choquet integrals are needed for proofs of theorem 1 and proposition 7:

Property 1 (Comonotonic additivity)**.** *For all $X,Y\in\mathscr{A}$ and $E\in\Sigma$, if X and Y are comonotonic on E, that is $[X(s)-X(s')][Y(s)-Y(s')]\geq 0$ for all $s,s'\in E$, then $I_E(X)+I_E(Y)=I_E(X+Y)$.*

Comonotonic additivity means that a Choquet DM behaves like a Bayesian one when dealing with comonotonic random variables. This is commonly stated with $\nu(.)$ instead of $\nu_E(.)$, and then it requires X and Y be comonotonic on S. Nevertheless, this property is trivially satisfied for $\nu_E(.)$ and we omit the proof.

Proof of theorem 1 We first prove that statements (i) and (ii) are equivalent. Statement (i) holds if and only if

$$I_E(X) = I(X/1_E=1)\nu(E), \tag{14}$$

that holds if $I(X/1_E=1)\leq I(X/1_E=0)$ or if $I(X/1_E=1)\geq I(X/1_E=0)$. Eq. (14) holds if and only if

$$I_E\left(\frac{X}{\nu(E)}\right) = I(X/1_E=1) \tag{15}$$

hence we have proved that (i) if and only if (iii). Now we prove that statements (ii) and (iii) are equivalent. Statement (ii) holds if and only if

$$I_E[X-I(X/1_E)] = 0 \tag{16}$$

that is equivalent to

$$I_E\begin{pmatrix} X-I(X/1_E=1) & if & s\in E \\ X-I(X/1_E=0) & if & s\in E^c \end{pmatrix} = 0 \tag{17}$$

Since the random variable $-I(X/1_E)$ yields the outcome $-I(X/1_E=1)$ for all $s\in E$, it is comonotonic with the r.v. X on E hence, by comonotonic additivity, eq. (17) becomes

$$I_E(X) = I_E[I(X/1_E=1)] \tag{18}$$

that is equivalent to $I_E(X)=I_E[I(X/1_E)]$ and then we have proved that (ii) if and only if (iii). □

We denote by $\overline{\nu_E}(.)$ the conjugate capacity of $\nu_E(.)$ such that for all $A\in\Sigma$, $\overline{\nu_E}(A)=\nu(E)-\nu(A^c\cap E)$. Hence we denote by $\overline{I_E}(.)$ the Choquet integral w.r.t. $\overline{\nu_E}(.)$. The second property, proposed for instance by Denneberg (1994), is purely technical.

Property 2 (Asymmetry)**.** *For all $X\in\mathscr{A}$ and $E\in\Sigma$, $I_E(-X)=-\overline{I_E}(X)$*

Proof of proposition 7 We have:

$$I_E[I(X/1_E) - X] = 0 \tag{19}$$

that is equivalent to

$$I_E \begin{pmatrix} I(X/1_E = 1) - X & if & s \in E \\ I(X/1_E = 0) - X & if & s \in E^c \end{pmatrix} = 0 \tag{20}$$

Since the random variable $I(X/1_E)$ yields the outcome $I(X/1_E = 1)$ for all $s \in E$, it is comonotonic with the r.v. $-X$ on E hence, by comonotonic additivity, eq. (20) becomes

$$I_E[I(X/1_E = 1)] + I_E(-X) = 0 \tag{21}$$

if and only if, by asymmetry, $I(X/1_E = 1)\nu(E) = \overline{I_E}(X)$, that is equivalent to

$$I(X/1_E = 1) = \overline{I_E}\left(\frac{X}{\nu(E)}\right) \tag{22}$$

Finally, note that $\overline{\nu_E}(.)/\nu(E)$ is equal to $[\bar{\nu}(.\cup E^c) - \bar{\nu}(E^c)]/[1 - \bar{\nu}(E^c)]$ and thus it is equivalent to apply the DS rule to $\bar{\nu}(.)$. □

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