The bias in a standard measure of herding

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Abstract
We address the Lakonishok, Shleifer and Vishny (LSV) herding measure. Frey, Herbst and Walter (FHW) have shown by empirical simulations that LSV is biased. Using a theoretical model we provide a formal explanation of this bias, and show that a corrected herding measure depends on some unobservable parameters. This suggests that assessing herding intensity with this kind a more difficult task than considered up to now in the empirical literature.
1. Introduction

Because of their potential weight on market transactions, institutional investors (pension funds, mutual funds, banks, insurance companies ...) have received particular attention in the financial literature. More specifically, fund managers can easily observe each other and their compensation contracts, reputation or career concern provide strong incentives to mimic or herd. A large theoretical and empirical literature has thus focused on the herding propensity of institutional investors.

Lakonishok, Shleifer and Vishny (1992) (hereafter LSV) proposed an indicator to empirically assess herding among institutional investors. The LSV indicator uses portfolio data to measure herding as an excessive concentration of transactions of a given group of investors on the same side of the market. LSV defines herding as the excess proportion of them buying (selling) a given stock in a given quarter. This excess is computed referring to the normal proportion of buyers (sellers) of all market stocks between fund managers. The LSV measure relies on portfolio data and is easy to implement. It also allows refinement in the analysis of institutional herding, for particular subgroups of investors or stocks. For these reasons, it has been widely used in the empirical literature dedicated to herding by institutional investors, for which portfolio data are easily available (Lakonishok et al. 1992, Grinblatt, Titman and Wermers 1995, Oehler 1998, Wermers 1999, Oehler and Chao 2000, Borensztein and Gelos 2003, Voronkova and Bohl 2005, Wylie 2005, Walter and Weber 2006, Lobao and Serra 2007, Do, Tan and Westerholm 2008, Puckett and Yan 2008 and Boyd, Buyukahin, Harris, and Haigh 2009).

As far as we know, there exist only a very few papers that criticize LSV for its lack of internal consistency. Frey, Herbst and Walter (2007) (FHW hereafter) have shown by Monte Carlo simulations that the LSV measure is accurate only if there is no herding, and is biased downward otherwise. The goal of this paper is to provide an explanation of this bias. Using a theoretical approach, we show that LSV bias is positively linked with the level of herding. We also show that a “correct” measure of herding depends on unobservable parameters. This suggests that assessing herding intensity is a more difficult task than considered up to now in the empirical literature.

This paper is organized as follows. Section 2 presents a simple descriptive model of herding which is used to analyze the properties of herding measures. In section 3, we present the LSV
measure, point out its bias, propose a correction and comment the bias properties. Section 4 concludes.

2. A simple descriptive model of herding structure

To study the properties of herding indicators, we first need to define the herding configuration that can be expected in a standard equity market. Most papers (for example FHW in their simulations\(^4\)) consider what we will call integral and symmetrical herding: all stocks are subject to the same level of herding with equal probabilities to be excessively bought or sold.

In the real world, during a given period, some stocks may not be subject to herding, while some sub-groups of stocks are potentially more subject to buy or sell herding than others. For example herding may be more likely to happen and possibly higher for stocks that are more difficult to evaluate because investors have less information about them. This is in accordance with a large strand of literature findings that small capitalization stocks have higher level of herding\(^5\).

To illustrate the bias of standard herding indicator, we construct a very simplified descriptive model: we consider only three groups of stocks with different herding level.

First, the stock \(i\) has a probability \(\pi_{o,i}\) to be bought in the same proportion than the market (hence, in this case, there will be no herding). In other words, we allow a situation that we can call “partial herding”, in which some stocks are subject to herding while others are not.

Second, the probability to be excessively bought relatively to the market (denoted \(\pi_{b,i}\)) is not necessary equal to the probability to be excessively sold (denoted \(\pi_{s,i}\)). These probabilities can differ from period to period. One can imagine that the proportion of stocks that are excessively sold relative to the whole market can be sensitive to the state of financial markets or to the economic situation. Hence, the level of herding can be different on both sides of the market: we denote \(h^b_t\) the level of buy-side herding and \(h^s_t\) the level of sell-side herding.

Moreover, denote by \(b_{i,t}\) the observed number of buy transactions and \(n_{i,t}\) the total number of transactions in stock \(i\) during a period \(t\), then \(b_{i,t}\) follows a binomial distribution with parameters \(n_{i,t}\) and \(p_{i,t}\) where \(p_{i,t}\) is the probability that the stock \(i\) is bought in period \(t\) by an active fund manager,

\[
\begin{align*}
    p_{i,t} &= p_{i} + h^b_t \quad \text{(buy-side herding)} \\
    p_{i,t} &= p_{i} - h^s_t \quad \text{(sell-side herding)} \\
    p_{i,t} &= p_{i} \quad \text{(no herding)}
\end{align*}
\]

(1)

The mean level of herding is defined as the weighted sum of the herding levels on each side of the market:

\[
h_t = \pi_{b,i} h^b_t + \pi_{s,i} h^s_t
\]

In summary, in this model, three kinds of states (or realizations) can be observed for each stock in period \(t\): the no herding state (its realizations are called hereafter “no herding stocks”) the buy-herding state (the “buy-side stocks”) and the sell-herding state (the”sell-side stocks”) each category of stocks having a proportion defined by the probability of each state.

3. The LSV herding measure: description, bias and correction

3.1. The LSV indicator: definition and assessment under no herding

\(^4\) In their theoretical model, FHW allow each stock to have an individual level of herding during a given period.

\(^5\) See Wermers (1999) for example.
The herding measure of LSV is defined as:

$$HLSV_{i,t} = LSV1_{i,t} - AF_{i,t} = \frac{b_{i,t}}{n_{i,t}} - p_i - AF_{i,t}$$

where $\frac{b_{i,t}}{n_{i,t}}$ is the observed proportion of buy transactions for stock $i$ in $t$. Thus $LSV1_{i,t}$ measures the absolute gap between this proportion and the expected proportion in the no-herding case. As the estimated herding in a given stock group is the mean of $HLSV_{i,t}$ in this group, the absolute value in the first term avoids a sign compensation between buy and sell side herding. $AF_{i,t}$ is an adjustment factor. As explained below, it implies that in case of no-herding, HLSV is null.

As $b_{i,t}$ follows a binomial law, the realized proportion of buying transactions for stock $i$ in period $t$ is $\frac{b_{i,t}}{n_{i,t}} = p_{i,t} + \varepsilon_{i,t}$ where $\varepsilon_{i,t}$ is an independent error term with a zero mean and a variance equal to $p_{i,t}(1-p_{i,t})/n_{i,t}$. In case of no-herding, as $p_{i,t} = p_t$ the indicator can be rewritten as: $HLSV_{i,t} = |\varepsilon_{i,t}| - AF_{i,t} = |\varepsilon_{i,t}| - E[\varepsilon_{i,t}]$

In the no herding case even if $\varepsilon_{i,t}$ is centered, its absolute value is not, and even if herding is null, $LSV1$ is always positive. In fact, the adjustment factor $AF_{i,t}$ is the expected value of $LSV1$ in case of no herding.

Therefore, as in this case $E(HLSV_{i,t})=0$, the LSV measure is unbiased.

Finally, given the law of $b_{i,t}$, the adjustment factor is given by:

$$AF_{i,t} = \sum_{k=0}^{n_{i,t}} \text{proba}(b_{i,t} = k) \left[ \frac{k}{n_{i,t}} - p_i \right] = \sum_{k=0}^{n_{i,t}} \left( \frac{n_{i,t}}{k} \right) p_i^k (1-p_i)^{n_{i,t}-k} \left[ \frac{k}{n_{i,t}} - p_i \right]$$

As illustrated by FHW using Monte Carlo simulations, the HLSV indicator is unbiased under the null hypothesis of no herding. But FHW also show empirically that in any other configuration the measure is biased. The aim of the following section is to theoretically explain this point.

### 3.2. A general expression of adjustment factor

As explained above, the adjustment factor is required because of the absolute value in LSV1 which is designed to avoid sign compensation between buy-side and sell-side herding. Hereafter, we will show that while for each kind of stocks (buy and sell herding, no herding) $\frac{b_{i,t}}{n_{i,t}} - p_i$ is an unbiased estimator of the herding level, its absolute value LSV1 is not: its expectation is higher than the herding level. As the spread between the expectation of the absolute value of any random variable and its expectation $m$, decreases with the absolute value of $m$, the adjustment factor required to obtain an unbiased estimator, should decrease when the herding is increasing. But, as the LSV adjustment factor does not depend on the herding level (see relation (3)), hence remains constant whatever the herding intensity, the LSV herding measure is consequently biased.

The aim of this section is to give a more encompassing expression of adjustment factor. We first have to construct three adjustment terms, corresponding to the three possible configurations: buy-side herding, sell-side herding and no herding.

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6 Notice that the adjustment factor has to be computed at the stock level because it depends on the number of transactions of the considered stock during the period.
3.2.1. Buy side herding

Let us denote \( X = \frac{b_{it}}{n_{it}} - p_i . \)

Here we have \( X = p_{it} + \varepsilon_{it} - p_i = h_{it}^b + \varepsilon_{it} . \)

\( E_b(X) \) is the conditional expectation of \( X \) given the herding is in the buy side.

Since \( \varepsilon_{it} \) is centered, \( E_b(X) = h_{it}^b \) and then \( X \) is an unbiased estimator of \( h_{it}^b \). But the absolute value of \( X \), LSV1 is not.

Let us state the link between \( E_b(X) \), the herding level in the buy side group, and \( E_b(|X|) \) to obtain the necessary adjustment factor and to get an unbiased estimator of \( h_{it}^b \).

From

\[
E_b(|X|) = E_b(X) + 2E_b(|X|/X<0).probab(X<0) = h_{it}^b + 2E_b(|X|/X<0).probab(X<0)
\]  

(4)

3.2.2. Sell-side herding

Here we just have to remember that \( X = -h_{it}^s + \varepsilon_{it} \) and then \( E_s(X) = -h_{it}^s \). Using the same approach as above, we can show that:

\[
E_s(|X|) = h_{it}^s + 2E_s(|X|/X>0).probas(X>0)
\]  

(5)

3.2.3. No herding

In this case, we have now \( X = \varepsilon_{it} \) and \( E_0(X) = 0 \)

Again we have:

\[
E_0(|X|) = h_{it}^0
\]  

(6)

3.2.4. Expression of the adjustment factor

As LSV1 is the absolute value of \( X \), we can compute its expectation from relations (4), (5) and (6), using respective weights \( \pi_{b,t} \), \( \pi_{s,t} \) and \( \pi_{0,t} \):

\[
E(LSV1) = E(|X|) = \pi_{b,t}E_b(|X|) + \pi_{s,t}E_s(|X|) + \pi_{0,t}E_0(|X|)
\]  

where \( h_{it} = \pi_{b,t}h_{it}^b + \pi_{s,t}h_{it}^s \)

It easily comes that:

\[
h_{it} = E(LSV1) - AFC_{i,t}
\]

where

\[
AFC_{i,t} = \pi_{0,t}AF0_{i,t} + 2\pi_{b,t}AFB_{i,t} + 2\pi_{s,t}AFS_{i,t}
\]  

(7)

In which \( AF0_{i,t} \), \( AFB_{i,t} \) and \( AF0_{i,t} \) are the adjustment terms associated with no-herding, buy-side herding and sell-side herding respectively. Replacing \( X \) by its value, the expressions of these terms are respectively:

\[
AFB_{i,t} = E_b\left( \frac{b_{it}}{n_{it}} - p_i \right) \left( \frac{b_{it}^0}{n_{it}} - p_{i<0} \right).probab\left( \frac{b_{it}}{n_{it}} - p_{i<0} \right)
\]

\[
AFS_{i,t} = E_s\left( \frac{b_{it}}{n_{it}} - p_i \right) \left( \frac{b_{it}^0}{n_{it}} - p_{i>0} \right).probas\left( \frac{b_{it}}{n_{it}} - p_{i>0} \right)
\]

\[
AF0_{i,t} = E_0\left( \frac{b_{it}}{n_{it}} - p_i \right)
\]
In case of buy-side herding, the terms to subtract are those where \( \frac{b_{t,j}}{n_{t,j}} - p_t \) is negative i.e. those for which \( b_{t,j} < n_{t,j}p_t \). As \( b_{t,j} \) follows a binomial law the theoretical expression of the adjustment term in this case:

\[
AF_{B,t} = \sum_{k=1}^{n_{t,j}} \text{proba}_b(b_{t,j} = k) \left( \frac{k}{n_{t,j}} - p_t \right) = \sum_{k=1}^{n_{t,j}} \binom{n_{t,j}}{k} \left( p_t + h_t \right)^k \left( 1 - (p_t - h_t) \right)^{n_{t,j} - k} \left( \frac{k}{n_{t,j}} - p_t \right)
\]  

(8)

Concerning the sell-side herding, as the terms to subtract are those for which \( b_{t,j} > n_{t,j}p_t \) we have:

\[
AF_{S,t} = \sum_{k=n_{t,j}p_t}^{n_{t,j}} \text{proba}_s(b_{t,j} = k) \left( \frac{k}{n_{t,j}} - p_t \right) = \sum_{k=n_{t,j}p_t}^{n_{t,j}} \binom{n_{t,j}}{k} \left( p_t - h_t \right)^k \left( 1 - (p_t - h_t) \right)^{n_{t,j} - k} \left( \frac{k}{n_{t,j}} - p_t \right)
\]  

(9)

In the no-herding case we find the « traditional » adjustment term (see relation (5))

\[
AF_{0,t} = \sum_{k=1}^{n_{t,j}} \binom{n_{t,j}}{k} p^t_t (1 - p_t)^{n_{t,j} - k} \left( \frac{k}{n_{t,j}} - p_t \right)
\]  

(10)

To conclude, we obtain an unbiased measure of herding intensity with the “corrected” LSV measure:

\[
HLSV_{C,t} = LSV_{1,t} - AFC_{t} = \left( \frac{b_{t,j}}{n_{t,j}} - p_t \right) - AFC_{t}
\]  

(11)

where \( AFC_{t} \) is defined by relations (7), (8), (9) and (10).

Lastly, it is worth noting that the corrected measure requires, even in a very simple model, a prior estimation of, not only the probability vector \( \{ \pi_{b_{t,j}}, \pi_{s_{t,j}}, \pi_{s_{t,j}} \} \) but also of the herding level. It means that we have to know the herding level to estimate it. We will go back over this point on conclusion. However, the corrected expression of the adjustment factor can shed light on the properties of LSV bias.

### 3.3. Remarks on the bias

The theoretical value of the bias of LSV can be computed by the difference between AFC and AF. It’s easy to show that all the results obtained by simulation by FHW are confirmed:

- First, the LSV measure is very well suited to test the null hypothesis of no herding.
- Second, since herding exists, LSV underestimates systematically it, and the bias is increasing with the herding level. This property is simply explained by the fact that the probability that a buy-side (respectively a sell-side) herding stock have a negative (respectively a positive) value for \( \frac{b_{t,j}}{n_{t,j}} - p_t \) decreases when herding is increasing. Then, given \( n_{t,j}, p_{t,j}, \pi_{b_{j}}, \pi_{s_{j}} \), \( AFC_{t} \) decreases with \( h_t \) and as the adjustment factor of LSV remains constant, the bias rises.
- Third, the LSV bias decreases (but remains positive) for higher numbers of transactions. As the variance of \( \frac{b_{t,j}}{n_{t,j}} \) is decreasing with \( n_{t,j} \), the required adjustment should decline when the number of transactions on a stock grows, which is true for both adjustment factors. But again, when herding grows, the adjustment should decrease. Yet, this is not true for HLSV.

Considering cases where herding can be partial and/or asymmetrical, we show two more properties of the LSV bias:

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7 More details about these computations can be found in Bellando (2010).
- The LSV bias decreases with the weight of no herding stocks. This result is easy to explain as LSV is designed to the no herding case. When the probability of no herding grows, the adjustment factor required (AFC) comes closer to the LSV adjustment factor AF and both converge as $\pi_{o,j}$ approaches one.

- The LSV bias reduces also with asymmetry. Technically, the LSV1 expectation for a given herding level increases with asymmetry and as the AF of LSV does not change, the downward bias is reduced.

4. Concluding remarks

As illustrated in FHW, the well-known LSV measure is only relevant in the case of no herding\(^8\). One of the main contributions of this paper is to provide a theoretical rationale for this property. We use a simple descriptive model of herding to derive the exact value of adjustment term required to properly estimate herding. But the correction we propose is not very tractable. As it requires a prior knowledge of the herding level and the probabilities to belong to each category of stocks (sell-side, buy-side, no herding), it suggests that there is much work left to be done concerning the empirical evaluation of herding behavior and offer a stimulating research agenda.

References


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\(^8\) Bellando (2011) shows that the new indicator proposed by FHW in the same paper is accurate only under strong assumptions. She proposes a corrected version of their indicator and observes that even if the true herding value belongs to an interval defined by LSV (lower bound) and FHW (upper bound), the spread between both measures may be very wide.


