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Preemption and rent equalization in the adoption of new technology: comment

Min-Hung Tsay
Department of International Business, National Taiwan University, Taipei 106, Taiwan.

Abstract
In this comment, we show that the existence of the preemption equilibrium in Fudenberg and Tirole (Review of Economics Studies, vol. 52, PP. 383-401, 1985)'s continuous-time games of timing is not guaranteed under their assumptions.
1. Comment

We direct this comment to the paper by Fudenberg and Tirole (1985). The authors develop a new framework for modeling continuous-time games of timing, and show that in a duopoly, the threat of preemption in the adoption of new technology will equalize two firms’ rents. The results of preemption and rent equalization have been adopted by a number of papers: Fudenberg and Tirole (1987), Tirole (1988), Choi (1996), Weeds (2002), Adner and Zemsky (2005), Ostrovsky and Schwarz (2005), Honoré and Paula (2010), and Shen and Villas-Boas (2010). However, by constructing a counterexample, we show that the existence of the *preemption equilibrium* is not guaranteed under Assumptions 1 and 2 in their paper.

We adopt Fudenberg and Tirole (1985)’s notation throughout our comments, and consider the following duopoly case of the model. Two identical firms, denoted by firm 1 and firm 2, exist in the industry. At time 0, a cost reducing innovation is announced. For $i \in \{1, 2\}$, let $\pi_0(m)$ be the net cash flow of firm $i$ when $m$ firm(s) have adopted the innovation, but firm $i$ has not. Let $\pi_1(m)$ be firm $i$’s net cash flow when $m$ firm(s) including $i$ have adopted. $T_i$ denotes firm $i$’s adoption date; and $c(t)$ is the present value of the cost of implementing the innovation on line by time $t$. Without loss of generality, suppose that firm $i$ is the $i$-th to adopt, then we can represent firm $i$’s payoff, $V^i(T_i, T_j)$, as follows.

\[
V^i(T_i, T_j) = \begin{cases} 
\int_0^{T_i} \pi_0(0)e^{-rt}dt + \int_{T_i}^{T_j} \pi_1(1)e^{-rt}dt + \int_{T_j}^{\infty} \pi_1(2)e^{-rt}dt - c(T_i) & \text{if } T_i \leq T_j; \\
\int_0^{T_j} \pi_0(0)e^{-rt}dt + \int_{T_j}^{T_i} \pi_0(1)e^{-rt}dt + \int_{T_i}^{\infty} \pi_1(2)e^{-rt}dt - c(T_i) & \text{if } T_i > T_j, 
\end{cases}
\]  

where $j \in \{1, 2\}$ and $j \neq i$, and $r$ is the constant common interest rate.

We next introduce the duopoly version of the two assumptions they impose on the firm’s net cash flow and adoption cost of the innovation.

**Assumption 1.**

(i) $\pi_0(0) \geq \pi_0(1) > 0$ and $\pi_1(1) \geq \pi_1(2) > 0$, and

(ii) $\pi_1(1) - \pi_0(0) > \pi_1(2) - \pi_0(1)$.

**Assumption 2.**

(i) $\pi_1(1) - \pi_0(1) \leq -c'(0)$.

(ii) $\inf_{t \geq 0} \{c(t)e^{rt}\} < \left[\pi_1(2) - \pi_0(1)\right]/r$.
Moreover, we define $T$ the other firm adopts at time $2$:

$$[\pi_1(m) - \pi_0(m - 1)] e^{-rT_m^*} + c'(T_m^*) = 0.$$ 

Moreover, we define $^1$

$$L(t) = \begin{cases} V(t, T_2^*) & \text{if } t < T_2^*; \\ V(t, t) & \text{if } t \geq T_2^*, \end{cases}$$

and

$$F(t) = \begin{cases} V(T_2^*, t) & \text{if } t < T_2^*; \\ V(t, t) & \text{if } t \geq T_2^*, \end{cases}$$

to be the leader’s and the follower’s payoffs, respectively, when the former preempts the latter at time $t$, and let $M(t) = V(t, t)$ be the payoff of both firms when they adopt together at time $t$. Finally, let $\hat{T}_2 = \text{argmax}_{t \in \mathbb{R}_+} M(t)$.

Under Assumptions 1 and 2, the authors offer a necessary condition for the existence of the $(T_1^*, T_2^*)$-diffusion equilibrium $^2$ (i.e., there exists a unique preemption time $T_1$ in $(0, T_1^*)$ such that $L(T_1) = F(T_1)$) $^3$, and then show in their Proposition 2 that the $(T_1, T_2^*)$-diffusion equilibrium always exists. $^4$ We show that under Assumptions 1 and 2, the existence of $T_1$ is not guaranteed by constructing a numerical example.

**The counterexample.** Let $\theta, r, k \in \mathbb{R}_{++}$ be such that $\theta + r - k > 0$. Let the related parameters be defined as follows so that they satisfy Assumptions 1 and 2: $c(t) = e^{-(\theta+r)t}$, $\pi_0(0) = 4k/3$, $\pi_0(1) = k$, $\pi_1(1) = \theta + r + k$, and $\pi_1(0) = 0$.

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$^1$The function $V(t, t')$ is defined as the right-hand side of (1) with replacing $T_i$ and $T_j$ by $t$ and $t'$, respectively.

$^2$The $(T_1, T_2^*)$-diffusion equilibrium exhibits that one of the two firms adopts at $T_1$ and the other firm adopts at $T_2^*$ with probability one.

$^3$To show this, they first prove that $L(t) - F(t)$ is strictly quasi-concave, and then they improperly use Assumption 2(i) to claim that $L(0) < F(0)$. On the other hand, they also show that $L(T_2^*) = F(T_2^*)$ (from the definitions of $L(t)$ and $F(t)$), and $L(T_1^*) > F(T_1^*)$ (from their proposition 1). Therefore, these imply that there must exist a unique $T_1 \in (0, T_1^*)$ such that $L(T_1) = F(T_1^*)$ since $L(t)$ and $F(t)$ are continuous functions of $t$.

$^4$Two cases are analyzed in their Proposition 2. If $L(T_1^*) > M(\hat{T}_2)$, the authors show that the $(T_1, T_2^*)$-diffusion equilibrium is the unique equilibrium of the timing game; otherwise, diffusion equilibrium and joint-adoptions equilibrium coexist.
\[ \pi_1(2) = \theta + r + k/3. \] It is easy to see that Assumptions 1 and 2 are satisfied under this setting.

We now show that \( L(0) > F(0) \). Recall that \( \forall t \in [0, T_2^*] \),

\begin{equation}
L(t) = \int_0^t \pi_0(0)e^{-rt}ds + \int_t^{T_2^*} \pi_1(1)e^{-rs}ds + \int_{T_2^*}^{\infty} \pi_1(2)e^{-rs}ds - c(t); \quad (2)
\end{equation}

and

\begin{equation}
F(t) = \int_0^t \pi_0(0)e^{-rt}ds + \int_t^{T_2^*} \pi_0(1)e^{-rs}ds + \int_{T_2^*}^{\infty} \pi_1(2)e^{-rs}ds - c(T_2^*). \quad (3)
\end{equation}

Thus,

\[
L(0) - F(0) = \int_0^{T_2^*} [\pi_1(1) - \pi_0(1)]e^{-rs}ds - c(0) + c(T_2^*)
\]

\[
= \theta e^{-rT_2^*} \int_0^{T_2^*} [e^{rt} - e^{-\theta t}]dt > 0,
\]

where the inequality follows from \( T_2^* > 0 \).\(^5\) Finally, since \( L(0) > F(0) \) and the strict quasi-concavity of \( L(t) - F(t) \), \( L(T_1^*) > F(T_1^*) \) together with \( L(T_2^*) = F(T_2^*) \) implies the non-existence of \( T_1^* \).\(^6\)

We now use two numerical results to show that the counterexample includes two cases in Proposition 2 of Fudenberg and Tirole (1985). Let \((\theta, r, k) =

\(^5\)To see that \( T_2^* > 0 \) in this example, since \( T_2^* \) is defined to be the solution of the following equation:

\[
[\pi_1(2) - \pi_0(1)]e^{-rt} + c'(t) = 0,
\]

(i.e., \((\theta + r + 2k/3)e^{-rt} - (\theta + r)e^{-(\theta + r)t} = 0 \) in this example) by solving this equation, we obtain that

\[
T_2^* = -\frac{1}{\theta} \ln \left(1 - \frac{2k}{3(\theta + r)}\right) > 0.
\]

Similarly,

\[
T_1^* = -\frac{1}{\theta} \ln \left(1 - \frac{k}{3(\theta + r)}\right); \quad \text{and}
\]

\[
T_2 = -\frac{1}{\theta} \ln \left(1 - \frac{k}{(\theta + r)}\right).
\]

Hence, we conclude that \( T_2 > T_2^* > T_1^* > 0 \) in this example.

\(^6\)To see this, suppose, by contradiction, that there exists \( T_1 \in (0, T_1^*) \) such that \( L(T_1) = F(T_1) \). Since \( L(T_1^*) = F(T_1^*) \), there must exist two disjoint intervals in \([0, T_1^*] \) such that the function \( L(t) - F(t) \) is decreasing w.r.t. \( t \) in the first one, and increasing w.r.t. in the other. Note that \( L(T_2^*) = F(T_2^*) \) implies that \( L(t) - F(t) \) must decrease in a subinterval of \([T_1^*, T_2^*] \), which is a contradiction of the strictly quasi-concavity of \( L(t) - F(t) \).
(2, 0.05, 1.5). It can be show that \( L(T_1^*) \leq M(\hat{T}_2) \) (i.e. Case B of Proposition 2 in Fudenberg and Tirole (1985).) Let \((\theta, r, k) = (1, 2.1, 2)\). It can be shown that \( L(T_1^*) > M(\hat{T}_2) \) (i.e. Case A of Proposition 2 in Fudenberg and Tirole (1985).)\(^7\) Figures 1 and 2 depict the dynamics of \( L(t) \), \( F(t) \) and \( M(t) \) in these two case, respectively. We can see, from the two figures, that \( L(0) > F(0) \). Thus the \((T_1, T_2^*)\)-diffusion equilibrium does not exist.

\(^7\)In fact, \( L(T_1^*) = 50.36 < 50.38 = M(\hat{T}_2) \) and \( L(T_2^*) = 1.3 > 1.28 = M(\hat{T}_2) \) in the first and second cases, respectively.

Figure 1: The Case of \( L(T_1^*) \leq M(\hat{T}_2) \)
Figure 2: The Case of $L(T_1^*) > M(\hat{T}_2)$
References


