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Contest with cooperative behavior: a note

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Abstract

The point of departure of this paper is that players in a contest may have mixed motives. On one hand, players have the interest of winning the contest and taking the prize. On the other hand, they could be better off taking part in a contest which implies some cooperative behaviour. This paper presents a contest model characterized by: (1) the existence of a second kind of effort here termed 'cooperative effort'; (2) an asymmetry in the evaluation of the stake; (3) a degree of responsiveness to cooperative efforts. By comparing a basic contest model with the above-mentioned model, it has been shown that players may be better off in a contest which involves some cooperative behaviour. As the asymmetry in the evaluation of the stake becomes larger and larger, even a smaller degree of responsiveness to the aggregate cooperative efforts would make players better off. Eventually, a contest involving also cooperative efforts is less balanced than a pure contest.

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1. Introduction

A contest is a strategic interaction in which players compete for a prize by making irreversible outlays. Contest models are grounded on Tullock (1980) and the most comprehensive on contest theory is Konrad (2009). The point of departure of this paper is that players in a contest may have mixed motives. On one hand, participants have the interest of winning the contest and taking the prize. On the other hand, they could be better off taking part in a contest which implies some cooperative behaviour. For example, players would be better off if the waste of resources used is less pronounced than the highest possible. This might be pursued whenever the players commit themselves to cooperate. In brief, it is possible to maintain that, albeit in a contest, players may behave both competitively and cooperatively. In particular, cooperative behaviour of players depends upon the behavioural environment to which they adapt so shaping their actions. That is, players are ecologically rational as in the definition provided by Smith (2008): "*[...] the behavior of an individual, a market, an institution, or other social system involving collectives of individuals is ecologically rational to the degree that it is adapted to the structure of its environment [...]*"[1]. With specific regard to contests, in the presence of a cooperative environment the competition may be prevented from becoming extremely harmful and even socially wasteful. It is worth noting that players may have an interest in cooperation even if they still retain a proper willingness to win the contest.

A fitting real-life example can be drawn from managerial competition within firms. In fact, in large firms the management may apply an internal competition between workers and managers in order to assign a promotion or to promote efficiency. In the presence of the harshest competition possible, players may put excessive efforts into the contest, thereby determining a detrimental and dissipative scenario for the firm. In such a case, a competitive structure reward scheme can be tempered by a cooperative structure in order to mitigate a possible pernicious impact of the contest. In fact, managers may have an interest in winning the contest for promotion, but they also may have the interest of avoiding unproductive and wasteful competition.

In sum, we can say that players in the contest may have a payoff function which encloses both competitive and cooperative efforts. In other words, the interaction is structured competitively and cooperatively. Evidently, this requires proper modelling. When an interaction is structured competitively there must be negative correlations between players' rewards. This is modelled in contest models by means of a Contest Success Function (henceforth CSF), which has been axiomatized by Skaperdas (1996) and Clark and Riis (1998). But when interactions are designed cooperatively there must be positive correlations between agents' rewards. In our context, cooperative efforts of players are assumed to generate a positive joint outcome which enters positively the interdependent utility functions of players. That is, outlays in cooperative efforts produce a public good, namely the cooperative environment. Eventually, there is a measure which captures to which extent the aggregate cooperative efforts of players translate into a positive contribution to final outcome. This is modelled following Lee and Kang (1998) who analyse collective contests in which aggregate efforts generate externalities to the participants.

Hence, formally a contest characterized by cooperative efforts would enclose two types of efforts, namely competitive and cooperative efforts. In contest theory the use of a two kinds of efforts is not a novelty. Baik and Shogren (1995) study a contest with spying. Konrad (2000) enriches a model of rent-seeking considering the interaction between two types of efforts: (i) the standard rent-seeking efforts; (ii) a sabotaging effort in order to reduce the effectiveness of other players' efforts. Haan and Schoonbeek (2003) model a contest

[1] Smith (2008), p. 36.

characterized by a bid and lobbying efforts. Epstein and Hefeker (2003), model a contest where players evaluate differently the stake and use a second instrument to create an advantage for the player with the higher stake. Caruso (2008) presents a model where players evaluate differently the stake and use a second instrument along with reciprocal concessions.

This short paper is structured as follows. In a first section, a basic model of contest with asymmetric evaluation of the stake is presented. In the following paragraph, a contest characterized by cooperative efforts is designed. Eventually, a comparison between the two types of contests is presented. Conclusions summarize the results.

2. The basic model

There are two risk-neutral players, indexed by $i = 1,2$. They have different evaluations of a contested stake denoted by $x_i \epsilon (0,\infty), i = 1,2$. Let me assume that player 1 has a higher evaluation than player 2, namely $x_1 > x_2$. Let $d\epsilon(0,1)$ denote the degree of asymmetry between the stakes of the two players, namely $d = x_2/x_1$. As $d \to 0$ the asymmetry in evaluation of the prize becomes larger and larger. Let $p_i, i = 1,2$ denote the probability of player i of winning the contest. The probabilities of winning for player 1 and player 2 are given respectively by:

$$p_1(g_1, g_2, s) = \frac{g_1}{g_1+sg_2} \quad (1)$$
$$p_2(g_1, g_2, s) = \frac{sg_2}{g_1+sg_2} \quad (2)$$

Where $g_1\epsilon(0,\infty)$ e $g_2\epsilon(0,\infty)$ denote the efforts exerted by player 1 and player 2 respectively with $\frac{\partial p_i}{\partial g_i} > 0, \frac{\partial p_i}{\partial g_j} < 0, i = 1,2, i \neq j$. The parameter $s\epsilon(0,\infty)$ indicates the effectiveness of abilities of player 2 against those of player 1. That is, following Rosen (1986) players are heterogeneous in abilities and they both know other's ability. Whenever $s\epsilon(0,1)$ player 2 is less effective with respect to player 1. In fact, $\partial p_1/\partial s < 0$ and $\partial p_2/\partial s > 0$, namely the probability of winning of player 1 is decreasing in s, whereas the probability of winning of player 2 is increasing in s. I assume convex cost for competitive efforts. The payoff functions are given by:

$$\pi_1 = p_1 x_1 - g_1^2; \quad (3)$$
$$\pi_2 = p_2 x_2 - g_2^2; \quad (4)$$

Players maximize (3) and (4) with respect to $g_i, i = 1,2$[2]. Using $d = x_2/x_1$ the Nash equilibrium choices can be written as follows:

$$g_1^* = \frac{\sqrt{2}\sqrt{s}}{2(1+sd^{\frac{1}{2}})} x_1^{1/2} d^{1/4} \quad (5)$$
$$g_2^* = \frac{\sqrt{2}\sqrt{s}}{2(1+sd^{\frac{1}{2}})} x_1^{1/2} d^{3/4} \quad (6)$$

In equilibrium, the level of total effort denoted by TG is given by:

[2]The F.O.C are:
$\left(g_2 s x_1/(g_1+g_2 s)^2\right) - 2g_1 = 0$ and $\left(g_1 s x_2/(g_1+g_2 s)^2\right) - 2g_2 = 0$

and the S.O.C. $-\frac{2g_2 s x_1}{(g_1+g_2 s)^3} - 2 < 0$ and $-\frac{2g_1 s^2 x_2}{(g_1+g_2 s)^3} - 2 < 0$ hold.

$$TG^* = \left(d^{1/2}+1\right)\frac{\sqrt{2}\sqrt{s}}{2(1+sd^{\frac{1}{2}})}x_1^{1/2}d^{1/4} \quad (7)$$

Total effort is increasing inx_1. In addition, TG^* is increasing in s if and only if $d^{1/2}s < 1$. This condition always holds for both $d\epsilon(0,1)$ and $s\epsilon(0,1)$. The winning probabilities for player 1 and player 2 are respectively:

$$p_1^*(g_1^*, g_2^*) = \frac{1}{1+sd^{1/2}} \quad (8)$$

$$p_2^*(g_1^*, g_2^*) = \frac{sd^{1/2}}{1+sd^{1/2}} \quad (9)$$

The contest balance (defined as the odds) is denoted by CB:

$$CB = \frac{p_1^*}{p_2^*} = \frac{1}{sd^{1/2}} \quad (10)$$

The contest balance is decreasing in both s and d. The payoffs accruing to player 1 and player 2 are respectively:

$$\pi_1^*(g_1^*, g_2^*) = \frac{(sd^{1/2}+2)}{2\left(sd^{1/2}+1\right)^2}x_1 \quad (11)$$

$$\pi_2^*(g_1^*, g_2^*) = \frac{s\left(2sd^{1/2}+1\right)}{2\left(sd^{1/2}+1\right)^2}x_1 d^{3/2} \quad (12)$$

Player 1 attains a higher payoff than player 2 if and only if $s < \frac{\left(d^2+14d+1\right)^{1/2}-d+1}{4d^{3/2}}$.

3. Contest and cooperation

Henceforth, another contest model involves a second kind of effort denoted by $z_i\ \epsilon(0,\infty), i = 1,2$ and labelled as *cooperative effort*. The positive contribution of aggregate cooperative efforts is captured by $q(z_1 + z_2)$ where $q\epsilon(0,\infty)$ proxies a degree of responsiveness to cooperative environment. As noted above, this borrows from Lee and Kang (1998) who analyse collective contests in which aggregate efforts generate externalities to the participants The probabilities of winning for player 1 and player 2 are denoted respectively by p_1^s and p_2^s and can be defined by:

$$p_1^s\left(g_1, g_2, z_1, z_{2,} s\right) = \frac{g_1 z_1}{z_1 g_1 + s z_2 g_2} \quad (13)$$

$$p_2^s\left(g_1, g_2, z_1, z_{2,} s\right) = \frac{s g_2 z_2}{z_1 g_1 + s z_2 g_2} \quad (14)$$

The multiplicative relation between competitive and cooperative efforts in the CSF implies that they are interdependent. That is, any change in the competitive efforts also induces a change in cooperative efforts and vice versa. In this kind of contest, the cooperative scheme cannot be disentangled from the competitive structure. There is no possibility of null cooperation. In other words, this kind of modelling captures the idea that players have an interest in cooperation even if they still retain a proper willingness to win the contest.

Let me assume that cooperative efforts exhibit linear cost. Eventually, the payoff functions for player 1 and player 2 are respectively:

$$\pi_1^s = p_1^s x_1 - g_1^2 - z_1 + q(z_1 + z_2) \quad (15)$$

$$\pi_2^s = p_2^s x_2 - g_2^2 - z_2 + q(z_1 + z_2) \quad (16)$$

Maximizing (15) and (16) with respect to both g_i and z_i for $i = 1,2$, the Nash equilibrium choices of competitive efforts are:

$$g_1^* = \frac{\sqrt{2}\sqrt{s}}{2(sd^{\frac{3}{2}}+1)} x_1^{1/2} d^{3/4} \quad (17)$$
$$g_2^* = \frac{\sqrt{2}\sqrt{s}}{2(sd^{\frac{3}{2}}+1)} x_1^{1/2} d^{5/4} \quad (18)$$

with $g_1^{s*} > g_2^{s*}$. In equilibrium, the optimal choices of cooperative efforts are:

$$z_1^{s*} = \frac{s}{(1-q)(sd^{3/2}+1)} x_1 d^{3/2} \quad (19)$$
$$z_2^{s*} = \frac{s}{(1-q)(sd^{3/2}+1)} x_1 d^{5/2} \quad (20)$$

The S.O.C. (see appendix) for an interior solution dictate $s > 1/(5d^{3/2})$. The aggregate contest efforts are:

$$TG^{s*} = (d^{1/2} + 1)\frac{2^{1/2}s^{1/2}}{2(sd^{3/2}+1)} x^{1/2} d^{3/4} \quad (21)$$

Aggregate contest efforts are decreasing in s only in the presence of particular combination of s and d, namely $\partial TG^{s*}/\partial s < 0 \Leftrightarrow s > 1/d^{3/2}$.The probabilities of winning for player 1 and player 2 are respectively:

$$p_1^{s*}(g_1^*, g_2^*, z_1^*, z_2^*) = 1/(sd^{3/2} + 1) \quad (22)$$
$$p_2^{s*}(g_1^*, g_2^*, z_1^*, z_2^*) = sd^{3/2}/(sd^{3/2} + 1) \quad (23)$$

The contest balance is:

$$CB^* = \frac{p_1^{s*}}{p_2^{s*}} = \frac{1}{sd^{3/2}} \quad (24)$$

Eventually, the expected payoffs accruing to the players are:

$$\pi_1^{s*} = \frac{(2qsd^{5/2} + sd^{3/2}(q-1) - 2(q-1))}{2(1-q)(sd^{3/2}+1)^2} x_1 \quad (25)$$
$$\pi_2^{s*} = \frac{sd^{3/2}\left(2sd^{\frac{5}{2}}(q-1) + d(1-q) - 2q\right)}{2(q-1)(sd^{3/2}+1)^2} x_1 \quad (26)$$

4. Contest Comparison

Hereafter a comparison between the two types of contests is presented. Player 1 would be better off in the second type of contest ($\pi_1^{s*} > \pi_1^*$) if $q > \frac{(d^3s^2+2sd^{5/2}+d^2s^2+D)}{(3d^3s^2+6sd^{5/2}+d^2(s^2+2)+D)}$ with $D = 4sd^{3/2} + 5d - 2sd^{1/2} - 3$. As $d \to 1$ the latter condition simplifies to $q > 1/2$, whilst as $d \to 0$ the condition no longer holds. Player 2 would be better off ($\pi_2^{s*} > \pi_2^*$) if and only if: $q(3d^3s^2 + S - d(2s^2 + 1) - 6d^{1/2}s - 3) < (3d^3s^2 + S - d - 2d^{1/2}s - 1)$ with

$S = 2sd^{5/2} - 5d^2s^2 - 4sd^{3/2}$. As $d \to 1$ the latter condition simplifies to $q > 1/2$. As $d \to 0$ the condition for player 2 simplifies to $q > 1/3$. That is, as the asymmetry in the evaluation of the stake becomes larger and larger, even a smaller degree of responsiveness to the joint cooperative outcome would make players better off. Put differently, in a contest any cooperative behaviour may be supported by the asymmetry in the evaluation of the stake. Evidently, total competitive efforts are unambiguously lower, ($TG^{*s} < TG^*$).

Eventually, consider the contest balance and assume a perfectly balanced contest as a reference point ($CB = 1$). The Euclidean distance between the computed measure of CB and the reference point can be used to consider to which extent a contest deviates from a perfect balanced position. The Euclidean distance from the reference point in the pure contest scenario is denoted by ED and it is given by:

$$ED = \sqrt{(CB-1)^2} = \frac{|sd^{1/2}-1|}{sd^{1/2}} \tag{29}$$

whereas for the contest it is denoted by ED^s:

$$ED^s = \sqrt{(CB^s-1)^2} = \frac{|sd^{3/2}-1|}{sd^{3/2}} \tag{30}$$

Since $s\epsilon(0,1)$ and $d\epsilon(0,1)$, it follows that $ED^s > ED$. Put in simpler words, a contest involving also cooperative efforts is less balanced than a pure contest.

5. Conclusions

This paper presented a contest model characterized by: (1) the existence of a second kind of effort here termed ‘cooperative effort’; (2) an asymmetry in the evaluation of the stake; (3) a degree of responsiveness to cooperative efforts. By comparing a basic contest model with the above-mentioned model, it has been shown that players may be better off in a contest which involves also some aggregate cooperative behaviour. In particular, as the asymmetry in the evaluation of the stake becomes larger and larger, even a smaller degree of responsiveness to the aggregate cooperative efforts would make players better off. Put differently, in a contest involving some cooperative behaviour, players are better off when their incentives differ widely enough. Eventually, a contest involving also cooperative efforts is less balanced than a pure contest. As noted above, this model of contest may be applied to a large number of situations in which players have mixed motives. On one hand, players have the main interest in winning the contest. In addition, players may have an interest in a contest less dissipative than the pure competitive one.

Appendix

To check whether the critical points (17), (18), (19) and (20) constitute a Nash Equilibrium I compute the Hessian matrices for both players. Consider first $\pi_1^{S*}(g_1^*, g_2^*, z_1^*, z_2^*)$ and eventually the Hessian matrix (omitting superscripts) is given by:

$$H_1 = \begin{pmatrix} \frac{\partial \pi_1}{\partial g_1 g_1} & \frac{\partial \pi_1}{\partial z_1 g_1} \\ \frac{\partial \pi_1}{\partial g_1 z_1} & \frac{\partial \pi_1}{\partial z_1 z_1} \end{pmatrix} =$$

$$\begin{pmatrix} -\frac{4x_1^{3/2}}{sx_2^{3/2} + x_1^{3/2}} - 2 & \frac{\sqrt{2}(q-1)\left(x_1^{3/2} - sx_2^{3/2}\right)}{\sqrt{s}x_1^{5/4}x_2^{3/4}} \\ \frac{\sqrt{2}(q-1)\left(x_1^{3/2} - sx_2^{3/2}\right)}{\sqrt{s}x_1^{5/4}x_2^{3/4}} & -\frac{2(q-1)^2\left(sx_2^{3/2} + x_1^{3/2}\right)}{sx_1 x_2^{3/2}} \end{pmatrix}$$

Let H_{1k} denote the k_{th} order leading principal submatrix of H_1 for $k = 1,2$. The determinant of the k_{th} order leading principal minor of H_{1k} is denoted by $|H_{1k}|$. The leading principal minors alternate signs as follows:

$$|H_{11}| < 0,\ |H_{12}| > 0 \Leftrightarrow s < \frac{5}{d^{3/2}} \qquad \text{(A.1)}$$

Then, consider $\pi_2^{s*}(g_1^*, g_2^*, z_1^*, z_2^*)$ and the Hessian matrix is given by:

$$H_2 = \begin{pmatrix} \frac{\partial \pi_2}{\partial g_2 g_2} & \frac{\partial \pi_2}{\partial z_2 g_2} \\ \frac{\partial \pi_2}{\partial g_2 z_2} & \frac{\partial \pi_2}{\partial z_2 z_2} \end{pmatrix} =$$

$$\begin{pmatrix} \frac{4x_1^{3/2}}{sx_2^{3/2} + x_1^{3/2}} - 6 & \frac{\sqrt{2}(q-1)\left(-x_1^{3/2} + sx_2^{3/2}\right)}{\sqrt{s}x_1^{3/4}x_2^{5/4}} \\ \frac{\sqrt{2}(q-1)\left(-x_1^{3/2} + sx_2^{3/2}\right)}{\sqrt{s}x_1^{3/4}x_2^{5/4}} & -\frac{2(q-1)^2\left(sx_2^{3/2} + x_1^{3/2}\right)}{sx_2 x_1^{3/2}} \end{pmatrix}$$

Also in this case, let H_{2k} denote the k_{th} order leading principal submatrix of H_1 for $k = 1,2$. The determinant of the kth order leading principal minor of H_2 is denoted by $|H_{1k}|$. The leading principal minors alternate in sign as follows:

$$|H_{21}| < 0,\ |H_{22}| > 0 \Leftrightarrow s > \frac{1}{5d^{3/2}} \qquad \text{(A.2)}$$

Since condition (A.2) is stricter than condition (A.1) to have an interior solution for equilibrium only condition (A.2) must hold.

References

Ansink E., Weikard H. (2009) “Contested water rights” *European Journal of Political Economy* 25, 247-260.

Baik K.H., Shogren J.F. (1995) “Contests with Spying” *European Journal of Political Economy* 11, 441-451.

Caruso R. (2008) “Reciprocity in the Shadow of Threat” *International Review of Economics* 55, 91-111.

Clark D.J., Riis C. (1998) “Contest Success Functions: an extension” *Economic Theory*, 11, 201-204.
Dixit, A. (1987) “Strategic Behavior in Contests” *The American Economic Review*, 77, 891-898.
Epstein, G.S., Hefeker, C. (2003) “Lobbying Contests with Alternative Instruments” *Economics of Governance*, 4, 81-89.
Haan, M.A. Schoonbeek, L. (2003) “Rent Seeking with Efforts and Bids” *Journal of Economics* 79, 215-235.
Konrad, K. (2009) *Strategy and Dynamics in Contests*, Oxford University Press: Oxford.
Konrad, K. (2000) “Sabotage in Rent-Seeking” *Journal of Law, Economics and Organization*, 16, 155-165.
Lee, S., Kang J. H. (1998) “Collective Contests with Externalities” *European Journal of Political Economy*, 14, 727-738
Rosen, S. (1986) “Prizes and Incentives in Elimination Tournaments” *The American Economic Review*76, 701-715.
Skaperdas, S. (1996) “Contest Success Functions” *Economic Theory*, 7, 283-290.
Smith, V. (2008) *Rationality in Economics: constructivist and ecological forms*, Cambridge University Press: Cambridge
Tullock, G. (1980) “Efficient Rent Seeking” in *Toward a Theory of the Rent-seeking Society*, by Buchanan, J. M., Tollison R., D., Tullock G., Eds., Texas A&M University, College Station, 97-112.