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Portfolio optimization using a parsimonious multivariate GARCH model: application to the Brazilian stock market

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Abstract

We apply a parsimonious multivariate GARCH speci cation based on the Fama-French-Carhart factor model to generate high-dimensional conditional covariance matrices and to obtain shortselling-constrained and unconstrained minimum variance portfolios. An application involving 61 stocks traded on the S~ao Paulo stock exchange (BM&FBovespa) shows that the proposed speci cation delivers less risky portfolios on an out-of-sample basis in comparison to several benchmark models, including existing factor approaches.

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1. Introduction

Obtaining covariance matrices for portfolios with a large number of assets remains a fundamental challenge in many areas of financial management, such as asset pricing, portfolio optimization and market risk management. Many of the the initial attempts to build models for conditional covariances, such as the VEC model of Bollerslev et al. (1988) and the BEKK model of Engle and Kroner (1995), among others, suffered from the so-called curse of dimensionality. In these specifications, the number of parameters increase very rapidly as the cross-section dimension grows, thus creating difficulties in the estimation process and entailing a large amount of estimation error in the resulting covariance matrices.

In this context, factor models emerge as promising alternatives to circumvent the problem of dimensionality and to alleviate the burden of the estimation process. The idea behind factor models is to assume that the co-movements of financial returns can depend on a small number of underlying variables, which are called factors. This dimensionality reduction allows for a great flexibility in the econometric specification and in the modeling strategy; see Santos and Moura (2011). In fact, alternative approaches for conditional covariance matrices based on factors models have been proposed in the literature. Generally, these models differ in their assumptions regarding the characteristics of the factors. For instance, Alexander and Chibumba (1996) and Alexander (2001) obtain common factors from statistical techniques such as principal components analysis (PCA) whereas Chan et al. (1999) use common factors extracted from asset returns. Engle et al. (1990), Alexander and Chibumba (1996), Alexander (2001), and Vrontos et al. (2003) assume that factors follow a GARCH process, whereas Aguilar and West (2000) and Han (2006) consider a stochastic volatility (SV) dynamics. Moreover, van der Weide (2002) extends previous studies by assuming that factors are not mutually orthogonal.

In this paper, we use a flexible approach to obtain conditional covariance matrices proposed by Santos and Moura (2011) to model a well known extension of the Fama and French's (1993) 3-factor model proposed by Carhart (1997). In particular, the model allows for a parsimonious multivariate specification for the covariances among factors based on a conditional correlation model and consider alternative univariate GARCH specifications to model the volatility of individual assets.

We apply the proposed model to obtain in-sample and out-of-sample one-step-ahead forecasts of the conditional covariance matrix of 61 assets traded in the São Paulo stock exchange BM&FBovespa during the sample period, and use the estimated matrices to compute short selling-constrained and unconstrained minimum variance portfolios. The performance of the proposed model is compared to that of alternative benchmark models, including existing factor approaches. The results indicate that the proposed model delivers less risky portfolios in comparison to the benchmarks.

The paper is organized as follows. In Section 2 we describe the model specification and give details regarding estimation and related models. Section 3 discusses an application in the context of portfolio optimization and proposes a methodology to perform out-of-sample evaluation. Finally, Section 4 brings concluding remarks.

2. The model

The extension of the Fama and French's (1993) 3-factor model proposed by Carhart (1997) is given by,

$$y_{it} = \alpha_{it} + \beta_{1i}(R_m - R_f) + \beta_{2i}SMB_t + \beta_{3i}HML_t + \beta_{4i}PR1YR_t + \varepsilon_{it} \tag{1}$$

where y_{it} is the return of asset i at time t, $R_m - R_f$ is the excess return of the value-weight return on 61 stocks traded in the BM&FBovespa minus the one-month CDI rate[1] and SMB, HML and $PR1YR$ are returns on value-weighted, zero-investment, factor-mimicking portfolios for size, book-to-market equity, and one-year momentum in stock returns, respectively. See Fama and French (1993) and Carhart (1997) for details regarding the construction of these factor-mimicking portfolios.

[1] CDI is the average rate that Brazilian banks charge when lending to other banks; see Buchholz et al. (2012) for details.

Here we assume that the factors and the $\varepsilon_{it} \sim N(0, h_{it})$ are heteroskedastic, so that the conditional covariance matrix, H_t, of the vector of returns in (1) is given by

$$H_t = \beta \Omega_t \beta' + \Xi_t \tag{2}$$

where Ω_t is a symmetric positive definite conditional covariance matrix of the factors, and Ξ_t is a diagonal covariance matrix of the residuals from the factor model in (1), i.e. $\Xi_t = diag(h_{1t}, \ldots, h_{Nt})$, where $diag$ is the operator that transforms the $N \times 1$ vector into a $N \times N$ diagonal matrix and h_{it} is the conditional variance of the residuals of the i-th asset. Note that the positivity of the covariance matrix H_t in (2) is guaranteed as the two terms in the right-hand side are positive definite.

To model Ω_t, the conditional covariance matrix of the factors in (2), alternative specifications can be considered, including multivariate GARCH models (see Bauwens et al., 2006; Silvennoinen and Teräsvirta, 2009, for comprehensive reviews) and stochastic volatility models Harvey et al. (1994); Aguilar and West (2000); Chib et al. (2009). In this paper, we consider the dynamic conditional correlation (DCC) model of Engle (2002)[2], which is given by:

$$\Omega_t = D_t R_t D_t \tag{3}$$

where $D_t = diag\left(\sqrt{h_{f_{1t}}}, \ldots, \sqrt{h_{f_{kt}}}\right)$, $h_{f_{kt}}$ is the conditional variance of the k-th factor, and R_t is a symmetric positive definite conditional correlation matrix with elements $\rho_{ij,t}$, where $\rho_{ii,t} = 1$, $i, j = 1, \ldots, K$. In the DCC model the conditional correlation $\rho_{ij,t}$ is given by

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}} \tag{4}$$

where $q_{ij,t}$, $i, j = 1, \ldots, K$, are collected into the $K \times K$ matrix Q_t, which is assumed to follow GARCH-type dynamics,

$$Q_t = (1 - \alpha - \beta)\, \bar{Q} + \alpha z_{t-1} z'_{t-1} + \beta Q_{t-1} \tag{5}$$

where $z_{f_t} = (z_{f_{1t}}, \ldots, z_{f_{kt}})$ with elements $z_{f_{it}} = f_{it}/\sqrt{h_{f_{it}}}$ being the standardized factor return, $\bar{Q}$ is the $K \times K$ unconditional covariance matrix of z_t and α and β are non-negative scalar parameters satisfying $\alpha + \beta < 1$.

We follow a similar modeling strategy of Cappiello et al. (2006) and consider alternative univariate GARCH-type specifications to model the conditional variance of the factors, $h_{f_{kt}}$, and the conditional variance of the residuals, h_{it}. In particular, we consider the GARCH model of Bollerslev (1986), the asymmetric GJR-GARCH model of Glosten et al. (1993), the exponential GARCH (EGARCH) model of Nelson (1991), the threshold GARCH (TGARCH) model of Zakoian (1994), the asymmetric power GARCH (APARCH) model of Ding et al. (1993), the asymmetric GARCH (AGARCH) of Engle (1990), and the nonlinear asymmetric GARCH (NAGARCH) of Engle and Ng (1993). In all models, we use their simplest forms where the conditional variance only depends on one lag of past returns and past conditional variances.

2.1. Estimation

The model is estimated in a multi-step procedure. First, the factor loadings in (1) are estimated via ordinary least squares (OLS). Second, the conditional covariance matrix of the factors in (3) is obtained by fitting a DCC model to the time series of factor returns assuming Gaussian innovations. The DCC parameters are estimated using the composite likelihood (CL) method proposed by Engle et al. (2008). Finally, we consider alternative univariate GARCH specifications to obtain conditional variances of the residuals of the factor model. For each residual series, we pick the specification that minimizes the Akaike Information Criterion (AIC).

[2]Recent applications of the DCC model in problems such as asset allocation, value-at-risk forecasting, and volatility transmission can be seen in Billio et al. (2006), Lee et al. (2006), and Dajcman et al. (2012).

2.2. Forecasting

One-step-ahead forecasts of the conditional covariance matrices based the model can be obtained as:

$$H_{t|t-1} = \beta\Omega_{t|t-1}\beta' + \Xi_{t|t-1}, \tag{6}$$

where β, $\Omega_{t|t-1}$, and $\Xi_{t|t-1}$ are, respectively, the vector of coefficients for each factor, one-step-ahead forecasts of the conditional covariance matrix of the factors computed according to (3), and one-step-ahead forecasts of the conditional residual variances computed according to a GARCH-type model and collected into the diagonal matrix $\Xi_{t|t-1}$.

2.3. Benchmark models

We consider two well known benchmark specifications for the conditional covariance matrix of returns. The first benchmark model is the orthogonal GARCH (OGARCH) model of Alexander and Chibumba (1996) and Alexander (2001),

$$H_t = W\Lambda_t W', \tag{7}$$

where W is a $N \times K$ matrix of eigenvectors of the first $K \leq N$ orthogonal factors obtained via principal components analysis (PCA) and Λ_t is a diagonal covariance matrix of the conditional variances of the principal components, i.e. $\Lambda_t = diag\left(h_{PC_{1t}}, \ldots, h_{PC_{kt}}\right)$ where h_{PC_t} follows a GARCH model.

The second benchmark model is the Risk Metrics model, which consists of an exponentially-weighted moving average scheme to model conditional covariances. In this approach, the conditional covariance matrix is given by

$$H_t = (1-\lambda)Y_{t-1}Y'_{t-1} + \lambda H_{t-1}, \tag{8}$$

with the recommended value for the model parameter for daily returns being $\lambda = 0.94$.

3. Application to portfolio optimization

To evaluate the performance of the proposed model in comparison to benchmark models we consider the minimum variance portfolio (MVP) problem. We examine the properties of MVP under two alternative weighting schemes: unconstrained; and short-sales constrained. In the unconstrained case, the MVP can be formulated as:

$$\begin{aligned}&\min_{w_t} w_t H_{t|t-1} w_t\\&\text{subject to}\\&w_t'\iota = 1\end{aligned} \tag{9}$$

where ι is a $N \times 1$ vector of ones. The solution to the unconstrained MVP problem in (9) is given by:

$$w_t = \frac{H^{-1}_{t|t-1}\iota}{\iota' H^{-1}_{t|t-1}\iota}. \tag{10}$$

In the short-sales constrained case, we add in (9) a restriction to avoid negative weights, i.e. $w_t \geq 0$. Previous research has shown that imposing such constraints may substantially improve performance, mostly by reducing portfolio turnover, see Jagannathan and Ma (2003), among others. In the constrained case, optimal MVP weights are obtained using numerical methods.

3.1. Data and implementation details

To evaluate the performance of the model vs. benchmark models, we use a data set composed of daily observations of 61 stocks that were traded in BM&FBovespa from January 4, 2000 until December 31, 2010, summing up to 2766 observations. Returns are computed as the differences in log prices.

The first 1722 observations are used to estimate the parameters of all models and to obtain in-sample forecasts, whereas the last 1000 observations are used to obtain out-of-sample forecasts. These forecasts are nonadaptative, i.e. the parameters estimated in the in-sample period were kept fixed in the out-of-sample period.

It is worth pointing out two technical details regarding the implementation of the benchmark models. First, the Risk Metrics approach does not involve any unknown coefficients as we set $\lambda = 0.94$. Second, when implementing the OGARCH model we consider alternative number of factors. In particular, we vary the number of factors from 1 up to N. To facilitate the exposition and discussion of results, we report the results for the OGARCH only for the best performing specification.

3.2. Methodology for evaluating performance

We examine the portfolios' performance in terms of the variance of returns ($\hat{\sigma}^2$), Sharpe ratio (SR) and turnover. These statistics are computed as follows:

$$\hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T-1}(w_t' R_{t+1} - \hat{\mu})^2$$

$$SR = \frac{\hat{\mu}}{\hat{\sigma}}, \text{ where } \hat{\mu} = \frac{1}{T-1}\sum_{t=1}^{T-1} w_t' R_{t+1}$$

$$\text{Turnover} = \frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{j=1}^{N}\left(\left|w_{j,t+1} - w_{j,t^+}\right|\right),$$

where w_{j,t^+} is the portfolio weight in asset j at time $t+1$ but *before* re-balancing and $w_{j,t+1}$ is the desired portfolio weight in asset j at time $t+1$. As pointed out by DeMiguel et al. (2009b), turnover as defined above can be interpreted as the average fraction of wealth traded in each period.

To test the statistical significance of the difference between the variances and Sharpe ratios of the returns for two given portfolios, we follow DeMiguel et al. (2009a) and use the stationary bootstrap of Politis and Romano (1994) with B=1,000 bootstrap resamples and expected block length b=5.[3] The resulting bootstrap p-values are obtained using the methodology suggested in Remark 3.2 of Ledoit and Wolf (2008).

3.3. Results

Table 1 reports the in-sample and out-of-sample portfolio variance, the Sharpe ratio and the portfolio turnover of the short-sales constrained and unconstrained minimum variance portfolio policies obtained with covariance matrices generated by the flexible multivariate factor GARCH model (hereafter FlexFGARCH) proposed by Santos and Moura (2011), and by the benchmark models. Stars in the tables indicate that the portfolio variance and the Sharpe ratio are statistically different with respect to those obtained with the Risk Metrics model at a confidence level of 10%.

The results in terms of portfolio variance indicate that the FlexFGARCH model delivers statistically lower portfolio variance in comparison to the benchmarks, for both in- and out-of-sample periods, and for both constrained and unconstrained policies. For instance, in the out-of-sample period the FlexFGARCH model achieves a portfolio variance of 0.836 for the unconstrained policy, which is substantially (and statistically) lower than the portfolio variance delivered by the OGARCH and the Risk Metrics models (2.849 and 1.560, respectively). The only exception to these results is the out-of-sample performance of the constrained MVP, in which the FlexFGARCH model delivers a statistically similar portfolio variance in comparison to the benchmark.

The results for the Sharpe ratios indicate that the FlexFGARCH model deliver a better risk-adjusted performance in comparison to the benchmark models. For instance, the FlexFGARCH model delivered a SR of 0.07 for the unconstrained policy during the out-of-sample period, whereas the Risk Metrics model delivered a SR of 0.03. However, the differences in SR are not statistically significant. In terms of portfolio turnover,

[3] We performed extensive robustness checks regarding the choice of the block length, using a range of values for b between 5 and 250. Regardless of the block length, the test results for the variances and Sharpe ratios are similar to those reported here.

we observe that the FlexFGARCH specification yields MVP with lower turnover in several cases, specially for the unconstrained policy. Finally, we observe that the portfolio turnover associated to the constrained policies tends to be lower in comparison to that of unconstrained policies. This result is in line with previous empirical literature such as DeMiguel et al. (2009a).

Alternative re-balancing frequencies

The results discussed above are based on the assumption that investors adjust their portfolio on a daily basis. The transaction costs incurred with such frequent trading can possibly deteriorate the net portfolio performance. Obviously this effect can be reduced by adjusting the portfolio less frequently, such as on a weekly or monthly basis, which in fact is done in practice by many institutional investors. A drawback of re-balancing the portfolio less frequently is that the portfolio weights become outdated, which may harm its performance. We examine the performance of the MVP under alternative re-balancing frequencies. Tables 2 and 3 show the results for weekly and monthly re-balancing frequencies, respectively. As expected, we find that lowering the re-balancing frequency results in a substantial reduction in portfolio turnover. We observe that the FlexFGARCH model delivers the lowest turnover and the lowest portfolio variance in all cases, except in the constrained out-of-sample portfolio with weekly re-balancing. Summarizing the results in Tables 1 to 3, we find that lowering the re-balancing frequency leads to similar risk-adjusted performance in terms of SR and substantially lower portfolio turnover.

4. Concluding remarks

Factor models are currently established as an alternative to alleviate the problem of dimensionality and the burden of the estimation process when modeling covariance matrices of portfolios containing a large number of assets. In this paper, we use the well known extension of the Fama and French's (1993) 3-factor model proposed by Carhart (1997) to parsimoniously model the conditional covariance matrix of 61 stocks traded at BM&FBovespa. Our approach achieves great flexibility by allowing a parsimonious specification for the common factors and alternative specifications the individual assets in the portfolio.

We apply the proposed model to obtain in-sample and out-of-sample one-step-ahead forecasts of the conditional covariance matrix, and use the estimated matrices to compute short selling-constrained and unconstrained minimum variance portfolios. The performance of the proposed model is compared to that of alternative benchmark models, including existing factor approaches. The results indicate that the proposed model delivers less risky portfolios in comparison to the benchmark models. Moreover, we show that the results are robust to the portfolio re-balancing frequency.

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Table 1: Minimum variance portfolio performance

The Table reports the average daily portfolio variance, the Sharpe ratio and the portfolio turnover of the short-sales constrained and unconstrained minimum variance portfolio policies obtained with covariance matrices generated by the FlexFGARCH model, the OGARCH model, and the Risk Metrics model. The asterisks indicate that the portfolio variance and the Sharpe ratio are statistically different with respect to those obtained with the Risk Metrics model at a confidence level of 10%.

	Variance	Sharpe ratio	Turnover	Variance	Sharpe ratio	Turnover
In-sample	Constrained			Unconstrained		
FlexFGARCH	0.642^{*}	0.097	0.093	0.586^{*}	0.104	0.159
OGARCH	1.910^{*}	0.068	0.032	2.116^{*}	0.067	0.070
Risk Metrics	0.958	0.074	0.173	1.858	0.074	0.929
Out-of-sample	Constrained			Unconstrained		
FlexFGARCH	1.051	0.056	0.108	0.836^{*}	0.070	0.179
OGARCH	2.533^{*}	0.027	0.028	2.849^{*}	0.024	0.179
Risk Metrics	0.806	0.053	0.028	1.560	0.031	0.938

Table 2: Minimum variance portfolio performance with weekly re-balancing

The Table reports the average daily portfolio variance, the Sharpe ratio and the portfolio turnover of the short-sales constrained and unconstrained minimum variance portfolio policies obtained with covariance matrices generated by the FlexFGARCH model, the OGARCH model, and the Risk Metrics model. The asterisks indicate that the portfolio variance and the Sharpe ratio are statistically different with respect to those obtained with the Risk Metrics model at a confidence level of 10%.

	Variance	Sharpe ratio	Turnover	Variance	Sharpe ratio	Turnover
In-sample	Constrained			Unconstrained		
FlexFGARCH	0.664^{*}	0.106^{*}	0.043	0.604^{*}	0.113^{*}	0.072
OGARCH	1.910^{*}	0.068	0.014	2.117	0.067	0.016
Risk Metrics	1.075	0.069	0.091	1.973	0.066	0.475
Out-of-sample	Constrained			Unconstrained		
FlexFGARCH	1.056	0.058	0.050	0.865^{*}	0.072	0.081
OGARCH	2.553^{*}	0.026	0.013	2.854^{*}	0.022	0.031
Risk Metrics	0.888	0.041	0.093	1.583	0.019	0.486

Table 3: Minimum variance portfolio performance with monthly re-balancing

The Table reports the average daily portfolio variance, the Sharpe ratio and the portfolio turnover of the short-sales constrained and unconstrained minimum variance portfolio policies obtained with covariance matrices generated by the FlexFGARCH model, the OGARCH model, and the Risk Metrics model. The asterisks indicate that the portfolio variance and the Sharpe ratio are statistically different with respect to those obtained with the Risk Metrics model at a confidence level of 10%.

	Variance	Sharpe ratio	Turnover	Variance	Sharpe ratio	Turnover
In-sample	Constrained			Unconstrained		
FlexFGARCH	0.688^{*}	0.108	0.017	0.633^{*}	0.115	0.028
OGARCH	1.912^{*}	0.068	0.006	2.125^{*}	0.065	0.014
Risk Metrics	0.902	0.077	0.041	1.796	0.092	0.209
Out-of-sample	Constrained			Unconstrained		
FlexFGARCH	1.050	0.064	0.021	0.892^{*}	0.077	0.032
OGARCH	2.552^{*}	0.026	0.006	2.854^{*}	0.023	0.014
Risk Metrics	1.310	0.037	0.047	1.997	0.032	0.224