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A note on the equivalence of the Blanchard and Quah (1989) and Sims (1980) identification procedures

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Abstract

We show that the two-variable VAR model of Blanchard and Quah (1989) produces results identical to those based on the Sims (1980) orthogonalization if the first-ordered variable is not caused in the long run by the second-ordered variable. Some illustrative examples are provided to demonstrate the usefulness of this result.

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1. Introduction

In an influential study, Blanchard and Quah (BQ, 1989) developed a vector autoregression (VAR) model that identifies the effects of aggregate supply (AS) and aggregate demand (AD) shocks on real output and the unemployment rate. Since then, numerous applications and extensions have followed. The BQ model employs as an identifying assumption the long-run output neutrality condition in which an AD shock has no long-run effects on real output. This assumption represents an important point of departure from the Sims (1980) orthogonalization, which places a recursive structure on the contemporaneous relationships among the variables in the model.

Nevertheless, we show that the BQ model produces results identical to those based on the Sims orthogonalization if the first-ordered variable (real output) is not caused in the long run by the second-ordered variable (unemployment rate). This proposition can be applied to any two-variable model irrespective of whether the variables are $I(0)$ or $I(1)$. For example, while the unemployment rate is assumed to be $I(0)$ in the BQ model, the second-ordered variable can also be an $I(1)$ process, as in a model of real output and inflation (Quah and Vahey, 1995; Cecchetti and Rich, 2001; Cover *et al*. 2006). Section 2 formally derives the conditions under which BQ and Sims identification procedures are equivalent in the BQ model of output and the unemployment rate. Some empirical examples are offered in Section 3. Section 4 concludes the paper.

2. Equivalence between BQ and Sims identification procedures

In the context of BQ, consider a reduced-form VAR model, given as

$$A(L)z_t = e_t \tag{1}$$

where $z_t = (\Delta y_t, u_t)'$, y_t is real output, u_t is the unemployment rate, $\Delta \equiv (1-L)$ is the first difference operator, L is the lag operator, $A(L) = I - \sum_{i=1}^{p} A_i L^i$, $e_t = (e_{1t}, e_{2t})'$ is a vector of reduced-form shocks and is *iid* with a mean of zero and a covariance matrix of $E(e_t e_t') = \Sigma = [(\sigma_{11}, \sigma_{12})' \ (\sigma_{12}, \sigma_{22})']$. The constant term is suppressed for the sake of illustration. The vector moving average (VMA) representation for Equation (1) can be expressed as

$$z_t = C(L)e_t \tag{2}$$

where $C(L) = C_0 + C_1 L + C_2 L^2 + \cdots$, $C_0 = I$, and $C(1) = \sum_{i=0}^{\infty} C_i = [(c_{11}, c_{21})' \ (c_{12}, c_{22})']$.

Subject to identification, a structural VMA representation corresponding to Equation (2) is given as

$$z_t = \Gamma(L)\varepsilon_t \tag{3}$$

where $\Gamma(L) = \Gamma_0 + \Gamma_1 L + \Gamma_2 L^2 + \cdots$, $\Gamma(1) = \sum_{i=0}^{\infty} \Gamma_i = [(\gamma_{11}, \gamma_{21})' \ (\gamma_{12}, \gamma_{22})']$, and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is a vector of structural shocks governing the economy. Following BQ, ε_{1t} and ε_{2t} are denoted as an aggregate supply (AS) shock and an aggregate demand (AD) shock, respectively. These are assumed to have a mean of zero and a covariance matrix of $E(\varepsilon_t \varepsilon_t') = \Omega = [(1, \omega_{12})' \ (\omega_{12}, 1)']$, where each structural shock is normalized to have unit variance without a loss of generality. From Equations (2) and (3), the relationships between the reduced-form and structural parameters are

$$\Gamma(L) = C(L)\Gamma_0 \quad (4)$$

and

$$\Gamma_0\varepsilon_t = e_t . \quad (5)$$

For the exact identification of the structural shocks, the four parameters in Γ_0 must be uniquely determined. BQ utilize two identifying assumptions: uncorrelatedness between ε_{1t} and ε_{2t} and the long-run output neutrality condition. The first assumption leads to $\omega_{12} = 0$, and $\tilde{\Gamma}_0\tilde{\Gamma}_0^{'} = \Sigma$ in Equation (5) then yields three restrictions on $\tilde{\Gamma}_0$, where '$\tilde{}$' denotes the BQ procedure. The second assumption implies that the element (1,2) of the long-run impact matrix $\tilde{\Gamma}(1)$ in Equation (4) is equal to zero. This provides the final identifying restriction on $\tilde{\Gamma}_0$ by setting the (1,2) element of $C(1)\tilde{\Gamma}_0$ to zero. Accordingly, solving $\tilde{\Gamma}_0\tilde{\Gamma}_0^{'} = \Sigma$ under the long-run neutrality condition gives the four parameters in $\tilde{\Gamma}_0$. These are given as

$$\tilde{\Gamma}_0 = \begin{bmatrix} (\theta/\lambda)(c_{11}\sigma_{11} + c_{12}\sigma_{12})/(c_{11}\sigma_{12} + c_{12}\sigma_{22}) & -(1/\lambda)(c_{12}\sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}) \\ \theta/\lambda & (1/\lambda)(c_{11}\sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}) \end{bmatrix} \quad (6)$$

where $\theta = (c_{11}\sigma_{12} + c_{12}\sigma_{22})$ and $\lambda = \sqrt{c_{11}^2\sigma_{11} + 2c_{11}c_{12}\sigma_{12} + c_{12}^2\sigma_{22}}$.

Once $\tilde{\Gamma}_0$ is estimated, the impulse responses and the forecast-error variances of the series can be computed from Equation (4).

Sims orthogonalization applies the Choleski decomposition to the covariance matrix of reduced-form errors, i.e. $\hat{\Gamma}_0\hat{\Gamma}_0^{'} = \Sigma$, where '$\hat{}$' denotes the Sims procedure. This is equivalent to imposing the uncorrelatedness condition between ε_{1t} and ε_{2t} and the restriction that ε_{2t} has no contemporaneous impact on real output. The parameters in $\hat{\Gamma}_0$ are obtained as follows:

$$\hat{\Gamma}_0 = \begin{bmatrix} \sqrt{\sigma_{11}} & 0 \\ \sigma_{12}/\sqrt{\sigma_{11}} & \sqrt{(\sigma_{11}\sigma_{22} - \sigma_{12}^2)/\sigma_{11}} \end{bmatrix} \quad (7)$$

The following proposition can now be established:

Proposition. The BQ and Sims identification schemes are equivalent if the first-ordered variable is not caused in the long run by the second-ordered variable.

Proof. The first-ordered variable is not caused in the long run by the second ordered variable if $A_{12}(1) = 0$, where $A_{12}(1)$ is the (1,2) element of $A(1) = I - \sum_{i=1}^{p} A_i$ in Equation (1). This can be assessed statistically in the usual manner (e.g., by Granger causality tests). Because $A(1)^{-1} = C(1)$, it is also true that $C_{12}(1) = c_{12} = 0$ in Equation (2), giving C(1) a lower triangular matrix. Substituting $c_{12} = 0$ into Equation (6) results in $\tilde{\Gamma}_0$ becoming identical to $\hat{\Gamma}_0$ in Equation (7). This leads to $\tilde{\Gamma}(L) = \hat{\Gamma}(L)$ in Equation (4) and hence, the responses are all identical between the BQ and Sims procedures. For example, the long-run impact matrix of structural shocks is equally given as

$$\tilde{\Gamma}(1) = C(1)\tilde{\Gamma}_0 = \begin{bmatrix} c_{11}\sqrt{\sigma_{11}} & 0 \\ (c_{21}\sigma_{11} + c_{22}\sigma_{12})/\sqrt{\sigma_{11}} & c_{22}\sqrt{(\sigma_{11}\sigma_{22} - \sigma_{12}^2)/\sigma_{11}} \end{bmatrix} = C(1)\hat{\Gamma}_0 = \hat{\Gamma}(1).$$

Two remarks merit attention. First, it is obvious that the proposition holds for any two-variable model irrespective of whether the variables are *I*(0) or *I*(1). For example, the second-ordered variable can be an *I*(1) process, unlike the BQ model. Models of two *I*(1) variables are also permissible. An exception is when these variables are cointegrated. In this case, Ribba (1997) and Fisher and Huh (1999) showed that if the first-ordered variable is weakly exogenous to the cointegrating relationship, the BQ and Sims identification schemes are equivalent. An interesting question is if their procedures may be applied to the BQ model above because the stationary unemployment rate itself can be regarded as the cointegrating relationship. If one proceeds as in the cointegrated case, the model will involve Δu_t rather than u_t. As Levtchenkova *et al*. (1996) point out, this has the effect of preserving only the information contained in $\Gamma_{11}(1) = \gamma_{11}$ (see Equation (3)) as $\Delta u_t = (1-L)\Gamma_{21}(L)\varepsilon_{1t}$ and, when L=1, any information about $\Gamma_{21}(1) = \gamma_{21}$ would disappear from such a model. Consequently, the BQ and Sims identification schemes do not necessarily produce identical results. Second, the proposition may also be applied to models consisting of more than two variables. The BQ and Sims identification schemes are equivalent if C(1) is lower triangular; that is, the first-ordered variable is not caused in the long run by any of the variables in the model, the second-ordered variable is not caused in the long run by any of the variables saving the first-ordered one, and so on.

3. Empirical illustration

This section provides two empirical examples. One is the BQ VAR model of US output and the unemployment rate. The other is the two-variable VAR model in Cover *et al*. (2006), which consists of US output and inflation. In this model, both variables were found to be *I*(1) processes with no cointegration, and the two structural shocks were identified using the BQ procedure. Definitions of the data series and their sources are as follows. The measure of output (y_t) is the log of real GDP. The unemployment rate (u_t) is for all civilians 16 years of age and older. The rate of inflation (π_t) is measured as the quarterly percentage change in the GDP deflator. All data were taken from FRED at the Federal Reserve Bank of St. Louis. The sample period is 1950:Q1 to 2011:Q1. Following the original contributions, the chosen lag lengths are p=8 for the former and p=10 for the latter. For the BQ model, a dummy is included to take into account a trend break in 1974 with a value of 1 up to 1973 and 0 afterwards. Data on output and inflation are differenced once to ensure stationarity.

Table I reports the testing results. For the BQ model, the t-test statistic strongly rejects the null hypothesis that the coefficients of the unemployment rate in the output equation are summed to zero. Because output can be caused by the unemployment rate in the long run, the BQ and Sims identification procedures would not produce the same results. As a robustness check, Figure 1 (upper panel) shows the responses of the variables to the structural shocks identified using the BQ and Sims orthogonalization, respectively. It is easy to see that the responses are indeed different depending on which scheme is used. Moving to the model of Cover *et al*., the null hypothesis that the coefficients of the inflation rate in the output equation are summed to zero cannot be rejected at conventional significant levels. Output is not caused in the long-run by inflation and hence, the BQ and Sims identification schemes produce identical results. The lower panel in Figure 1 confirms that the responses are indistinguishable between the two schemes across all occasions.

Table I. Results of causality tests

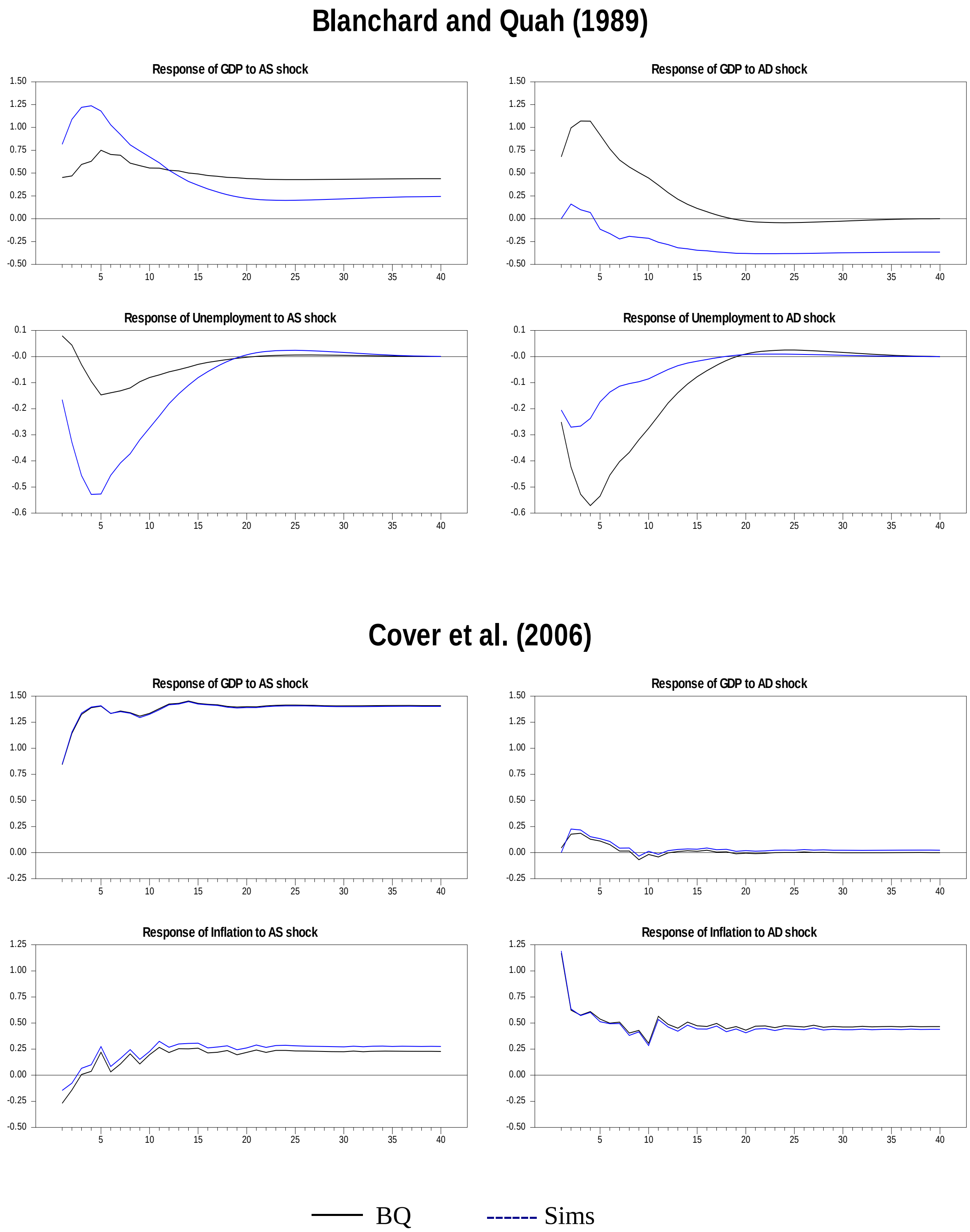
Blanchard and Quah (1989)		Cover *et al*. (2006)	
$A_{12}(1)$	t-test	$A_{12}(1)$	t-test
0.144 (0.05)	0.00	– 0.190 (0.25)	0.46

The first column reports the sum of the coefficients of the unemployment rate in the output equation for the BQ model. The figures in parentheses are the standard errors. The second column reports the marginal significance level (*p*-value) of the t-test statistic for the null hypothesis in which the sum of the coefficients is equal to zero (i.e., $A_{12}(1) = 0$). The third and fourth columns do the same for the sum of the coefficients of inflation in the output equation from the Cover *et al*. model.

4. Concluding remarks

In identifying aggregate supply and aggregate demand shocks, Blanchard and Quah (BQ, 1989) adopted a long-run restriction stating that aggregate demand shocks do not have a long-run effect on real output. This approach differs from the traditional method of Sims (1980), which assumes a contemporaneously recursive ordering of the variables. Yet, the present paper shows that if the first-ordered variable in the model is not caused in the long run by the second-ordered variable, the BQ and Sims identification schemes produce identical results. This proposition is applied to two empirical examples. In the model of output and the unemployment rate, the BQ and Sims identification schemes do not produce identical results, whereas they do in the model of output and inflation.

Figure 1. Impulse responses



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